Nucleon-pion-state contributions to nucleon correlation functions

Oliver Bär
Humboldt Universität zu Berlin

Outline

- Introduction
- ...
- ...

Joint Lattice Seminar HU Berlin - NIC DESY
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Propaganda concerning present day lattice simulations

- Progress over the last years:
  - Dynamical simulations with 3 or 4 flavors
  - Small lattice spacings $a$ in the range $0.05 \ldots 0.1$ fm
    often $O(a)$ improved actions and operators are used ...
  - Sufficiently large volumes with $M_\pi L \gtrsim 4$ to have FV effects under control
  - Small pion masses, sometimes even physical pion masses
  ➡ Systematic uncertainties are much better controlled nowadays
    particularly concerning the chiral extrapolation

- But: Some problems get more severe for small pion masses
  - Presence of multi-particle states in correlation functions
  - Signal-to-noise problem
Example: Nucleon 2pt-function

- Consider
  \[ C_2(t) = \int_{L^3} d^3x \langle N(\vec{x}, t) \overline{N}(0, 0) \rangle \]

- \( N \): Interpolating field, quantum numbers of the nucleon
- Projection to zero momentum
- Finite spatial volume

- Spectral decomposition (discrete spectrum)
  \[ C_2(t) = b_0 e^{-M_N t} + b_1 e^{-E_1 t} + b_2 e^{-E_2 t} \ldots \]

- For small pion masses and large volumes (i.e. small momenta)
  \[ E_1 \approx E_N + E_\pi < E_2 \approx M_N + 2M_\pi < E_3 \approx M_N^* \]

- Spectroscopy calculations get more complicated
  Variational method with multi-particle states ...
In practice: Use the effective mass

$$M_{\text{eff}}(t) = M_N + \frac{b_1}{b_0} \Delta E_1 e^{-\Delta E_1 t} + \frac{b_2}{b_0} \Delta E_2 e^{-\Delta E_2 t} + \ldots$$

Deviations from a constant due to excited states

$a \approx 0.086 \text{ fm}$

$M_\pi \approx 190 \text{ MeV}$

S. Dürr et al. 2009

fitting range $t \approx 0.6 \ldots 1.02 \text{ fm}$

$$\Delta E_i = E_i - M_N$$
Signal-to-noise problem prevents large $t$

Expectation for nucleons: $\frac{\text{signal}}{\text{noise}} \sim e^{- (M_N - \frac{3}{2} M_\pi) t}$  

Need to determine plateau for early times where excited states may contribute
Consequence

- The smaller the pion mass
  - the smaller the (useful) euclidean time extent
  - the larger the excited state contribution in correlation functions
  - analyze data using the *ansatz*

\[ M_{\text{eff}}(t) = M_N + \frac{b_1}{b_0} \Delta E_1 e^{-\Delta E_1 t} + \frac{b_2}{b_0} \Delta E_2 e^{-\Delta E_2 t} + \ldots \]

- But: Fits are difficult with \( b_k/b_0 \) and \( \Delta E_k \) as fit parameters

- Often used in practice: Theoretical expectations, e.g. \( \Delta E_1 = 2M_\pi \)
Constrained fits

Quote: ods to extract estimates for the mass of the ground state: First, a naive plateau fit is performed for each correlator, starting at a reasonably large Euclidean time; secondly, taking the plateau value as the starting point, a two-state fit including the leading excited state contribution is performed with a larger time range. Finally, another two-state fit with the same time range is performed for the baryonic channel, in which the gap between the ground and excited state is fixed to the theoretically expected gap of $2m_\pi$ using the measured pion mass. Examples of effective mass plots are shown
Topic of this talk

\[ M_{\text{eff}}(t) = M_N + \frac{b_1}{b_0} \Delta E_1 e^{-\Delta E_1 t} + \frac{b_2}{b_0} \Delta E_2 e^{-\Delta E_2 t} + \ldots \]

- Here: Computation of \( b_1/b_0 \) in ChPT for \( N\pi\)-states*

- useful for analyzing lattice data (constrained fit)?

- \textit{a posteriori} check of fit results and the theoretical expectation

- Important: LO ChPT makes a \textit{definite prediction} for \( b_1/b_0 \), it does not depend on LECs associated with interpolating fields

- Details and full results in arXiv:1503.03649

* \( N\pi\pi \)-state contribution not done!
**Excited-state contribution to $g_A$**

- Consider
  
  $$C_3(t, t') = \int d^3 x \int d^3 y \Gamma'_{k, \alpha \beta} \langle N_\beta(\vec{x}, t) A_k^3(\vec{y}, t') \bar{N}_\alpha(0, 0) \rangle$$

- Form ratio with 2pt-function
  
  $$R = \frac{C_3(t, t')}{{C_2(t)}}$$

  and consider large times
  
  $$R \xrightarrow{t \gg t' \gg 0} g_A + \tilde{b}_1 e^{-\Delta E_1(t-t')} + \tilde{b}_1' e^{-\Delta E_1 t'} + \tilde{c}_1 e^{-\Delta E_1 t} + \ldots$$

- Smaller time separations $t-t'$ and $t'$ present
  
  - less exponential suppression
  - expect larger excited-state contribution than in 2pt-function (?)
Again: constrained fits

Fitting strategies to extract the axial charge of the nucleon from lattice QCD

Georg M. von Hippel¹, Jiayu Hua¹, Benjamin Jäger¹,², Harvey B. Meyer¹,², Thomas D. Rae¹, Hartmut Wittig¹,²

Quote:

\[ R(t, t_s) = g_A^{\text{bare}} + c_1 e^{-\Delta t} + c_2 e^{-\Delta(t_s-t)} + c_3 e^{-\Delta t_s} + \ldots, \quad (t_s, t \to \infty) \quad (1.5) \]

where the energy gap, \( \Delta \), is the energy difference between the ground and the first excited state carrying the same quantum numbers as the nucleon at rest. In principle, the energy gap can be extracted from the two-point correlation function; however, we have found the extraction to be difficult with the data available to us. In some of the fits described below, we therefore choose the approximation \( \Delta = 2m_\pi \).
Here: Computation of the $\tilde{b}_1$, $\tilde{b}_1'$, $\tilde{c}_1$ due to $N\pi$-states within ChPT

But: Work in progress $\Rightarrow$ partial and preliminary results only

One more comment:

Signs of coefficients determine whether $R > g_A$ or $R < g_A$ for finite $t, t'$

Lattice calculations typically underestimate $g_A$
Lattice results for $g_A$

Axial charge versus pion mass, computed using $N_f = 2$ and $N_f = 2 + 1 + 1$ twisted mass fermions [9]; a mixed action with clover-improved Wilson valence quarks and $N_f = 2 + 1$ HISQ staggered sea quarks [19]; $N_f = 2 + 1$ domain wall fermions [24]; $N_f = 2 + 1$ clover-improved Wilson fermions, without [13] and with [16] smearing; and $N_f = 2$ clover-improved Wilson fermions [26–28]. Note that some of the same ensembles were used by RQCD and QCDSF, so that their errors may be correlated.

BENCHMARK OBSERVABLES

Given that full control over all systematics is still a work in progress for nucleon structure calculations, we rely on comparisons with experiment to help judge the quality of our calculations. Understanding what is required to obtain agreement with experiment for these “benchmark” observables, such as the axial charge and electromagnetic form factors, is essential for judging the quality of calculations of other observables.

Axial charge

The nucleon axial charge $g_A$ is defined via a neutron-to-proton transition matrix element,

$$\langle p | \bar{u} g_5 \gamma_5 u | n \rangle_i = g_A \langle p | \bar{u} \gamma_5 u | n \rangle_i,$$

and has long served as a benchmark for lattice calculations. It is a relatively simple quantity to compute, being a forward matrix element and an isovector quantity that doesn't require disconnected diagrams. Experimentally, it is well known from beta decay of polarized neutrons; the latest PDG value is $g_A = 1.2723^{+0.023}$. Obtaining agreement with the experimental value has proven difficult for lattice calculations. Those that include pion masses below 300 MeV are shown in Fig. 1. Note that, unless otherwise stated, the plotted data show the “raw” values from each lattice ensemble, renormalized but without any extrapolations to zero lattice spacing, infinite volume, or physical pion mass. In general, the lattice data tend to lie below the experimental value and show no strong dependence on the pion mass. In the past, the added uncertainty due to extrapolations to the physical pion mass meant that data that were below experiment could still be reconciled with it [29], but this becomes more difficult as calculations with near-physical pion masses become available.

The possibility of large excited-state contaminations affecting lattice calculations of $g_A$ has seen several studies in recent years. The Mainz group obtained agreement of their extrapolated value with experiment, when using the summation method to remove contributions from excited states, whereas the ratio method with a source-sink separation $T_\pi > 1.1$ fm produced a value below experiment [30]. Using similar methods, LHPC reported similar results for pion masses $m_\pi > 250$ MeV, but found that closer to the physical pion mass, removing excited states yielded...
Lattice results for $g_A$

Lattice calculations typically underestimate $g_A$.

**Figure 1.** Axial charge versus pion mass, computed using $N_f = 2$ and $N_f = 2 + 1 + 1$ twisted mass fermions [9]; a mixed action with clover-improved Wilson valence quarks and $N_f = 2 + 1$ HISQ staggered sea quarks [19]; $N_f = 2 + 1$ domain wall fermions [24]; $N_f = 2 + 1$ clover-improved Wilson fermions, without [13] and with [16] smearing; and $N_f = 2$ clover-improved Wilson fermions [26–28]. Note that some of the same ensembles were used by RQCD and QCDSF, so that their errors may be correlated.

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Outline of the rest of the talk

- Introduction ✓
- Setup: Covariant Baryon ChPT and the interpolating nucleon fields
- $N\pi$-state contribution to the 2pt-function and eff. mass
- $N\pi$-state contribution to $g_A$
- Conclusions
Setup: Baryon ChPT

Framework: Covariant Baryon ChPT (LO, Euclidean space time)
Gasser et al 1988, Becher, Leutwyler 1998, ...

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{N\pi}^{(1)} + \mathcal{L}_{\pi\pi}^{(2)} \]

\[ \mathcal{L}_{\pi\pi}^{(2)} = \frac{f^2}{4} \text{Tr}[\partial_{\mu} U \partial_{\mu} U^\dagger] + \frac{f^2 B}{2} \text{Tr}[\mathcal{M}(U + U^\dagger)] \]

\[ \mathcal{L}_{N\pi}^{(1)} = \overline{\Psi} \left( iD + M_N - i \frac{g_A}{2} \gamma_5 \right) \Psi \]

Leading interaction vertex

\[ \mathcal{L}_{\text{int}, \text{LO}} = \frac{ig_A}{2f} \overline{\Psi} \gamma_{\mu} \gamma_5 \sigma^a \Psi \partial_{\mu} \pi^a \Rightarrow \overline{\Psi} \Psi \pi \text{-Vertex} \]

\[ U = \exp \left[ \frac{i}{f} \pi^a \sigma^a \right] \]

\[ \Psi = \left( \begin{array}{c} p \\ n \end{array} \right) \]

\[ u = \sqrt{U} \]

\[ \phi = i[u^\dagger \phi u - u \phi u^\dagger] \]
Setup: Baryon ChPT

- Consider QCD/ChPT
  - in a finite box with spatial extent $L$
  - infinite time extent $T$ (for simplicity)

- Discrete spatial momenta, e.g. for periodic bc: $\vec{p}_n = \frac{2\pi}{L} \vec{n}$, $\vec{n} \in \mathbb{N}^3$

- Finite-volume propagators

\[
G^{ab}(x, y) = \delta^{ab} \frac{1}{L^3} \sum_{\vec{p}_n} \frac{1}{2E_\pi} e^{i\vec{p}(x-y)} e^{-E_\pi|x_0-y_0|}
\]

\[
P^{\pi}_n = \sqrt{\vec{p}_n^2 + M_\pi^2}
\]

\[
S^{ab}(x, y) = (\not{D} + M_N)G^{ab}(x, y)
\]

$pions$

$nucleons$

$pions$ $\rightarrow$ $nucleons$
Nucleon interpolating fields in QCD

- Nucleon interpolating fields in QCD
  We consider
  - local 3-quark operators
  - no derivatives

- Two independent operators
  Ioffe 1981, Espriu et al 1983

\[
N_1 = (\tilde{q} q) q \\
N_2 = (\tilde{q} \gamma_5 q) \gamma_5 q
\]

\[
q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \tilde{q} = q^T C \gamma_5 (i \sigma_2)
\]

- Transformation under chiral sym. and parity
  Nagata et al 2008

\[
N_i = N_{i,R} + N_{i,L} \xrightarrow{R,L} RN_{i,R} + LN_{i,L} \\
N_i = N_{i,R} + N_{i,L} \xrightarrow{P} \gamma_0 (N_{i,L} + N_{i,R})
\]
Nucleon interpolating fields in ChPT

- Map interpolating fields to ChPT
  \[ N_i = \alpha_i^{(0)} \left( u \Psi_R + u^\dagger \Psi_L \right) + \alpha_i^{(1)} \ldots + \alpha_i^{(1)} \ldots + \ldots \]

- Transform as the fields on the quark level

- Same chiral expansion for both operators \((i=1\and 2)\), different LECs only

- Expand leading term in powers of pion fields
  \[ N_i = 4\alpha_i^{(0)} \left( \Psi + \frac{i}{2f} \pi^a \sigma^a \gamma_5 \Psi + \ldots \right) \Rightarrow \Psi\pi\text{-Vertex} \]

\[ \Rightarrow \tilde{\alpha}_i \]
Smeared interpolating fields

- Often used in Lattice QCD: *Smeared interpolating fields* build from smeared quark fields

\[
q_{\text{sm}}(x) = \int d^4y K(x - y)q(y)
\]

\[
N_{1,\text{sm}} = (\bar{q}_{\text{sm}}q_{\text{sm}})q_{\text{sm}}
\]

\[
N_{2,\text{sm}} = (\bar{q}_{\text{sm}}\gamma_5q_{\text{sm}})\gamma_5q_{\text{sm}}
\]

- The gauge covariant kernel \( K \)
  - is essentially zero for \(|x - y| > R_{\text{smear}}\) (``smearing radius'')
  - contains a delta function in time for *Gaussian smearing*
  - is truly 4-dimensional for the *Gradient Flow* [G"usken 1989, Alexandrou et. al. 1991]
  - is diagonal in spinor space

\( \Rightarrow \) Smeared fields \( \uparrow \) transform as unsmeared ones

\( \Rightarrow \) map to the same *pointlike* fields as their unsmeared counterparts provided

\[ R_{\text{smear}} \ll \frac{1}{M_\pi} \]

Different LECs only!
Why smeared fields?

Smeared fields suppress excited-state contributions!

Gaussian smearing with $R_{\text{smear}} \approx 0.3 \text{fm}$
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- Introduction
- Setup: Covariant Baryon ChPT and the interpolating nucleon fields
- $N\pi$-state contribution to the 2pt-function and eff. mass ★ You are here
- $N\pi$-state contribution to $g_A$
- Summary and conclusions
2pt-function: $N$ contribution

- 2pt-function we are interested in (always $t > 0$)

$$C_{\pm,ij}(t) = \int_{L^3} d^3x \langle N_{\pm,i}(\vec{x}, t) \overline{N}_{\pm,j}(0, 0) \rangle$$

Projection to definite parity using $\Gamma_{\pm} = \frac{1}{2}(\gamma_0 \pm 1)$:

$$N_{\pm} = \Gamma_{\pm} N$$
$$\overline{N}_{\pm} = \overline{N}\Gamma_{\pm}$$

- Use LO expressions for the interpolating fields

Feynman diagram

$C_{-,ij}(t) = 0$ for symmetry reasons

$$C_{+,ij}(t) = 2\tilde{\alpha}_i\tilde{\alpha}^*_j e^{-M_N t}$$

single-particle state contribution
real for $i=j$
2pt-function: $N\pi$ contribution

- Feynman diagrams for the $N\pi$ contribution:
  - Standard PT calculation (in position space)
  - Integration over vertex position $z, z'$
  - Momentum conservation
    \[ \vec{p}_N = -\vec{p}_\pi \]
  - Exponential decay
    \[ E_{\text{tot}} = E_N + E_\pi \]
    (ignore other contributions !)
2pt-function: $N\pi$ contribution

Result

$E_{\text{tot}} = E_N + E_\pi$

\[
C_{+,ij}(t) = 2\tilde{\alpha}_i\tilde{\alpha}_j^* \frac{3}{8(fL)^2} \sum_{\vec{p}} \frac{1}{E_\pi L} \frac{E_N - M_N}{2E_N} \left[ 1 - g_A \frac{E_{\text{tot}} + M_N}{E_{\text{tot}} - M_N} \right]^2 e^{-E_{\text{tot}}|t|}
\]

- Expected $1/L^3$ suppression of a 2-particle state
- $C_{+,ij} = 0$ for vanishing momentum (symmetry!)
- Dependency on the details of the interpolating fields in prefactors only
- Same prefactor as the 1-particle-state contribution
- Degeneracy due to spatial symmetries:

  \[
  \sum_{\vec{p}} \rightarrow \sum_{p_n} m_n \quad \text{multiplicity}
  \]
  \[
  m_1 = 6, \quad m_2 = 12, \ldots
  \]

(neg. parity $\rightarrow$ arXiv:1503.03649)
2pt-function: Combined result

\[ C_{+,ij}(t) = 2\tilde{\alpha}_i \tilde{\alpha}_j^* e^{-M_N t} \left[ 1 + \sum_{p_n} c_n^+ e^{-(E_{\text{tot},n} - M_N) t} \right] \]

\[ c_n^+ = \frac{3m_n}{8(f L)^2 E_{\pi,n} L} h_n^+ \]

\[ h_n^+ = \frac{E_{N,n} - M_N}{2E_{N,n}} \left[ 1 - g_A \frac{E_{\text{tot},n} + M_N}{E_{\text{tot},n} - M_N} \right]^2 \]

\( n=1: \quad = b_1/b_0 \) from introduction

- 4 unknowns in the coefficients: \( f/M_N \quad g_A \quad M_\pi/M_N \quad M_\pi L \)
- Universal result for all interpolating fields (to LO)!
- For estimates use \( f/M_N \approx f_{\text{exp}}/M_{N,\text{exp}} \quad g_A \approx g_{A,\text{exp}} \)
- Note: \( c_n^+, h_n^+ \) are positive
- Naive dimensional analysis \( \rightarrow h_n^+ = O(1) \)
2pt-function: coefficients $h_1$ and $h_2$

$$h^+_n = \frac{E_{N,n} - M_N}{2E_{N,n}} \left[ 1 - g_A \frac{E_{\text{tot},n} + M_N}{E_{\text{tot},n} - M_N} \right]^2$$

O(1), as expected

Remark: not so for 3-particle-state contribution in mesonic correlators

OB, M. Golterman, 2011
2pt-function: coefficients $c_1$ and $c_2$

\[
c_n^+ = \frac{3m_n}{8(fL)^2 E_{\pi,n}L} h_n^+
\]

\[
\begin{align*}
M_\pi L = 3 \\
M_\pi L = 4 \\
M_\pi L = 5
\end{align*}
\]

Roughly $\approx 0.1$ for $M_\pi L = 4$

$M_\pi/M_N = 0.2$

Monday, April 20, 15
\( N\pi \) contribution to effective mass

\[
M_{N,\text{eff}} = M_N \left[ 1 + \sum_{p_n} d_n^+ e^{-(E_{\text{tot},n} - M_N)t} \right]
\]

\[
\sum_{p_n \leq p_{n_{\text{max}}}} d_n^+ e^{-(E_{\text{tot},n} - M_N)t}
\]

\[
\begin{array}{c|cccc}
\hline
\text{\( n_{\text{max}} \)} & \text{0.51} & \text{0.68} & \text{0.85} & \text{1.02} \\
\hline
1 & 0.018 & 0.012 & 0.009 & 0.006 \\
2 & 0.043 & 0.028 & 0.018 & 0.012 \\
3 & 0.054 & 0.034 & 0.022 & 0.014 \\
\hline
\end{array}
\]

\[ E_{\text{tot},n} = E_{N,n} + E_{\pi,n} \]

Typical time range (e.g. Dürr et al)

\( 0.5 \text{ fm} \lesssim t \lesssim 1 \text{ fm} \)

few percent contribution whether visible or not depends on the statistical error in the lattice data
Errors

- LO calculation only ➡ error ?
  Higher order calculation
  - is in principle straightforward
  - does depend on LECs associated with the nucleon interpolating fields

- Impact of other excited states ?
  - 3-particle $N\pi\pi$ state: suppressed by additional factor $\frac{1}{2(fL)^2M_{\pi}L}$ probably too small to be relevant (?)
  - 1-particle $N^*$ state: ?

- Smeared fields mapped on point-like ChPT fields ➡ error ?

⇒ Error estimate for the LO result: ? %
Looking again at the BMW data

- roughly 5% stat error in nucleon channel
- no indication of the $N\pi$-state contribution
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- $N\pi$-state contribution to $g_A$  
  \textcolor{red}{\textit{PRELIMINARY}}\n  \textit{work in progress}
- Summary and conclusions
3pt-function and $g_A$

- **3pt-function we are interested in**
  \[
  C_3(t, t') = \frac{i}{2} (1 + \gamma_0) \gamma_k \gamma_5 \int d^3x \int d^3y \Gamma'_{k,\alpha\beta} \langle N_\beta(\vec{x}, t) A^3_k(\vec{y}, t') N_\alpha(\vec{0}, 0) \rangle
  \]
  \[
  \text{pos. parity: } \frac{N}{N} = \frac{N_+}{N_+}
  \]

  _axial vector current_

- **Ratio**
  \[
  R = \frac{C_3(t, t')}{C_2(t)}
  \]

- **Use LO expression for the fields**
  \[
  C_3(t, t') = g_A C_2(t) \quad \Rightarrow \quad R = g_A
  \]
Feynman diagrams for the 3pt-function

- Feynman diagrams for the nucleon-pion-state contributions

![Feynman diagrams](image)

- Interpolating field
- Axial vector current
- Interaction vertex

\[
C_{3},a = ig A^2 |\bar{\epsilon}|^2 e^{M t} (31)
\]

Obviously this is equal to \( ig A \) times the LO result for the 2-pt function. Thus, taking the ratio \( R_{LO} = C_{3},LO / C_{2},LO \) we finally find \( R_{LO} = ig A \).

B. Nucleon-pion-state contributions

1. Generalities

The nucleon-pion contribution to the 3-pt function stems from the diagrams in figure 1. These diagrams are obtained if two NLO parts of either the interpolating fields (fig. d) or the axial vector current (figs. b and c) is used. In addition there are diagrams involving one or two vertex insertions.
$N\pi$ contribution to $g_A$

$$R = g_A + \sum_{p_n} \left( b_n e^{-\Delta E_n (t-t')} + \tilde{b}_n e^{-\Delta E_n t'} + c_n e^{-\Delta E_n t} \right)$$

Some general results:

- The coefficients $b_n, \tilde{b}_n, c_n$ depend only on $f/M_N, g_A, M_\pi/M_N, M_\pi L$

  ➞ universal result for all interpolating fields (to LO) !
$N\pi$ contribution to $g_A$

\[ R = g_A + \sum_{p_n} \left( b_n e^{-\Delta E_n (t-t')} + \tilde{b}_n e^{-\Delta E_n t'} + c_n e^{-\Delta E_n t} \right) \]

\[ \Delta E_n = E_{\text{tot},n} - M_N \]

\[ c_n = c_n^{3\text{pt}} - c_n^{+,2\text{pt}} = \frac{3m_n}{8(fL)^2 E_\pi L} \left( h_n^{3\text{pt}} - h_n^{+,2\text{pt}} \right) \]

\[ h_n^{3\text{pt}} \propto \left( \frac{E_{N,n} - M_N}{E_{N,n}} \right) \]
$N\pi$ contribution to $g_A$

\[ h_{1}^{+,2\text{pt}} \]

\[ h_{1}^{3\text{pt}} - h_{1}^{+,2\text{pt}} \approx -h_{1}^{+,2\text{pt}} \]
$N\pi$ contribution to $g_A$

\begin{align*}
 h_2^{+,2pt} & \approx -h_2^{+,2pt} \\
 h_3^{3pt} & - h_2^{+,2pt} \approx -h_2^{+,2pt}
\end{align*}
$N\pi$ contribution to $g_A$

$$R = g_A + \sum_{p_n} \left( b_n e^{-\Delta E_n (t-t')} + \tilde{b}_n e^{-\Delta E_n t'} + c_n e^{-\Delta E_n t} \right)$$

$$\Delta E_n = E_{tot,n} - M_N$$

$$c_n = c_n^{3\text{pt}} - c_n^{+\text,+2\text{pt}} = \frac{3m_n}{8(fL)^2 E_\pi L} \left( h_n^{3\text{pt}} - h_n^{+\text,+2\text{pt}} \right)$$

- To good approximation we find $c_n \approx -c_n^{+\text,+2\text{pt}}$
- We already know that $c_n^{+\text,+2\text{pt}}$ is positive

$\Rightarrow$ this contribution makes $R$ smaller than $g_A$

very likely true even when higher orders are taken into account
\( N\pi \) contribution to \( g_A \)

\[
R = g_A + \sum_{p_n} \left( b_n e^{-\Delta E_n (t-t')} + \tilde{b}_n e^{-\Delta E_n t'} + c_n e^{-\Delta E_n t} \right)
\]

- we find \( b_n = \tilde{b}_n \) (as expected by symmetry)
- Many diagrams, contributions with positive and negative sign:

\[
b_n = \frac{3m_n}{8(fL)^2 E_\pi L} g_n
\]

\[
g_n = \left(1 - \frac{M_N}{E_{N,n}}\right) \left[1-g_A \frac{E_{\text{tot},n} + M_N}{E_{\text{tot},n} - M_N} + \frac{3}{4} g_a \left[\frac{E_{\text{tot},n} + M_N}{E_{\text{tot},n} - M_N}\right]^2 \ldots\right]
\]

\( \Delta E_n = E_{\text{tot},n} - M_N \)

dominant
$N\pi$ contribution to $g_A$

$$b_n = \frac{3m_n}{8(fL)^2 E_\pi L} g_n$$

Preliminary conclusion: $b_n > 0$

work in progress...
Summary and outlook

- ChPT provides useful information concerning $N\pi$-state contributions in nucleon correlation functions.

- To LO the LECs associated with the nucleon interpolating fields do not contribute.
  - LO results are fairly universal (e.g. with respect to smearing).
  - LO results have some predictive power.

- $N\pi$-state contributions to:
  - 2pt-function are at the few-percent level.
  - 3pt-function and $g_A$ are at the ...... -percent level.

Outlook:

- $N\pi\pi$-state contribution

- Other observables: Momentum fraction $<x>_{u-d}$
  Electromagnetic form factors
  ...

Monday, April 20, 15
Additional slides
Comparison with finite volume effects

\[ R = g_A \left( 1 + \delta_{N\pi} \left( M_\pi L, M_\pi / M_N \right) \right) \]

\[ g_A(L) = g_A(\infty) \left( 1 + \delta_{FV} (M_\pi L) \right) \]

combined finite volume/excited state effects
Comment on arXiv:1503.06329

- Three weeks ago: Preprint arXiv:1503.06329 by Brian Tiburzi, *Chiral corrections to Nucleon Two- and three-Point Correlation Functions*

- Difference: Uses heavy baryon ChPT

- Comparison with results presented here
  - Use $E_N \approx M_N + \frac{p^2}{2M_N}$
  - Expand in $\frac{E_\pi}{M_N}$
  - Obtain the same result for the 2pt function

Note: Typically pion energy is not small with $\frac{E_\pi}{M_N} \approx \frac{1}{2}$

Good approximation?
Our ChPT results for the effective mass

\[ m_{a,\text{eff}} = -\frac{d}{dt} \log C_{PP}^a(t) \]

\[ = m_a \left( 1 + k_a e^{-2m_\pi t} \right) \]

with coefficient \( k_a \) determined from results for PP correlator given before.

Example: pionic effective mass (\( a = 1 \)) gives

\[ k_1 = \frac{90}{512(f_\pi L)^4(m_\pi L)^2} \]

relevant parameters for our estimate
Relevant parameters for "O7 lattice"

\[ m_\pi = 270 \text{ MeV} \]
\[ m_\pi L = 4.21 \]
\[ f_\pi L = 1.58 \]
\[ \Rightarrow \quad k_1 \approx 1.6 \times 10^{-3} \]

- 3-pion state contribution is at the per mille level! (much smaller than the statistical error)
- Curvature in plot not due to the 3-pion state!
Relevant parameters for "O7 lattice"

\[ m_\pi = 270 \text{ MeV} \]
\[ m_\pi L = 4.21 \]
\[ f_\pi L = 1.58 \]
\[ \Rightarrow k_1 \approx 1.6 \times 10^{-3} \]

Naive fit with \( k_1 \) as free fit parameter yields \( k_1 \approx 2 \times 10^{-1} \)