Open vs Periodic Boundary Conditions in the Deconfined Phase

A Dream of Spring

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Topology in a Nutshell

Open-Boundary Conditions

An Attempt: Switches

Topological Susceptibility

Conclusions and Outlooks
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Conclusions and Outlooks
Classical gauge theories

Fibre bundles
Classical gauge theories

Fibre bundles

Can be 'twisted'!
Classical gauge theories

Fibre bundles

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Classical gauge theories

Fibre bundles

Can be 'twisted'!

de Rham's Theorem

\[ \frac{\partial}{\partial x} \]

Top. invariants

\[ \mathbb{P}[\mathcal{F}] \]
Classification

SU(N) bundles in 4D

\[ q = -\frac{1}{8\pi^2} \text{Tr} (F \wedge F) \]

de Rham's Theorem

Top. invariants

\[ \frac{\partial}{\partial x} \]

\[ \mathbb{P}[\mathcal{F}] \]
Classification

\[ \text{SU}(N) \text{ bundles in } 4D \]

\[ q = -\frac{1}{8\pi^2} \text{Tr} (F \wedge F) \]

Topological charge:

\[ Q = -\frac{1}{16\pi^2} \int_M F_{\mu\nu} \tilde{F}^{\mu\nu} \]
Some Solutions on $\mathbb{T}^4$

$A_\mu = \frac{2\pi}{a} \sum_\nu x^\nu b_{\mu\nu} \mathbf{N}$ \quad $b_{\mu\nu} \in \mathbb{N}$

$F_{\mu\nu} = \frac{2\pi}{a^2} (b_{\mu\nu} - b_{\nu\mu}) \mathbf{N}$

$Q = \frac{1}{2} \text{Tr} \mathbf{N}^2 \left((b_{01} - b_{10})(b_{23} - b_{32}))\right)$

$-(1 \leftrightarrow 2) + (1 \leftrightarrow 3)$

---

Classification

$SU(N)$ bundles in 4D

$2^{nd}$ Chern class: $q = -\frac{1}{8\pi^2} \text{Tr} (F \wedge F)$

Topological charge:

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Some Solutions on $\mathbb{T}^4$

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$$-(1 \leftrightarrow 2) + (1 \leftrightarrow 3)$$

Remarks

- For compact spaces $Q \in \mathbb{Z}$
- For non-compact spaces $Q \in \mathbb{R}$
- Continuum story

Refs: [Avis,Isham 1978]
[DeWitt,Hart,Isham 1979]
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Motivations

Axion Cosmology

Clustering

$U(1)$ Problem

Issues

Physical: High-T Suppression

Algorithmical: Topological Freezing

No more top. transitions!
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Physical: High-T Suppression

Algorithmical: Topological Freezing

No more top. transitions!

Lat. accomodates < and < inst.

Damped by the Plasma
High-T Suppression

Lat. accommodates $< \text{ and } < \text{ inst.}$

\[ \text{Damped by the Plasma} \]

Freezing

Continuum limit

Distinct top. sectors emerge

Access. by small disp. in conf. space

$\tau_{\text{autocorrelation}}(Q) \gg 0$
Freezing

Continuum limit

Distinct top. sectors emerge

Access. by small disp. in conf. space

\[ t_{\text{autocorrelation}}(Q) \gg 0 \]

Solutions

• Meta-Dynamics

• Reweighting

• Multiscale Equilibration

• Master-Field

• Open-Boundary Conditions

• ...

Refs: [Laio et al., 2015; Moore et al., 2018]
[Endres et al., 2015; Luscher, 2017]
[Luscher et al. 2011]
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Topological Susceptibility

Conclusions and Outlooks
Solutions

- Meta-Dynamics

- Reweighting

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... 

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OBC

\[ T^4 \]

Open-up the torus!
Open-up the torus!

OBC

\[ \mathbb{T}^4 \]

\[ T \not= 0 \]

\[ F_{0\mu}(x)|_{x_0=0} = F_{0\mu}(x)|_{x_0=l_0} = 0 \]

\[ F_{i\mu}(x)|_{x_i=0} = F_{i\mu}(x)|_{x_i=l_i} = 0 \]
Open-up the torus!

\[ T^4 \]

\[ F_{0\mu}(x)|_{x_0=0} = F_{0\mu}(x)|_{x_0=l_0} = 0 \]

\[ T \neq 0 \]

\[ F_{i\mu}(x)|_{x_i=0} = F_{i\mu}(x)|_{x_i=l_i} = 0 \]
\[ \beta = \frac{6}{\gamma} \]

\[ T \approx 1.28T_c \]
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\[ \langle Q^2 \rangle \] vs \[ \sqrt{\frac{6}{\beta}} \]

\[ \beta = 6.165 \]
\[
\beta = 6.580
\]

\[
\sqrt{\frac{6}{\beta}} = \langle \frac{Q^2}{V} \rangle [\text{GeV}^4]
\]

\[
T \approx 1.28T_c
\]
\[ \beta \approx 6.930 \]
Wilson Flow

\[ \tau = \frac{459}{900} \]

\[ \tau = \frac{1950}{900} \]

\[ \frac{dQ}{\tau_{wils}} \]

\[ |Q| = 1 \]

\[ |Q| = 2 \]
Wilson Flow

WF and $Q$

$\tau = 459$

$\tau = 1950$

$\frac{dQ}{\tau_{wils}}$

$|Q| = 1$

$|Q| = 2$

Finite Size Effects

\[ V_s = L_s^3 \]
Finite Size Effects

\[ V_s = L_s^3 \quad V_s' = L_s'^3 \]
\[ V_s = L_3 \]

\[ V_s' = L_3' \]

\[ \langle W^{4 \times 4}_{\text{loops}} \rangle \]
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Switches

Do $n_{PBC}$ sweeps with PBC

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Switches

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Switch PBC to OBC in some direction(s)
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\[ \ldots \]
Switches

Do $n_{PBC}$ sweeps with PBC

Switch PBC to OBC in some direction(s)

Do $n_{OBC}$ sweeps

Switch back

Do $n_{PBC}$ sweeps

... 

Repeat $n_{trans}$ times
Switches

Do $n_{PBC}$ sweeps with PBC

Switch PBC to OBC
in some direction(s)

Do $n_{OBC}$ sweeps

Switch back

Do $n_{PBC}$ sweeps

... 

Repeat $n_{\text{trans}}$ times

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**Switches**

Do $n_{PBC}$ sweeps with PBC

**Switch PBC to OBC** in some direction(s)

Do $n_{OBC}$ sweeps

Switch back

Do $n_{PBC}$ sweeps

...\[ \text{Repeat } n_{\text{trans}} \text{ times} \]
$T \approx 1.5T_c, 16 \times 64^3, \beta = 6.872$

- Generates higher $Q$'s
Facts

\[ T \approx 1.5T_c, 16 \times 64^3, \beta = 6.872 \]

- Generates higher Q's
- Oversamples
\[ T \approx 1.5T_c, \ 16 \times 64^3, \ \beta = 6.872 \]

- Generates higher Q's
- Oversamples

Break det. balance

\[ // \text{tempering?} \]
\[ T \approx 1.5 T_c, 16 \times 64^3, \beta = 6.872 \]

- Generates higher \( Q \)'s
- Oversamples

\[ Q \]

Break det. balance

Which dist? OBC?

// tempering?
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**Topological Susceptibility**

Conclusions and Outlooks
\[ \chi_t = \frac{1}{V} \frac{d^2}{d\theta^2} \ln Z(\theta) \bigg|_{\theta=0} \]
\[ = \frac{1}{V} \left\langle \int dx q(x) \int dy q(y) \right\rangle \]
\[ = \frac{1}{V} \left\langle \int dx_0 \int dy q(x_0) q(x_0 + y) \right\rangle \]
\[ = \frac{1}{V} \left\langle \int dx_0 \int dy G_{qq}(x_0, x_0 + y) \right\rangle \]

[Bonati, d'Elia, 2017; Refs. therein]
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[Bonati, d'Elia, 2017; Refs. therein]
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[Bonati, d'Elia, 2017; Refs. therein]
\( \chi_t \)

\[
\chi_t = \frac{1}{V} \left. \frac{d^2}{d\theta^2} \ln \mathcal{Z}(\theta) \right|_{\theta=0}
\]

\[
= \frac{1}{V} \left\langle \int dx q(x) \int dy q(y) \right\rangle
\]

\[
= \frac{1}{V} \left\langle \int dx_0 \int dy q(x_0) q(x_0 + y) \right\rangle
\]

\[
= \frac{1}{V} \left\langle \int dx_0 \int dy G_{qq}(x_0, x_0 + y) \right\rangle
\]

[Bonati, d'Elia, 2017; Refs. therein]
$\chi_t$

\[ \mathcal{S} \text{ to } \mathcal{S}: \]
\[ \chi_t = \left\langle \int_{\Omega_{\text{cut}}} dy G_{qq}(x_{\text{mid}}, x_{\text{mid}} + y) \right\rangle \]

Subvolume av.:
\[ \chi_t = \left< Q^2 \right|_{\Omega_{\text{sub}}} \]
$S$ to $S$: 

$$\chi_t = \left\langle \int_{\Omega_{cut}} dy G_{qq}(x_{mid}, x_{mid} + y) \right\rangle$$

Subvolume av.: 

$$\chi_t = < Q^2 > \big|_{\Omega_{sub}}$$

Benchmark

$64^3 \times 6$, $\beta = 6.139$

$$a\chi^{1/4} = 0.030 \pm 0.0005$$

[Berkowitz et al., 2015]
\[ \alpha x(r)^{1/4} \]

Correlators

\[ \langle \frac{q^2}{V} \rangle_{PBC} \]
\[ a \chi(r)^{1/4} \]

Correlators

\[ \langle \frac{q^2}{V} \rangle_{PBC} \]

\[ PBC^4 \]

\[ ar \]

\[ 10^{-2} \]
\[
\alpha \chi(r)^{1/4}
\]

Correlators

\[
\langle \frac{\psi^2}{V} \rangle_{PBC}
\]

\[
OBC^3 \times PBC
\]

\[
PBC^4
\]
\[ \alpha(x(r)^{1/4}) \]

Correlators

\[ \langle \frac{\varphi^2}{V} \rangle_{PBC} \]

\[ OBC^3 \times PBC \]

\[ PBC^4 \]

\[ OBC_x \times PBC^3 \]

\[ ar \]

\[ 0 \rightarrow 30 \]

\[ 0 \rightarrow 0.1 \times 10^{-2} \]

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Preliminary

\[ \langle \frac{\varphi^2}{V} \rangle \bigg|_{PBC} \]

Correlators

\[ a x(r)^{1/4} \]

\[ OBC_x \times PBC^3 \]

\[ ar \]

\[ 10^{-2} \]
\[
\alpha \chi(r)^{1/4}
\]

Correlators

\[
\langle \frac{\mathcal{Q}^2}{V} \rangle_{PBC}
\]

\[
OBC^3 \times PBC \quad PBC^4
\]

\[
OBC_x \times PBC^3 \quad PBC^4, Q = 0
\]
$PBC = PBC, |Q| < 2$
A. Florio, Zeuthen, 11/06/18
$a \chi(r)^{1/4}$

Subvolumes

$\langle \frac{q^2}{V} \rangle_{PBC}$

$OBC^3 \times PBC$

$PBC^4$

$OBC_x \times PBC^3$
Subvolumes

\[ a \chi(r)^{1/4} \]

\[ \langle \frac{Q^2}{V} \rangle_{PBC} \]

\[ OBC^3 \times PBC \]

\[ PBC^4 \]

\[ OBC^x \times PBC^3 \]

\[ PBC^4, Q = 0 \]
\[ \langle \frac{Q^2}{V} \rangle_{PBC} \mid OBC^3 \times PBC \]

\[ OBC \times PBC^3 \]

\[ PBC^4, Q = 0 \]

\[ PBC^4, |Q| < 2 \]
OBC

PBC

\[ q(x) \]

\[ OBC \]

\[ PBC \]
**Benchmark**

\[ 64^3 \times 6, \beta = 6.139 \]

\[ \alpha \chi^{1/4} = 0.030 \pm 0.0005 \]

[Berkowitz et al., 2015]

**Larger Lattices**

\[ 64^3, 96^3 \times 16, \beta = 6.872 \]
Preliminary

\[ \langle \frac{Q^2}{V} \rangle \bigg|_{PBC} \]
\[ \alpha \chi(r)^{1/4} \]

Subvolumes

\[ \langle \frac{Q^2}{V} \rangle \big|_{PBC} \]

\[ PBC^4 \]
Subvolumes

\[ \alpha \chi(r)^{1/4} \]

\[ \langle \frac{Q^2}{V} \rangle_{PBC} \quad OBC^3 \times PBC \quad PBC^4 \]

\[ ar \]

\[ 0 \quad 10 \quad 20 \quad 30 \quad 40 \]

\[ 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \quad 3 \times 10^{-2} \]
Subvolumes

\[ aX(r)^{1/4} \]

\[ \langle \frac{Q^2}{V} \rangle_{PBC} \]

\[ OBC^3 \times PBC \]

\[ OBC_x \times PBC^3 \]

\[ PBC^4 \]
Preliminary

\[ \langle \frac{q^2}{V} \rangle \bigg|_{\text{PBC}} \]

\[ \text{OBC}^3 \times \text{PBC} \]

\[ \text{OBC}_x \times \text{PBC}^3 \]

\[ \text{PBC}^4 \]
What's next?

• Improve statistics
What's next?

- Improve statistics
- Wilson flow dep.
Todo's

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- Wilson flow dep.
Todo's

- Improve statistics
- Wilson flow dep.
- Model $Q$ diffusion?
  [Mawhinney et al., 2014]
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Take Away

• OBC may be used for thermal simulations

• May not fully solve the topological freezing

• "Local" $\chi_t$ differentiates between freezing and high-T suppression

• $\chi_t$ at very high-T?
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Thank you!

(... and https://www.pinterest.com/pin/571746115169525297/ for the Moebius strip)

(but more Yannis Burnier for his early-on collaboration)