A new mechanism for dynamical nonperturbative particle mass generation

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Introduction

A nonperturbative mechanism for the generation of a finite quark mass

Might be a viable nonperturbative alternative to the Higgs mechanism

Motivated by naturalness, mass hierarchy and flavor problems of the Standard Model

Wilson fermions: the critical Wilson term explicitly breaks chiral symmetry, and this triggers a spontaneous dynamical chiral symmetry breaking

However, the associated mass term cannot be disentangled from the $1/a$ divergence of the critical mass

Idea: add scalar fields, coupled to a SU(2) fermion doublet via Yukawa and Wilson-like terms (toy model)

Conjecture: for some critical value of the Yukawa coupling, in the phase in which $\langle\Phi\rangle \neq 0$ a light fermion mass can be dinamically generated

There are no $1/a$ divergences in the toy model, and one can observe this nonperturbative mass directly

This mass originates from peculiar nonperturbative operator mixings, triggered by the Wilson-like terms, which survive the removal of the ultraviolet cutoff

The structure of the toy model allows also easily to include electroweak interactions – and the finite masses of the W and Z can also be generated by the same nonperturbative mechanism

$\Rightarrow$ full BSM model of elementary particles (with massless neutrinos)

Could also be an interesting new approach to the mass hierarchy problem: the stronger is the strongest of the interactions a fermion is subjected to, the larger its mass – and could also explain why the mass of the top quark is so large

In the Standard Model even the order of magnitude of the masses of elementary fermions, weak gauge bosons and the Higgs is not understood, but simply fit to the experimental data

Here: common nonperturbative physical mechanism, and the magnitude of the masses is parametrically understood

The 125 GeV scalar could be a bound state of two electroweak bosons, the binding due to superstrongly interacting fermions

Wilson fermions

Wilson fermions: the Wilson term breaks chiral symmetry, and the renormalization of the quark mass requires a $m_{cr}\,\overline{\psi}\psi$ counterterm, which is linearly divergent and formally given by

$$m_{cr} = \frac{c_0}{a} + c_1\,\Lambda_{QCD} + O(a)$$

If we could tune in the Wilson action m_0 to exactly cancel only the c_0/a term, then $c_1\,\Lambda_{QCD}$ would play the role of a quark mass in the renormalized theory

To see this, consider the renormalized axial (nonsinglet) Ward identities of Wilson fermions [Bochicchio et al., 1985] :

$$\nabla_\mu \langle \hat{J}^f_{5\mu}(x)\hat{O}(0)\rangle = \langle \Delta^f \hat{O}(0)\rangle \delta(x) + 2(m_0 - \overline{M}(m_0))\langle P^f(x)\hat{O}(0)\rangle + \mathrm{O}(a)$$

$J^f_{5\mu}$ is the nonsinglet axial current and $\overline{M}$ the mixing coefficient between the axial variation of the Wilson term, O^f_5, and the pseudoscalar quark density, P^f:

$$\hat{O}^f_5(x) = Z_5\left[O^f_5(x) + \frac{2\overline{M}}{a}P^f(x) + \frac{Z_A - 1}{a}\nabla_\mu J^f_{5\mu}(x)\right]$$

where in general

$$\overline{M}(m_0) = \frac{c_0(1-d_1)}{a} + c_1(1-d_1)\Lambda_{QCD} + d_1 m_0 + \mathrm{O}(a)$$

with the coefficients c_0, c_1 and d_1 functions of the gauge coupling

Imposing $\overline{M}(m_0) = m_0$ one gets exactly $m_{cr} = c_0/a + c_1\,\Lambda_{QCD} + O(a)$

If we could set $m_0 = c_0/a$, the Ward identity would take the form

$$\nabla_\mu \langle \hat{J}^f_{5\mu}(x)\hat{O}(0)\rangle = \langle \Delta^f \hat{O}(0)\rangle \delta(x) - 2\,c_1(1-d_1)\Lambda_{QCD}\langle P^f(x)\hat{O}(0)\rangle + \mathrm{O}(a)$$

Then we can see that the term proportional to Λ_{QCD} plays the role of a nonperturbatively generated quark mass

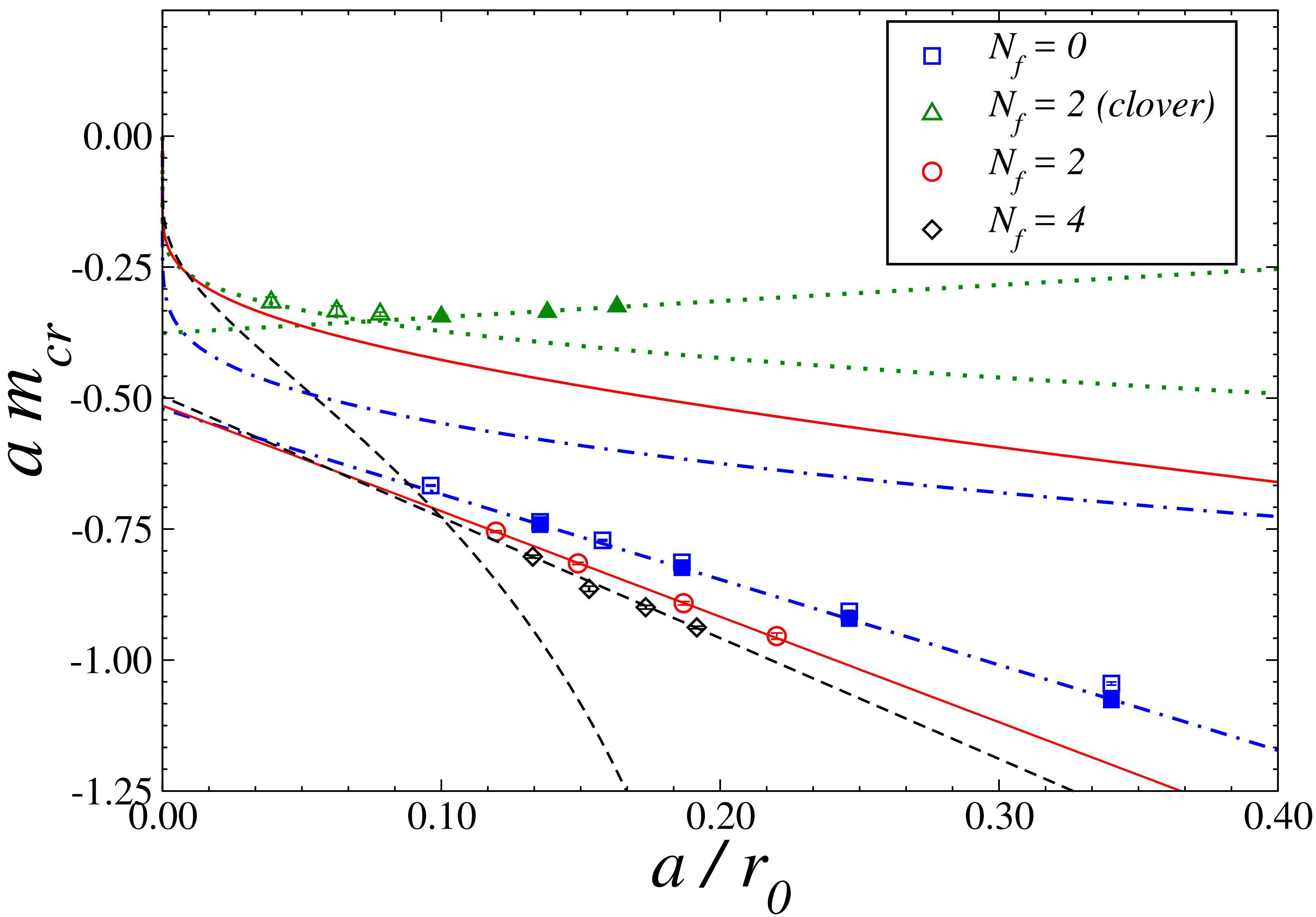
This mass is dynamically generated as a result of the interplay between $O(a)$ chiral breaking effects left over in the critical theory and power divergences of loop integrals where a Wilson vertex is inserted

But: these nonperturbative effects are not seen in standard lattice simulations, because one takes m_{cr} to be the value of m_0 at which the PCAC mass vanishes, which is the whole expression, not just c_0/a

The $c_1\Lambda_{QCD}$ term, if present, is eliminated away together with the $1/a$ term

Difficult fine tuning problem of separating a small (nonperturbative) contribution $c_1\Lambda_{QCD}$ from a large (perturbative) mass term c_0/a

Is there numerical evidence for a nonvanishing $c_1\Lambda_{QCD}$ term in simulations with Wilson fermions?



$N_f = 0$
$N_f = 2$ (clover)
$N_f = 2$
$N_f = 4$
0.00
-0.25
-0.50
-0.75
-1.00
-1.25
0.00
0.10
0.20
0.30
0.40
$a\, m_{cr}$
a / r_0

ETM twisted mass at maximal twist, except clover in one case; and 2-loop PT

Linear behavior: the slope is $c_1 \Lambda_{QCD} r_0$ – and does not vanish!

One cannot rigorously determine an a/r_0 range where the logarithmic a-dependence of am_{cr} and the higher order lattice artefacts that become important at large enough a values can be considered negligible

But, it is difficult to give a reasonable description of the nonperturbative points without a linear term of the kind $c_1 \Lambda_{QCD} a/r_0$ contained in m_{cr}

Moreover, the a/r_0 behavior of the 2-loop perturbative curves is very different from that of the nonperturbative data

Reliable fits give about 700, 900 and 1000 MeV for $c_1\Lambda_{QCD}$ in the three twisted mass cases – a mild N_f dependence

In the clover case, c_1 appears instead to be rather small, it looks almost flat

This is not surprising: a non-zero slope is a consequence of the chiral breaking terms in the Wilson action

But the chirality breaking effects are suppressed when one adds a clover term which is nonperturbatively tuned, and in this case c_1 must be much smaller

($O(a)$ chiral breaking effects will be absent only in on-shell quantities)

The existence in m_{cr} of nonperturbative $O(a\Lambda_{QCD})$ corrections should not come as a surprise

Indeed, there is an overwhelming evidence for similar cutoff effects in Wilson fermions simulations, where they are seen to affect the correlation functions from which physical quantities like masses, operator matrix elements, etc., are extracted

On the other hand, it is known that in the absence of spontaneous chiral symmetry breaking effects all (non-trivial) correlators of massless Wilson fermions would be automatically $O(a)$ improved – which is not the case

One expects a dependence of c_1 on the Wilson r parameter which is non-analytic – this is a footprint of the dynamical origin of the nonperturbative mass term $c_1 \Lambda_{QCD}$

Since the Wilson term is odd in r, c_1 should be proportional to the sign of r (times an r-even coefficient)

This behavior is analogous to what happens in QCD with the chiral condensate $\langle \overline{q} q \rangle$, which (in the infinite volume limit) is proportional to the sign of m_q

In both instances it is the dynamical breaking of chiral symmetry, triggered by either the (critical) Wilson term or by a nonzero mass term, that is responsible for the occurrence of such nonperturbative dynamical phenomena

We want now to argue that in Wilson QCD there is room for the appearance of a finite (up to logs) contribution in m_{cr}, like the term $c_1 \Lambda_{QCD}$ that we have discussed

Theoretical arguments

Two lines of reasoning:

- Symanzik expansion, and lattice fermion self-energy
- spectral density of the Dirac operator

Of course none of these two lines of reasoning can be rigorously pursued until the end. . . – otherwise we would be able to do exact nonperturbative mass calculations in a regularized field theory

But the converging results provided by the two approaches give us confidence that the numerical indications that we have discussed are a real feature of m_{cr}

First argument: delicate interplay between $O(a)$ corrections to quark and gluon propagators and vertices coming from the spontaneous breaking of chiral symmetry, and the power-like divergence of the loop integration in self-energy diagrams where one Wilson term vertex is inserted

Indeed, peculiar nonperturbative $O(a)$ corrections proportional to Λ_{QCD} (and independent of $m_0 - m_{cr}$) affect lattice correlators

They can be described in terms of formal $O(a)$ contributions in the Symanzik expansion of lattice correlators

In the limit $m_0 \to m_{cr}$ we can in general write

$$\langle O(x,x',...)\rangle\Big|^{L} = \langle O(x,x',...)\rangle\Big|^{C} - a\langle O(x,x',...)\int d^4z L_5(z)\rangle\Big|^{C} + \mathrm{O}(a^2)$$

$$O(x,x',...) \quad \Leftrightarrow \quad A^b_\mu(x)A^c_\nu(x')\,,\;\; q_{L/R}(x)\overline{q}_{L/R}(x')\,,\;\; q_{L/R}(x)\overline{q}_{L/R}(x')A^b_\mu(y)$$

where O is a (multi)local, formally chiral invariant operator and L_5 is the $d=5$ chiral breaking Symanzik local effective Lagrangian operator

$$L_5 = b_{\sigma F}\,\overline{q}(i\sigma\cdot F)q + b_{DD}\,\overline{q}(-D\cdot D)q$$

The $O(a)$ continuum correlators would vanish were it not for the chiral breaking (critical) Wilson term which triggers spontaneous chiral symmetry breaking

From symmetry and dimensional arguments one can write the nonperturbative $O(a)$ contributions as

$$\Delta G^{bc}_{\mu\nu}(k)\Big|^{L} = -\alpha_s\, a\Lambda_{QCD}\,\delta^{bc}\,\frac{\Pi_{\mu\nu}(k)}{k^2}\; f_{AA}\Big(\frac{\Lambda^2_{QCD}}{k^2}\Big)$$

$$\Delta S_{LL/RR}(k)\Big|^{L} = -\alpha_s\, a\Lambda_{QCD}\,\frac{ik_\mu(\gamma_\mu)_{LL/RR}}{k^2}\; f_{q\overline{q}}\Big(\frac{\Lambda^2_{QCD}}{k^2}\Big)$$

$$\Delta\Gamma^{b,\mu}_{Aq\overline{q}}(k,\ell)\Big|^{L} = \alpha_s\, a\Lambda_{QCD}\, ig_s\lambda^b\gamma_\mu f_{Aq\overline{q}}\Big(\frac{\Lambda^2_{QCD}}{k^2},\frac{\Lambda^2_{QCD}}{\ell^2},\frac{\Lambda^2_{QCD}}{(k+\ell)^2}\Big)$$

where $\Pi_{\mu\nu}(k)$ is the projector appropriate to the chosen gauge fixing condition

These nonperturbative $O(a)$ contributions do no appear on the free theory, and so they must be proportional to some power of α_s

The factor α_s comes from the fact that the quark or gluon emitted from the L_5 vertex has to be absorbed somewhere else in the diagram

The RGI scale Λ_{QCD} signals the nonperturbative nature of the effect

From the Symanzik analysis of lattice artefacts, such kinds of $O(a)$ expansions are expected to be valid for squared momenta small compared to $1/a^2$

Here we assume that these nonperturbative effects continue to exist up to large momenta, and conjecture the asymptotic behavior

$$f_{AA}\left(\frac{\Lambda_{QCD}^2}{k^2}\right) \stackrel{k^2\to\infty}{\longrightarrow} h_{AA}, \qquad f_{q\overline{q}}\left(\frac{\Lambda_{QCD}^2}{k^2}\right) \stackrel{k^2\to\infty}{\longrightarrow} h_{q\overline{q}}$$

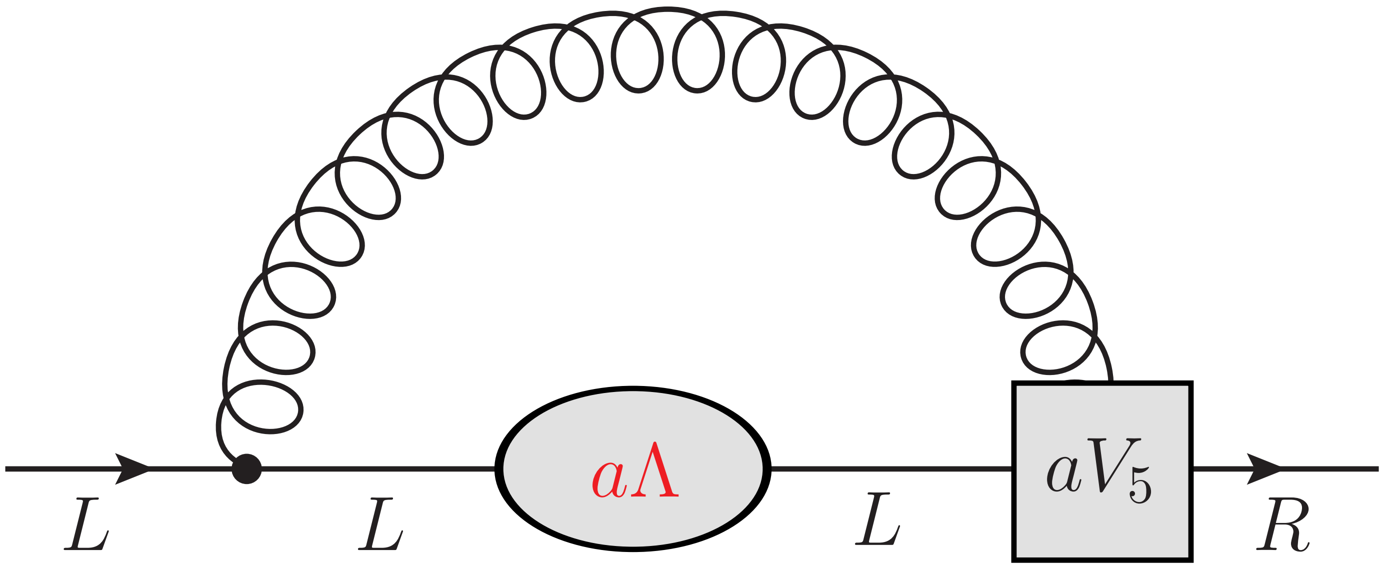
$$f_{Aq\overline{q}}\left(\frac{\Lambda_{QCD}^2}{k^2}, \frac{\Lambda_{QCD}^2}{\ell^2}\, \frac{\Lambda_{QCD}^2}{(k+\ell)^2}\right) \stackrel{k^2,\ell^2,(k+\ell)^2\to\infty}{\longrightarrow} h_{q\overline{q}}$$

where h_{AA} and $h_{q\overline{q}}$ are $O(1)$ constants, and the last two limits are related by gauge invariance

The constant large momentum behavior will be essential to generate a finite fermion mass contribution

To explicitly see how the finite $c_1\Lambda_{QCD}$ term can arise in m_{cr}, let us consider the $L-R$ component of the quark propagator and look for possible $O(a^0\Lambda_{QCD})$ nonperturbative mass-like contribution in lattice QCD

Example: the diagram



In the $a\to 0$ limit, the loop momentum counting gives (for small external momentum) factors $a\Lambda_{QCD}\alpha_s k_\mu/k^2$ and $1/k^2$ from the nonperturbative contribution to the quark propagator and the standard gluon propagator, and a factor ak_μ from the derivative coupling of the Wilson vertex

Assuming the constant asymptotic behavior, the multiplicative a^2 power is exactly compensated by the quadratic divergency of the loop integral

Including an α_s factor from the gluon loop, one gets a nonperturbative fermion mass term of the order

$$a\Lambda_{QCD} g_s^2 \alpha_s \int^{1/a} d^4k \, \frac{k_\mu}{k^2} \, \frac{1}{k^2} \, a k_\mu \quad \sim \, g_s^2 \, \alpha_s \Lambda_{QCD}$$

Other diagrams give similar nonperturbative mass contributions producing, to lowest order in the gauge coupling, the result $c_1 \sim \mathrm{O}(\alpha_s^2)$

To summarize: this argument shows that $O(a\Lambda_{QCD}\alpha_s)$ corrections to propagators and vertices have the potential of generating nonperturbative $O(\alpha_s^2 \Lambda_{QCD})$ corrections to the quark self-energy

We will not discuss here the argument based on the spectral density of the Dirac operator

Beyond Wilson

Central issue: would it be possible to subtract from m_0 not the whole critical mass (vanishing PCAC mass condition, restoration of the nonsinglet Ward identities), but only the c_0/a counterterm?

One should single out this nonperturbative finite mass from beneath the linearly divergent term which accompanies it

This is a naturalness problem, which for Wilson fermions has no solution – there is only one operator of dimension three with which O_5 can mix (P)

We need some extension of QCD . . . $\rightarrow$ add scalar particles

Have some useful symmetries – they are of primary importance for the solution of the fine tuning problem

If we want to avoid an undesirable fine tuning, we need to provide a good reason for choices like $m_0 = c_0/a$, or special values of the chirality restoring countertem parameters

Indeed, with this extension of QCD, the toy model, we have an enlarged chiral symmetry, and the fine tuning can be successfully done – a natural solution of the naturalness problem

Toy model

$$\mathcal{L}_{\rm toy}(Q,A,\Phi) = \mathcal{L}_{kin}(Q,A,\Phi) + \mathcal{V}(\Phi) + \mathcal{L}_{Wil}(Q,A,\Phi) + \mathcal{L}_{Yuk}(Q,\Phi)$$

with

$$\mathcal{L}_{kin}(Q,A,\Phi) = \frac{1}{4}(F\cdot F) + \overline{Q}_L \not{\mathcal{D}} Q_L + \overline{Q}_R \not{\mathcal{D}}\, Q_R + \frac{1}{2}\mathrm{tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right]$$

$$\mathcal{V}(\Phi) = \frac{\mu_0^2}{2}\mathrm{tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{tr}\left[\Phi^\dagger\Phi\right]\right)^2$$

$$\mathcal{L}_{Wil}(Q,A,\Phi) = \frac{b^2}{2}\rho\left(\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + \overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \mathcal{D}_\mu Q_L\right)$$

$$\mathcal{L}_{Yuk}(Q,\Phi) = \eta\left(\overline{Q}_L \Phi Q_R + \overline{Q}_R \Phi^\dagger Q_L\right)$$

where $\Phi = (\phi, -i\tau^2\phi^*)$, $Q_L = (u_L, d_L)^T$, $\mathcal{D}_\mu = \partial_\mu - i g_s \lambda^a A_\mu^a$,
$\overleftarrow{\mathcal{D}}_\mu = \overleftarrow{\partial}_\mu + i g_s \lambda^a A_\mu^a$

A non-abelian gauge model with fermion and scalar SU(2) doublets

A crucial role in the model is played by the $d=4$ Yukawa term and the Wilson-like $d=6$ operator – the fermion doublet is coupled to a complex scalar field via Yukawa and Wilson-like terms

The parameter ρ has no relevance for the naturalness argument here, but it will become important when eletroweak interactions are added

Symmetries of the toy model

$b^{-1} = \Lambda_{UV}$ is the ultraviolet cutoff

This model, which includes QCD, is renormalizable by power counting

Usual Lorentz, gauge, P, C, T symmetries . . .

The Lagrangian is invariant under global $\chi = \chi_L \otimes \chi_R$ transformations

$$\chi_L = \widetilde{\chi}_L \otimes \chi_L^{\Phi}, \quad \chi_R = \widetilde{\chi}_R \otimes \chi_R^{\Phi}$$

of the form

$$\begin{aligned}
\widetilde{\chi}_L : \quad & Q_L \longrightarrow \Omega_L\, Q_L \\
& \overline{Q}_L \longrightarrow \overline{Q}_L \Omega_L^{\dagger} \\
\widetilde{\chi}_R : \quad & Q_R \longrightarrow \Omega_R\, Q_R \\
& \overline{Q}_R \longrightarrow \overline{Q}_R \Omega_R^{\dagger} \\
\chi_L^{\Phi} : \quad & \Phi \longrightarrow \Omega_L\, \Phi \\
\chi_R^{\Phi} : \quad & \Phi \longrightarrow \Phi\, \Omega_R^{\dagger}
\end{aligned}$$

where $\Omega_L \in SU(2)_L$ and $\Omega_R \in SU(2)_R$

This chiral symmetry forbids any $1/a$ fermion mass term: a giant step towards solving the naturalness problem

Despite the presence of two chirality breaking operators, an exact symmetry acting on fermions and scalars prevents perturbative mass corrections

The conserved currents corresponding to the exact $\chi_L \otimes \chi_R$ symmetry are $(i = 1, 2, 3)$

$$J_\mu^{L\,i} = \overline{Q}_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{1}{2}\mathrm{tr}\Big[\Phi^\dagger \frac{\tau^i}{2}\partial_\mu \Phi - (\partial_\mu \Phi^\dagger)\frac{\tau^i}{2}\Phi\Big]$$
$$- \frac{b^2}{2}\rho\,\Big(\overline{Q}_L \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} Q_L\Big)$$
$$J_\mu^{R\,i} = \overline{Q}_R \gamma_\mu \frac{\tau^i}{2} Q_R - \frac{1}{2}\mathrm{tr}\Big[(\partial_\mu \Phi^\dagger)\Phi\frac{\tau^i}{2} - \frac{\tau^i}{2}\Phi^\dagger(\partial_\mu \Phi)\Big]$$
$$- \frac{b^2}{2}\rho\,\Big(\overline{Q}_R \frac{\tau^i}{2}\Phi^\dagger \mathcal{D}_\mu Q_L - \overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \frac{\tau^i}{2} Q_R\Big)$$

with the Ward identities

$$\partial_\mu \langle J_\mu^{L\,i}(x)\,\hat{O}(0)\rangle = \langle \Delta_L^i \hat{O}(0)\rangle \delta(x)$$
$$\partial_\mu \langle J_\mu^{R\,i}(x)\,\hat{O}(0)\rangle = \langle \Delta_R^i \hat{O}(0)\rangle \delta(x)$$

For generic values of the parameters $\mathcal{L}_{\mathrm{toy}}$ is instead **not** invariant under the purely fermionic chiral transformations $\widetilde{\chi}_L$ and $\widetilde{\chi}_R$, in which the scalar field does not take part

These transformations give rise to the (bare) Schwinger-Dyson equations

$$\partial_\mu \langle \widetilde{J}^{L\,i}_\mu(x)\,\hat{O}(0)\rangle = \langle \widetilde{\Delta}^i_L \hat{O}(0)\rangle \delta(x) - \eta\,\langle \Big(\overline{Q}_L \frac{\tau^i}{2}\Phi Q_R - \overline{Q}_R \Phi^\dagger \frac{\tau^i}{2} Q_L\Big)(x)\,\hat{O}(0)\rangle$$

$$-\frac{b^2}{2}\rho\,\langle\Big(\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2}\mathcal{D}_\mu Q_L\Big)(x)\,\hat{O}(0)\rangle$$

$$\partial_\mu \langle \widetilde{J}^{R\,i}_\mu(x)\,\hat{O}(0)\rangle = \langle \widetilde{\Delta}^i_R \hat{O}(0)\rangle \delta(x) - \eta\,\langle \Big(\overline{Q}_R \frac{\tau^i}{2}\Phi^\dagger Q_L - \overline{Q}_L \Phi \frac{\tau^i}{2} Q_R\Big)(x)\,\hat{O}(0)\rangle$$

$$-\frac{b^2}{2}\rho\,\langle\Big(\overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi^\dagger \mathcal{D}_\mu Q_L - \overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \frac{\tau^i}{2}\mathcal{D}_\mu Q_R\Big)(x)\,\hat{O}(0)\rangle$$

The **non**-conserved currents associated to the transformations $\widetilde{\chi}_L$ and $\widetilde{\chi}_R$ are

$$\widetilde{J}^{L\,i}_\mu = \overline{Q}_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{b^2}{2}\rho\Big(\overline{Q}_L \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} Q_L\Big)$$

$$\widetilde{J}^{R\,i}_\mu = \overline{Q}_R \gamma_\mu \frac{\tau^i}{2} Q_R - \frac{b^2}{2}\rho\Big(\overline{Q}_R \frac{\tau^i}{2}\Phi^\dagger \mathcal{D}_\mu Q_L - \overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \frac{\tau^i}{2} Q_R\Big)$$

They differ from the conserved ones, $J^{L\,i}_\mu$ and $J^{R\,i}_\mu$, only because they are missing the contribution bilinear in the scalar field coming from the kinetic term of Φ

Symmetry enhancement

The Wilson-like term plays a similar role to that of the Wilson term in standard lattice QCD with Wilson fermions

For this purpose one could also have chosen other operators, like $\overline{Q}_L \Phi\, i(\sigma \cdot F) Q_R + \overline{Q}_R \Phi^\dagger\, i(\sigma \cdot F) Q_L$

Lagrangian terms with $d = 6$ are part of the ultraviolet regularization of the model – we only need some $d \geq 6$ operator that breaks the purely fermionic chiral symmetry

So, the fermionic chiral symmetry is in general broken if $(\rho, \eta) \neq (0, 0)$

But there exist a critical value $\eta_{cr}(g_s^2, \rho, \lambda_0)$ of the Yukawa coupling for which also $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ becomes a symmetry of the theory!

η_{cr} is determined by enforcing the conservation of the currents $\widetilde{J}_\mu^{L,i}$ and $\widetilde{J}_\mu^{R,i}$

An extension of the Golterman-Petcher theorem of the Higgs-Yukawa model to this toy model, where fermions interact also with gauge fields

The symmetry restoration does not depend on the fine details of the ultraviolet regulator, only the numerical value of η_{cr} (and of other bare parameters) will change

Different phases with different physics

The physics of this model depends on the shape of the quartic potential of the scalar field

Two phases of the $\chi_L \otimes \chi_R$ symmetry, which can be realized

- à la Wigner, *or*
- à la Nambu-Goldstone

The physics of the model with enhanced $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ symmetry is completely different in these two phases:

- $\langle \Phi \rangle = 0$, single minimum, Wigner phase:

  no dynamical generation of mass

- $\langle \Phi \rangle \neq 0$, Mexican hat shape, Nambu-Goldstone phase:

  the $\widetilde{\chi}$ symmetry is dynamically broken by a nonperturbative mechanism (analogous to the generation of $c_1 \Lambda_{QCD}$ in lattice QCD)

The toy model appears to be the minimal setup for the generation of mass

Wigner phase

Wigner phase: $\mathcal{V}(\Phi)$ has a single minimum, and one gets $\langle \Phi \rangle = 0$

We expect the field Φ not to be able to provide a seed for dynamical $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ symmetry breaking

Nonperturbative terms *(which we later in the Nambu-Goldstone phase will denote by $\underline{\ldots}$)* are not expected to occur in the mixing pattern of the six-dimensional operators in this phase

So, the operator mixings are of the same form as those found in perturbation theory

The squared mass of Φ undergoes additive and multiplicative renormalization:

$$\hat{\mu}_\Phi^2 = Z_{\Phi^\dagger\Phi}^{-1}\left(\mu_0^2 - \frac{\tau}{b^2}\right)$$

where τ is a dimensionless function of g_s^2, λ_0, η and ρ

η_{cr} can only be a function of dimensionless bare parameters, and so it depends on the scalar squared mass only via the quantity $b^2 Z_{\Phi^\dagger\Phi}\hat{\mu}_\Phi^2 = b^2\mu_0^2 - \tau$, a negligible $O(b^2)$ effect

Symmetry enhancement

To understand the details of the $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ symmetry enhancement, let us renormalize the model and study the mixing pattern of the operators appearing in the bare Schwinger-Dyson equations

We look at the mixing pattern of the dimension-six operators

$$O_6^{L\,i} = \frac{1}{2}\rho\Big[\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi\mathcal{D}_\mu Q_R - \text{h.c.}\Big]$$

$$O_6^{R\,i} = \frac{1}{2}\rho\Big[\overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi^\dagger\mathcal{D}_\mu Q_L - \text{h.c.}\Big]$$

Like in Bochicchio et al., the symmetries of $\mathcal{L}_{\rm toy}$ mean that these mix with two $d = 4$ operators, plus a set of six-dimensional ones ($[O_6^{L\,i}]_{sub}$ and $[O_6^{R\,i}]_{sub}$):

$$O_6^{L\,i} = \Big[O_6^{L\,i}\Big]_{sub} + \frac{Z_{\widetilde{J}}-1}{b^2}\partial_\mu \widetilde{J}_\mu^{L\,i} - \frac{\overline{\eta}}{b^2}\Big[\overline{Q}_L \frac{\tau^i}{2}\Phi Q_R - \text{h.c.}\Big] + \underline{\ldots}$$

$$O_6^{R\,i} = \Big[O_6^{R\,i}\Big]_{sub} + \frac{Z_{\widetilde{J}}-1}{b^2}\partial_\mu \widetilde{J}_\mu^{R\,i} - \frac{\overline{\eta}}{b^2}\Big[\overline{Q}_R \frac{\tau^i}{2}\Phi^\dagger Q_L - \text{h.c.}\Big] + \underline{\ldots}$$

The $\underline{\ldots}$ denote possible nonperturbative contributions to the operator mixing

They are actually the main focus here, and we will soon discuss the circumstances of their possible appearance

We do not need to resolve the mixing among the different $d=6$ operators, they only give rise to negligible $O(b^2)$ effects

In deriving these equations the conservation laws $\partial_\mu J_\mu^{L\,i}=0$ and $\partial_\mu J_\mu^{R\,i}=0$ have been used to eliminate from the mixing pattern the purely Φ-dependent operators $\partial_\mu \mathrm{tr}\Big[\Phi^\dagger \frac{\tau^i}{2}\partial_\mu\Phi - (\partial_\mu\Phi^\dagger)\frac{\tau^i}{2}\Phi\Big]$ and $\partial_\mu \mathrm{tr}\Big[(\partial_\mu\Phi^\dagger)\Phi\frac{\tau^i}{2} - \frac{\tau^i}{2}\Phi^\dagger\partial_\mu\Phi\Big]$

For now we are renormalizing the Schwinger-Dyson equations, not the whole model

The critical value of η at which (up to irrelevant terms of order b^2) the transformations $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ become a symmetry of the theory can be consistently determined by imposing that the Schwinger-Dyson equations take the form of renormalized Ward identities of the $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ symmetry:

$$\partial_\mu \langle Z_{\widetilde{J}} \widetilde{J}^{L\,i}_\mu(x)\,\hat{O}(0)\rangle\Big|_{cr} = \langle \widetilde{\Delta}^i_L \hat{O}(0)\rangle\Big|_{cr} \delta(x) + \mathrm{O}(b^2)$$

$$\partial_\mu \langle Z_{\widetilde{J}} \widetilde{J}^{R\,i}_\mu(x)\,\hat{O}(0)\rangle\Big|_{cr} = \langle \widetilde{\Delta}^i_R \hat{O}(0)\rangle\Big|_{cr} \delta(x) + \mathrm{O}(b^2)$$

Now, in Wigner phase, the $\underline{\ldots}$ terms are set to zero

What happens is that taking η equal to the solution of the equation

$$\eta = \overline{\eta}(g_s^2, \rho, \lambda_0, \eta) \implies \eta = \eta_{cr}(g_s^2, \rho, \lambda_0)$$

makes the $\widetilde{\chi}_{L/R}$-variation of the $d=4$ Yukawa term cancel the $d=4$ operator that mixes with $-b^2 O_6^{L/R\,i}$, which is the $\widetilde{\chi}_{L/R}$-variation of the Wilson-like term in the action

As a consequence, in the right hand side of the Ward identities only $d \geq 6$ subtracted operators are left, which contribute irrelevant cutoff artefacts of order b^2

So, if one sets $\eta = \eta_{cr}$ the transformations $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ are promoted to symmetries of the action

This implies that also the transformations $\chi_L^\Phi : \Phi \longrightarrow \Omega_L \Phi$ and $\chi_R^\Phi : \Phi \longrightarrow \Phi \Omega_R^\dagger$ become symmetries of the action (always up to $O(b^2)$ effects): the scalar field actually **decouples** from the fermion and gauge boson degrees of freedom

To this order the corresponding currents $J_\mu^{L\,i} - \widetilde{J}_\mu^{L\,i}$ and $J_\mu^{R\,i} - \widetilde{J}_\mu^{R\,i}$, that involve only scalar fields, are now conserved

In a sense, at η_{cr} the Yukawa and the Wilson-like term compensate each other: the $\widetilde{\chi}_{L/R}$ variation of the Yukawa term cancels the dimension-four operator which mixes with the $\widetilde{\chi}_{L/R}$ variation of the Wilson-like term

Thus: we achieve cancellation of $\widetilde{\chi}$-symmetry breaking effects, and $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ is promoted to a symmetry of the action while the effective Yukawa coupling vanishes (up to irrelevant $O(b^2)$ cutoff artefacts)

Maximal enhancement of the chiral symmetry, charge algebra closure

At η_{cr} the Schwinger-Dyson equations become Ward identities

The $\widetilde{\chi}_L \times \widetilde{\chi}_R$ charge algebra closes, even though in the Nambu-Goldstone phase the corresponding Ward identities are broken by $O(\alpha_s^2 \Lambda_s)$ mass terms of nonperturbative origin

Decoupling of the scalar field:

- no Yukawa coupling in the effective action
- no $m_q \sim \langle \Phi \rangle$
- recovery of $\widetilde{\chi}_{L/R}$ at η_{cr}
- $\widetilde{\chi}_L \otimes \widetilde{\chi}_R = \chi_L \otimes \chi_R$ in correlators with quarks and gluons

That the scalars are completely decoupled from fermions and gluons dictates the form of the local part of the 1PI effective Lagrangian of the theory in the Wigner phase ($\hat{\mu}_\Phi^2 > 0$):

$$L_4^{Wig} = \frac{1}{4}(F \cdot F) + \overline{Q}_L \not{D} Q_L + \overline{Q}_R \not{D} Q_R$$

$$+\frac{1}{2}\mathrm{tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \frac{\hat{\mu}_\phi^2}{2}\mathrm{tr}\left[\Phi^\dagger \Phi\right] + \frac{\hat{\lambda}}{4}\left(\mathrm{tr}\left[\Phi^\dagger \Phi\right]\right)^2$$

Nambu-Goldstone phase

What are the physical properties of the model in the Nambu-Goldstone phase with the dimensionless Yukawa coupling kept at the critical value η_{cr}?

The $\chi_L \otimes \chi_R$ symmetry is reduced to its diagonal subgroup χ_V

Expand the scalar field around its vev: $\Phi(x) = (v + \sigma(x))1\!\!1_{2x2} + i\vec{\pi}(x) \cdot \vec{\tau}$, where $\vec{\pi}$ is a triplet of massless pseudoscalar Nambu-Goldstone bosons and σ is a scalar of mass m_σ of order $\langle\Phi\rangle$

Ignoring the fluctuations of Φ around its vev, the $d = 6$ term $\mathcal{L}_{Wil}$ with $b^2 v \to ar$ looks very much like the $d = 5$ Wilson term of Wilson fermions

We may then expect that the residual $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ breaking terms left-over at η_{cr}, where $\mathcal{L}_{Wil}$ is (partially) compensated by $\mathcal{L}_{Yuk}$, will trigger the phenomenon of dynamical chiral symmetry breaking

. . . just as it happens with Wilson fermions, where the (critical) Wilson term can trigger the chiral symmetry breaking

Indeed in this familiar case, since there is a residual explicit $O(a)$ breaking of chirality, we know that the phenomenon of spontaneous chiral symmetry breaking occurs even when m_0 is set at m_{cr} and the Wilson term gets (partially) compensated by the mass term

So, the $\widetilde{\chi}$ symmetry is dynamically broken by a nonperturbative mechanism (analogous to the generation of $c_1 \Lambda_{QCD}$ for Wilson fermions)

The new crucial thing with respect to Wilson QCD: now a new operator of dimension three appears in the renormalized Ward identities, which comes multiplied by a well-defined and naturally light fermion mass

The residual chiral breaking terms trigger the dynamical spontaneous breaking of the recovered chiral symmetry $\rightarrow$ nonperturbative finite mass (up to logs)

The fermion mass, of dynamical origin, is proportional to Λ_s and independent of the expectation value of the scalar field

It is because at the critical Yukawa coupling η_{cr} the fermion chiral transformations $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ become a symmetry of the theory that the dynamically generated mass is naturally light, and unrelated to the value of $\langle \Phi \rangle$

Also, the value of η_{cr} does not depend on the mass of the scalar field

We can use in the Nambu-Goldstone phase the same value of η_{cr} which has been previously determined in the Wigner phase

Toy model: after compensation of the Yukawa and Wilson-like terms, the residual $\widetilde{\chi}$-breaking contributions of $O(b^2\langle\Phi\rangle)$ induce a spontaneous $\widetilde{\chi}$-symmetry breaking and dynamically generate the nonperturbative mass

A nonperturbative mass operator appears in the effective action reflecting the spontaneous symmetry breaking of $\widetilde{\chi}_{L/R}$

We stress again that in the toy model the relevant chiral symmetry which is dynamically broken (triggered by the Wilson-like term) is $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$

Conjecture: $\widetilde{\chi}$-symmetry breaking induces nonperturbative vertex corrections, and in this way and a nonperturbative fermion mass emerges (which must be proportional to Λ_s)

Value of this mass: numerically unrelated to the expectation value of the scalar field – proportional to the Λ_{RGI} of the strong interactions of the fermions

The value of the mass depends on the details of the ultraviolet regularization of the model: universality breaking at the nonperturbative level

Symanzik expansion

To determine the properties of the critical theory in the Nambu-Goldstone phase, we study how nonperturbative terms coming from dynamical chiral symmetry breaking can affect correlators and in particular the building blocks that enter the quark self-energy diagrams

Let us look at the small b expansion of the gluon-gluon-scalar, $Q_{L/R}$-$\overline{Q}_{L/R}$-scalar, $Q_{L/R}$-$\overline{Q}_{L/R}$-gluon-scalar correlators that take the form

$$\langle O(x,x',\ldots)\rangle\Big|^{R} = \langle O(x,x',\ldots)\rangle\Big|^{F}$$

$$-b^2\langle O(x,x',\ldots)\int d^4z\,[\,L_6^{\widetilde{\chi}} + L_6^{\widetilde{\chi}}\,](z)\rangle\Big|^{F} + \mathrm{O}(b^4)$$

$$O(x,x',\ldots) \quad \Leftrightarrow \quad A_\mu^b(x)A_\nu^c(x')\sigma(y)\,,\; Q_{L/R}(x)\overline{Q}_{L/R}(x')\sigma(y)$$

$$Q_{L/R}(x)\overline{Q}_{L/R}(x')\sigma(y)A_\mu^b(y')\,\underline{\ldots}$$

$L_6^{\widetilde{\chi}}$ is the $d=6$ $\widetilde{\chi}$-breaking operator, and $L_6^{\widetilde{\chi}}$ the $d=6$ $\widetilde{\chi}$-conserving one

$|^{R}$ means that expectation values are taken in the ultraviolet regulated theory, $|^{F}$ that expectation values are taken in the formal theory

The renormalized correlation functions have a Symanzik-like expansion with only even powers of b

Terms odd in b are excluded because the Lagrangian has also the discrete symmetry $\mathcal{D}_d$, consisting in multiplying each field (of naive dimension d) by $e^{i\pi d} = (-1)^d$ and changing sign to its space-time argument

Like for Wilson fermions, the $O(b^2)$ terms with the insertion of $L_6^{\widetilde{\chi}}$ would vanish were it not for the nonperturbative phenomenon of dynamical chiral symmetry breaking

The resulting nonperturbative contributions to the gluon-gluon-scalar, $Q_{L/R}$-$\overline{Q}_{L/R}$-scalar, $Q_{L/R}$-$\overline{Q}_{L/R}$-gluon-scalar vertices will have the form

$$\Delta\Gamma^{bc\,\mu\nu}_{AA\Phi}(k,\ell)\Big|^{R} = b^2\Lambda_s\alpha_s\,\frac{\delta^{bc}}{2}T_{\mu\nu}\,F_{AA\Phi}\Big(\frac{\Lambda_s^2}{\mathrm{mom}^2}\Big)$$

$$\Delta\Gamma_{Q\overline{Q}\Phi}(k,\ell)\Big|^{R} = b^2\Lambda_s\,\alpha_s\,\frac{i}{2}\gamma_\mu(2k+\ell)_\mu\,F_{Q\overline{Q}\Phi}\Big(\frac{\Lambda_s^2}{\mathrm{mom}^2}\Big)$$

$$\Delta\Gamma^{b,\mu}_{Q\overline{Q}A\Phi}(k,\ell,\ell')\Big|^{R} = b^2\Lambda_s\,\alpha_s\,ig_s\lambda^b\gamma_\mu\,F_{Q\overline{Q}A\Phi}\Big(\frac{\Lambda_s^2}{\mathrm{mom}^2}\Big)$$

where $T_{\mu\nu} = [k(k+\ell)\delta_{\mu\nu} - k_\mu(k+\ell)_\nu] + [\mu \to \nu]$

and mom stands for any one of the kinematically appropriate momenta in the set $\{k,\ell,\ell',...,\ell'+\ell,k+\ell\}$

Similarly to Wilson fermions, the factor α_s comes from the fact that the quark or gluon line emitted from the $L_6^{\widetilde{\chi}}$ vertex has to be absorbed somewhere in the diagram

Like for Wilson fermions, we assume that the nonperturbative effects persist up to $\mathrm{mom}^2 = O(b^{-2})$, and conjecture the asymptotic behavior

$$F_{AA\Phi}\left(\frac{\Lambda_s^2}{\mathrm{mom}^2}\right) \stackrel{\mathrm{mom}^2\to\infty}{\longrightarrow} H_{AA}$$

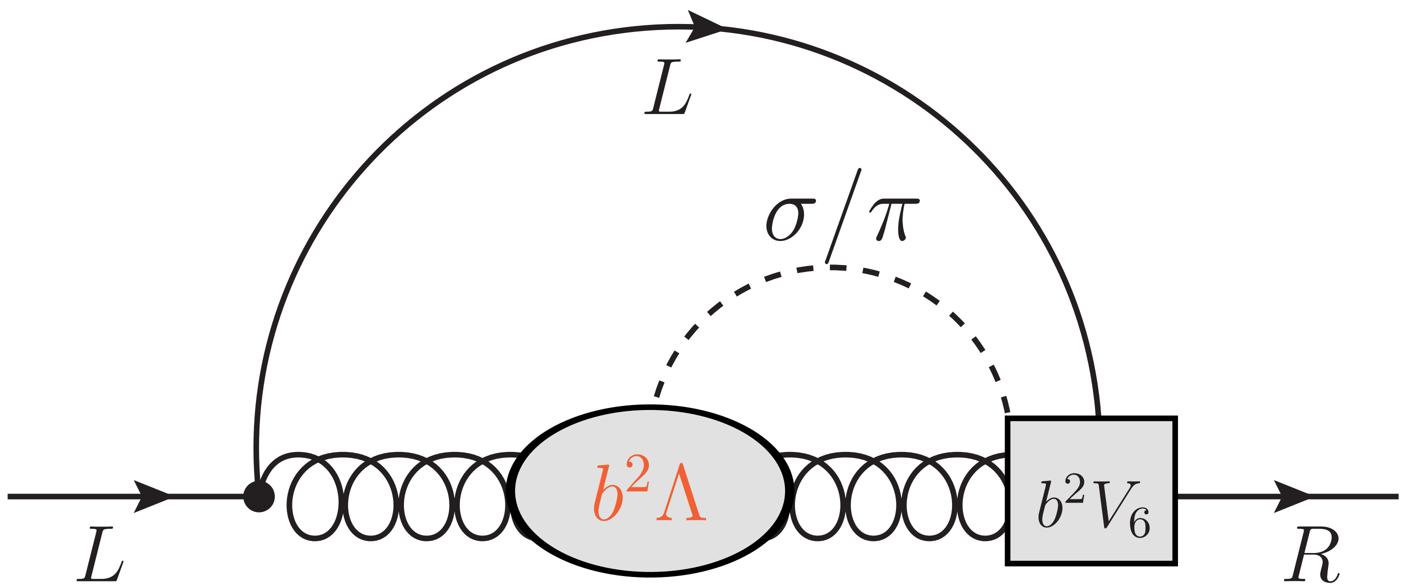
$$F_{Q\overline{Q}\Phi}\left(\frac{\Lambda_s^2}{\mathrm{mom}^2}\right) \stackrel{\mathrm{mom}^2\to\infty}{\longrightarrow} H_{Q\overline{Q}}$$

$$F_{Q\overline{Q}A\Phi}\left(\frac{\Lambda_s^2}{\mathrm{mom}^2}\right) \stackrel{\mathrm{mom}^2\to\infty}{\longrightarrow} H_{Q\overline{Q}}$$

where H_{AA} and $H_{Q\overline{Q}}$ are $O(1)$ constants, and the last two limits are related by gauge invariance

With these building blocks, we can now see how finite $O(g_s^2\alpha_s\Lambda_s)$ nonperturbative contributions to the fermion mass can arise

We look at some self-energy diagrams, for example



The finiteness of these contributions can be understood from a counting of loop momenta in the graphs

Neglecting external (compared to loop) momenta, one finds a double integral with factors $1/k^2$ and $1/(\ell^2 + m_\sigma^2)$ from the standard gluon and σ propagators, the factors $\gamma_\mu k_\mu / k^2$ and $\gamma_\nu (k+\ell)_\nu / (k+\ell)^2$ for the quark propagators, a factor $b^2 (k+\ell)_\lambda$ from the derivative coupling in $\mathcal{L}_{Wil}$, and a factor $b^2 \alpha_s \Lambda_s (2k+\ell)_\rho \gamma_\rho$ from the nonperturbative vertex $\Delta\Gamma_{Q\overline{Q}\Phi}(k,\ell)|^R$

Putting everything together, one gets in the $b \to 0$ limit a finite fermion mass term of the order

$$b^4 g_s^2 \alpha_s \Lambda_s \int^{1/b} d^4k \int^{1/b} d^4\ell \, \frac{1}{k^2} \gamma_\lambda \frac{\gamma_\mu k_\mu}{k^2} \frac{(2k+\ell)_\rho \gamma_\rho}{\ell^2 + m_\sigma^2} \frac{\gamma_\nu (k+\ell)_\nu}{(k+\ell)^2} (k+\ell)_\lambda$$

$$\sim g_s^2 \alpha_s \Lambda_s$$

The overall b^4 multiplicative factor is compensated by the quartic divergence of the two-loop integrals

The diagrams in the figure represent a subset of all the lowest order terms contributing to the fermion self-energy – they are the ones where only one σ propagator appears

To the same lowest order in g_s^2 there are infinitely many other contributions coming from diagrams that take into account the self-interaction of the Φ field and include in general scalar (σ and π) loops

The noperturbative mass

What form will have a nonperturbative mass term of such kind?

To be able to interpret the finite term as a quark mass the following must happen:

- No extra quark mass of order $v = \langle \Phi \rangle$ is left over as a consequence of the Higgs mechanism – a term of this kind would completely obscure our nonperturbative contribution in case $v \gg \Lambda_s$, or make it of little interest for predicting the value of the quark mass in case $v \sim \Lambda_s$
- The nonperturbative mass term must be $\chi_L \otimes \chi_R$ invariant, and have the correct symmetry properties so that it can appear in the effective Lagrangian in the Nambu-Goldstone phase
- The nonperturbative mass term is renormalization scale independent and its chiral variation can be accommodated in the right hand side of the restored $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ Ward identities

As for the first point, in the Nambu-Goldstone phase the equation determining η_{cr} becomes just a condition for the cancellation of the $v(\overline{Q}_R Q_L + \overline{Q}_L Q_R)$ quark mass term

The other points go as follows:

The exact $\chi_L \otimes \chi_R$ invariance forbids a mass term of the kind $\overline{Q}_R Q_L + \overline{Q}_L Q_R$ in the effective Lagrangian

So, how can we get a mass in this model?

We need to introduce the field

$$U = \frac{\Phi}{\sqrt{\Phi^\dagger \Phi}}$$

This is a dimensionless **non-analytic** function of Φ (its phase), with the same transformation properties under $\chi_L \otimes \chi_R$

It cannot be defined in the Wigner phase: we can have it only if $\langle \Phi \rangle \neq 0$

One can always write $U = \text{sign}\,(v+\sigma)\exp\,(i\vec{\tau}\vec{\zeta}/v)$, with $\vec{\zeta} = \vec{\pi}\,[1 + \mathrm{O}(\sigma/v, \vec{\tau}\vec{\pi}/v)]$

We have the expansion

$$U = \frac{\Phi}{\sqrt{\Phi^\dagger \Phi}} = \frac{(v + \sigma + i\vec{\tau}\cdot\vec{\varphi})}{\sqrt{(v+\sigma)^2 + \vec{\varphi}\cdot\vec{\varphi}}} = 1\!\!1 + i\frac{\vec{\tau}\vec{\varphi}}{v} + O(\frac{\sigma^2}{v^2}, \frac{\pi^2}{v^2})$$

Now we have all the ingredients to build a nonperturbative mass term which is also $\chi_L \otimes \chi_R$ invariant in the effective action density:

$$\Gamma^{NG}_{\rm loc} = L_4^{Wig}\Big|_{\hat\mu^2_\Phi<0} + C_2\,\Lambda_s^2\,\text{tr}\,[\partial_\mu U^\dagger \partial_\mu U] + C_1\,\Lambda_s\,[\overline{Q}_L U Q_R + \overline{Q}_R U^\dagger Q_L]$$

We see that there is also a kinetic term for the non-linear field U, which cannot be excluded on the basis of symmetry considerations

The appearance of the non-analytic field U is not surprising

In QCD nonperturbative effects like the nonvanishing of the chiral condensate are proportional to the sign of m_q – the sign of the coefficient of the chiral breaking term in the action

In lattice QCD at $m_0 = m_{cr}$, the seed for spontaneous chiral symmetry breaking is instead provided by the (critical) Wilson term – and so such nonperturbative effects are proportional to the sign of the Wilson coefficient r

We can see the toy model as an action where the Wilson coefficient has been elevated to a dynamical field, Φ

Indeed, the dynamically generated nonperturbative quark mass turns out to be proportional to U, the phase of Φ

The emergence of a nonperturbative mass term in the $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ Ward identities is a consequence of the quadratically divergent mixing of the dimension-six operators $O_6^{L\,i}$ and $O_6^{R\,i}$ with the nonperturbatively generated operators

$$C_1 \Lambda_s \Big(\overline{Q}_L \frac{\tau^i}{2} U Q_R - \text{h.c.} \Big)\,, \qquad C_1 \Lambda_s \Big(\overline{Q}_R \frac{\tau^i}{2} U^\dagger Q_L - \text{h.c.} \Big)$$

This is precisely the possible nonperturbative mixing which was alluded to by the ellipses $\underline{\ldots}$ before

Indeed, owing to $\chi_L \otimes \chi_R$ and other obvious symmetries, at $\eta = \eta_{cr}$ the renormalized Ward identities associated to the $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ transformations are conjectured to take the form

$$\partial_\mu \langle Z_{\widetilde{J}} \widetilde{J}_\mu^{L\,i}(x)\, \hat{O}(0) \rangle\Big|_{cr} = \langle \widetilde{\Delta}_L^i \hat{O}(0) \rangle\Big|_{cr} \delta(x)$$

$$+ C_1 \Lambda_s \langle (\overline{Q}_L \frac{\tau^i}{2} U Q_R - \text{h.c.})\, \hat{O}(0) \rangle\Big|_{cr} + \mathrm{O}(b^2)$$

$$\partial_\mu \langle Z_{\widetilde{J}} \widetilde{J}_\mu^{R\,i}(x)\, \hat{O}(0) \rangle\Big|_{cr} = \langle \widetilde{\Delta}_R^i \hat{O}(0) \rangle\Big|_{cr} \delta(x)$$

$$+ C_1 \Lambda_s \langle (\overline{Q}_R \frac{\tau^i}{2} U^\dagger Q_L - \text{h.c.})\, \hat{O}(0) \rangle\Big|_{cr} + \mathrm{O}(b^2)$$

This shows that in the critical theory, consistently with the form of the effective Lagrangian, $\Gamma^{NG}_{\rm loc}$, in the right hand side of these Ward identities a quark mass term occurs that is proportional to Λ_s, and not to the scalar field vev, $v = \langle\Phi\rangle$

To leading order in the gauge coupling we get

$$m_Q^{dyn}\Big|_{\rm LO} = C_1\Big|_{\rm LO}\Lambda_s = k_{\rm LO}\, g_s^2\alpha_s\,\Lambda_s$$

No unnatural fine tuning

For Wilson fermions instead this fine tuning procedure is not natural nor well defined

Since the $\widetilde{\chi}$-currents $Z_{\widetilde{J}}\widetilde{J}^{L\,i}_{\mu}$ and $Z_{\widetilde{J}}\widetilde{J}^{R\,i}_{\mu}$ are ultraviolet-finite (they are conserved up to $O(b^2)$ in the Wigner phase), to interpret the coefficient $C_1\Lambda_s$ as a mass this quantity must be renormalized by the inverse of the renormalization constant of the operators $\overline{Q}_L\frac{\tau^i}{2}UQ_R$

Final comments

There are two important scales here: Λ_s and $\langle\Phi\rangle$

Natural choice: $\Lambda_s \ll \langle\Phi\rangle \ll b^{-1}$

If instead $\Lambda_s \sim \langle\Phi\rangle$, we would be back to the situation where fermion masses are of the order of $\langle\Phi\rangle$, like in the Standard Model

The physics of the critical model at energies below $\langle\Phi\rangle$ is independent of $\langle\Phi\rangle$, because the σ particle which has a square mass $m_\sigma^2 \sim \hat{\lambda}\langle\Phi\rangle^2$ decouples

The condition $\langle\Phi\rangle \ll b^{-1}$ is needed to guarantee the independence of η_{cr} on the value of $\hat{\mu}_\Phi^2$ (and its sign) – so the step of $\widetilde{\chi}_L \times \widetilde{\chi}_R$ symmetry restoration, essential to solve the naturalness problem, is unambiguous

Nonperturbative fermion mass $\sim \Lambda_s$: natural and independent of $\langle\Phi\rangle$ if $\Lambda_s \ll \langle\Phi\rangle$

The dynamically generated fermion mass are much smaller than $\langle\Phi\rangle$ because at the critical Yukawa coupling η_{cr} the fermion chiral $\widetilde{\chi}_L \times \widetilde{\chi}_R$ transformations become a symmetry of the theory

The cancellation of the large quark mass term of order $\langle\Phi\rangle$ is guaranteed by the tuning of η

This nonperturbative mass generation mechanism fulfills ’t Hooft naturalness requirement: tuning η to η_{cr} enlarges (even in the Nambu-Goldstone phase) the symmetries of the theory

Indeed $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$, which acts only on fermions, is promoted to a symmetry

There are two sets of Goldstone bosons here, related to the two kinds of spontaneous chiral symmetry breaking occurring in this model:

- the spontaneous breaking of the exact $\chi_L \otimes \chi_R$ symmetry induced by $\langle \Phi \rangle \neq 0$

  (…and when χ_L is gauged to generate the electroweak interactions, these Goldstone bosons will become the longitudinal modes of the electroweak gauge bosons)

  (Electroweak extension: add W_μ, which transforms as $W_\mu \longrightarrow \Omega_L W_\mu \Omega_L^\dagger$)

- the dynamical breaking of the $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ symmetry that is restored at η_{cr}

  at variance with lattice QCD, the dynamically generated fermion mass is here of $O(\Lambda_s)$, and then the squared mass of the pseudoscalar mesons are of $O(\Lambda_s^2)$, and so comparable to that of other hadrons

Numerical studies on the lattice

We want to verify numerically if such nonperturbative mass terms are indeed present in the toy model

For this purpose a specific lattice ultraviolet regularization of the model must be adopted

Lattice simulations of this toy model are technically quite challenging: it contains fermions, gauge fields and scalars, all together

The first numerical study of models of this kind in the strong interaction regime

The mechanism for mass generation works also in the quenched approximation

Indeed, the diagrams that we have discussed before are still present in the quenched approximation

This is quite convenient because then scalar and gauge field configurations can be generated independently from each other:

$$Z = \int \mathcal{D}\Phi\mathcal{D}U\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\, e^{S[U]+S[\phi]} \det\left(D_{latt}[U,\phi]\right)$$

and then if $\det\left(D_{latt}[U,\phi]\right) = 1$ one has $Z = Z[\Phi]Z[U]$

The Wilson-like term is of dimension six and naively of order b^2, and this term cannot remove the doublers, and they do not decouple

Naive fermions, cheap and chirally invariant

2 flavors (u,d) times 16 doublers, even in the $b \to 0$ limit

This is fine for quenched studies, but staggered or overlap or doman-wall fermions will be needed in unquenched simulations

Quenched $\to$ exceptional configurations with spinors zero modes, which at large $|\eta|$ and $|\rho|$ are enhanced by the fluctuations of $\Phi \to$ add a twisted mass term $i\mu_Q b^4 \sum_x \overline{\psi}(x)\gamma_5\tau^3\psi(x)$, and at the end extrapolate to $\mu_Q = 0$

A theoretically clean IR cutoff, with a harmless soft breaking of $\chi_L \otimes \chi_R$ (and of $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ when restored)

In order to improve the signal, in $\overline{\psi} D_{lat} \psi$ we use a locally smeared Φ to reduce the noise: $D_{lat}[U, \Phi_{smeared}]$

The scalar field is replaced with its average over the Φ values at the sites corresponding to the 16 vertices of the hypercube of side $2b$ centered in x

Lattice regularization of the toy model:

$$S_{lat} = b^4 \sum_x \left\{ \mathcal{L}_{kin}^{YM}[U] + \mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) + \overline{\Psi} D_{lat}[U,\Phi]\Psi \right\}$$

$\mathcal{L}_{kin}^{YM}[U]$: SU(3) plaquette action

$$\mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) = \frac{1}{2}\,\mathrm{tr}[\Phi^\dagger(-\partial_\mu^*\partial_\mu)\Phi] + \frac{\mu_0^2}{2}\,\mathrm{tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{tr}\left[\Phi^\dagger\Phi\right]\right)^2$$

where $\Phi = \varphi_0 \mathbb{1} + i\varphi_j\tau^j$, and the Dirac operator is

$$\begin{aligned}(D_{lat}[U,\Phi]\Psi)(x) \quad &= \quad \gamma_\mu \widetilde{\nabla}_\mu \Psi(x) + \eta F(x)\Psi(x) - \frac{b^2}{2}\rho\, F(x)\widetilde{\nabla}_\mu\widetilde{\nabla}_\mu\Psi(x) \\ & -\frac{b^2}{4}\rho\left[(\partial_\mu F)(x)U_\mu(x)\widetilde{\nabla}_\mu\Psi(x+\hat{\mu}) + (\partial_\mu^* F)(x)U_\mu^\dagger(x-\hat{\mu})\widetilde{\nabla}_\mu\Psi(x-\hat{\mu})\right]\end{aligned}$$

with $F(x) \equiv [\varphi_0\mathbb{1} + i\gamma_5\tau^j\varphi_j](x)$

The lattice derivatives are

$$\nabla_\mu f(x) \equiv \frac{1}{b}(U_\mu(x)f(x+\hat{\mu}) - f(x)), \quad \nabla_\mu^* f(x) \equiv \frac{1}{b}(f(x) - U_\mu^\dagger(x-\hat{\mu})^f(x-\hat{\mu}))$$

$$\widetilde{\nabla}_\mu f(x) \equiv \frac{1}{2}(\nabla_\mu + \nabla_\mu^*)F(x)$$

With naive fermions one has to use symmetric derivatives in the action

Then the action is invariant under the spectrum doubling symmetry

$$\Psi(x) \to \Psi'(x) = e^{-ix\cdot\pi_H} M_H \Psi(x) \qquad \overline{\Psi}(x) \to \overline{\Psi}'(x) = \overline{\Psi}(x) M_H^\dagger e^{ix\cdot\pi_H}$$

which in momentum space exchanges the corners of the Brillouin zone

(H is an ordered set of four-vector indices $\{\mu_1, ..., \mu_h\}$, $(\mu_1 < \mu_2 < ... < \mu_h)$; for $0 \le h \le 4$ there are 16 four-vectors π_H with $\pi_{H,\mu} = \pi$ if $\mu \in H$, otherwise $\pi_{H,\mu} = 0$ and 16 matrices $M_H \equiv (i\gamma_5\gamma_{\mu_1})...(i\gamma_5\gamma_{\mu_h})$)

With symmetric derivatives, at tree level the Wilson-like term contribute only $O(b^2)$ effects

Beyond tree level the renormalization is like it would be in the $\rho = 0$ case

The spectrum doubling symmetry guarantees the equivalence between the various doublers – and that all of them have the same η_{cr}

So, at this η_{cr} the $\widetilde{\chi}$ symmetry gets simultaneously restored for all doublers

η_{cr} is well defined even when doublers are present, and independent from μ^2_{sub}, and so has the same value in the Wigner and Nambu-Goldstone phases

Our simulations:

- $\beta = 6/g_0^2 = 5.85$
- $b = 0.12$ fm
- $b\Lambda_s \sim 0.1$
- $b^2\mu_{sub}^2 = b^2(\mu^2 - \mu_{cr}^2) = 0.074(1) > 0$
- $\lambda_0 \sim 0.592$
- $\rho = 1.96$
- $16^3 \times 40$ sites
- 750 – 1400 gauge-scalar configurations

  (30 gauge configurations times 24 scalar configurations, and then 60 gauge configurations times 24 scalar configurations)

Simulations are done at several values of the Yukawa coupling η and of the twisted mass m_Q

The Wilson-like coupling ρ is not important if we are interested only to verify the mechanism – but is relevant for the value of the nonperturbative mass

D_{lat} is not γ_5-hermitian, because Φ is not constant – one needs then to invert D_{lat} and $D_{lat}^\dagger$ to get forward and backward quark propagators

To determine η_{cr}, compute, in the Wigner phase,

$$C_{\widetilde{J}\widetilde{D}}(x-y) \equiv \langle \widetilde{J}_0^{V3}(x)\widetilde{D}^{S3}(y)\rangle$$

where

$$\begin{aligned}
\widetilde{J}_0^{V3}(x) &= \widetilde{J}_0^{L3}(x) + \widetilde{J}_0^{R3}(x) \\
\widetilde{J}_0^{L/R\ 3}(x) &= \frac{1}{2}\Big[\overline{Q}_{L/R}(x-\hat{0})\gamma_0\frac{\tau_3}{2}U_0(x-\hat{0})Q_{L/R}(x) \\
&\qquad +\overline{Q}_{L/R}(x)\gamma_0\frac{\tau_3}{2}U_0^\dagger(x-\hat{0})Q_{L/R}(x-\hat{0})\Big] \\
\widetilde{D}^{S3}(y) &= \overline{Q}_L(y)\left[\Phi,\frac{\tau_3}{2}\right]Q_R(y) - \overline{Q}_R(y)\left[\frac{\tau_3}{2},\Phi^\dagger\right]Q_L(y)
\end{aligned}$$

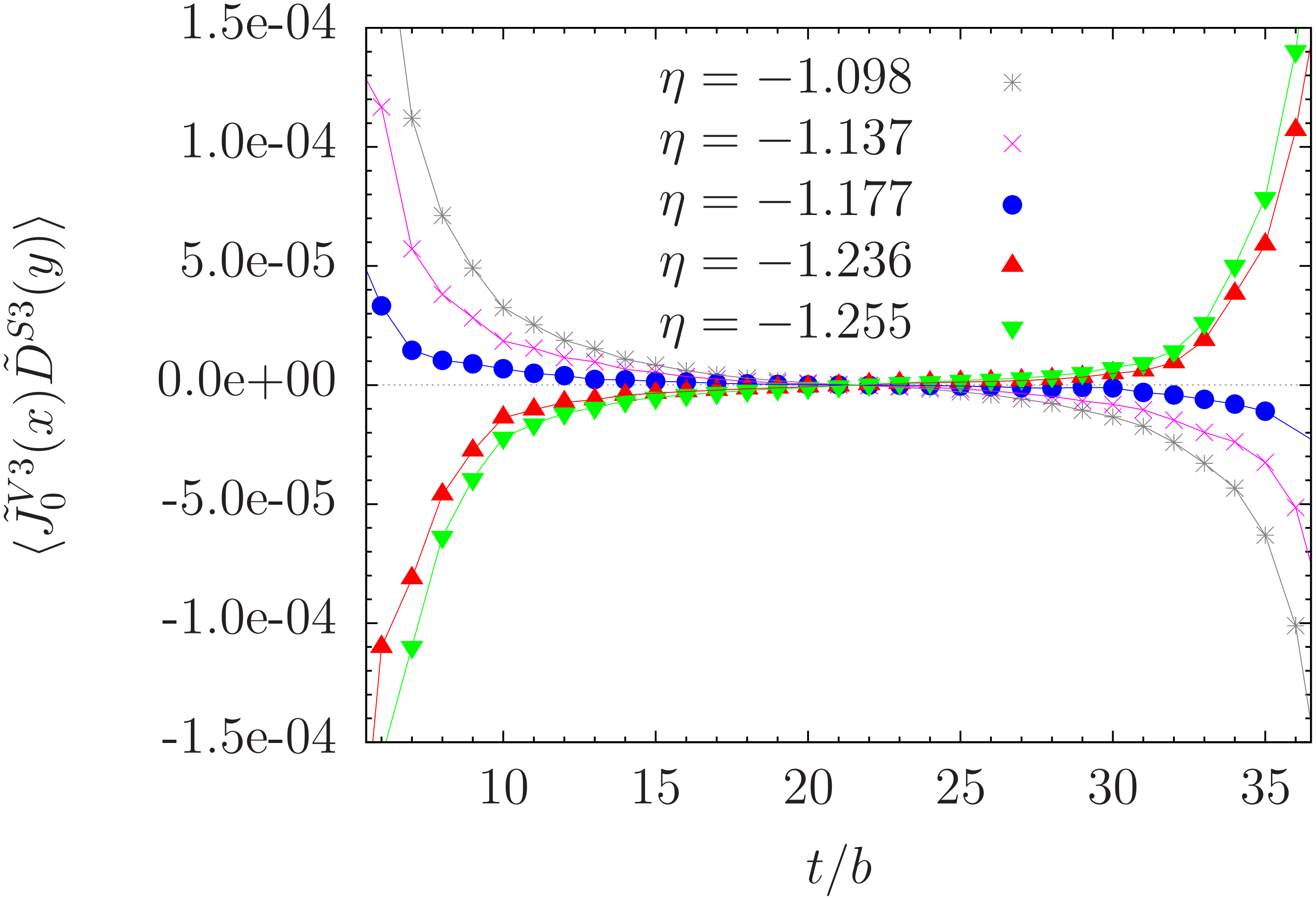
We have $\langle 0|\partial_0\widetilde{J}_0^{V3}|M_S\rangle \simeq f_{M_S}M_{M_S}^2$

When, varying η, this matrix element becomes zero at some particular η_{cr}, then the $\widetilde{\chi}$ symmetry is restored: $\partial_\mu\widetilde{J}_\mu^{V3} = (\eta-\eta_{cr})\widetilde{D}^{S3} + O(b^2)$

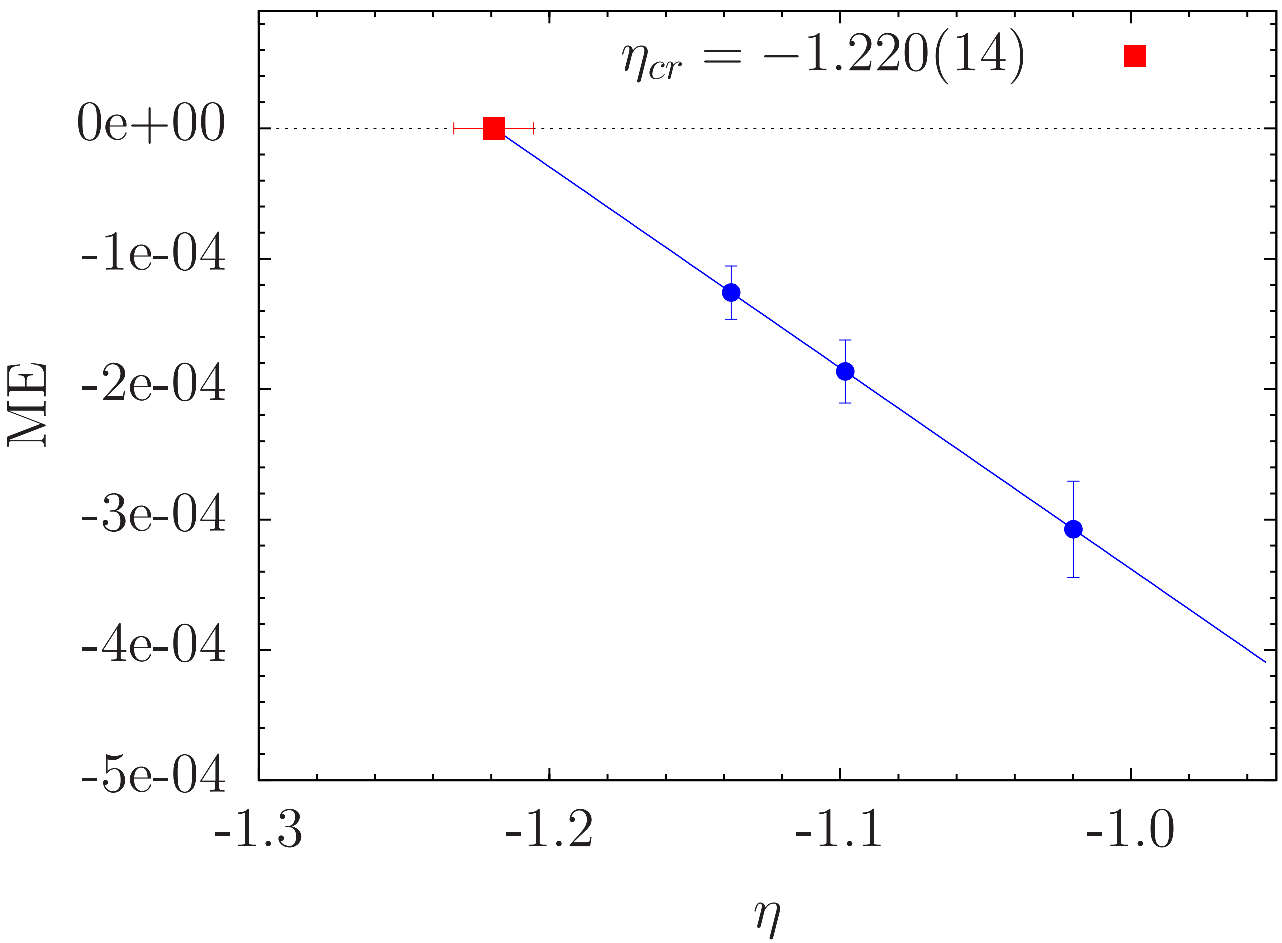
$\Rightarrow$ (preliminary)

$$\eta_{cr} = -1.220(14)$$

Example at $am_Q = 0.0224$:



After having extrapolated to $m_Q = 0$ the results at each value of η, one looks for the value η_{cr} at which the matrix element $\langle \widetilde{J}_0^{V3}(x)\widetilde{D}^{S3}(y)\rangle$ vanishes:



Now go to the Nambu-Goldstone phase: $b^2\mu^2_{sub} = -0.045(1) < 0$

All the other lattice parameters are kept the same as in the Wigner phase

For a generic values of η in this phase there are two kind of operators which break the $\widetilde{\chi}$ symmetry:

$$\Gamma^{NG} = \cdots + (\eta - \eta_{cr})(\overline{Q}_L \Phi Q_R - h.c.) + c_1 \Lambda_S (\overline{Q}_L U \Phi Q_R - h.c.)$$

Determine M_{PS} (from $\langle PP \rangle$ correlation functions, for example), and

$$m^{WTI} \equiv \frac{\langle 0 | \partial_0 \widetilde{J}_0^{A^\pm} | M_{PS\pm} \rangle}{\langle 0 | P^\pm | M_{PS\pm} \rangle} = (\eta - \eta_{cr}) v + C_1 \Lambda_s$$

which at η_{cr} gives a measure of the nonperturbative $\widetilde{\chi}$ breaking

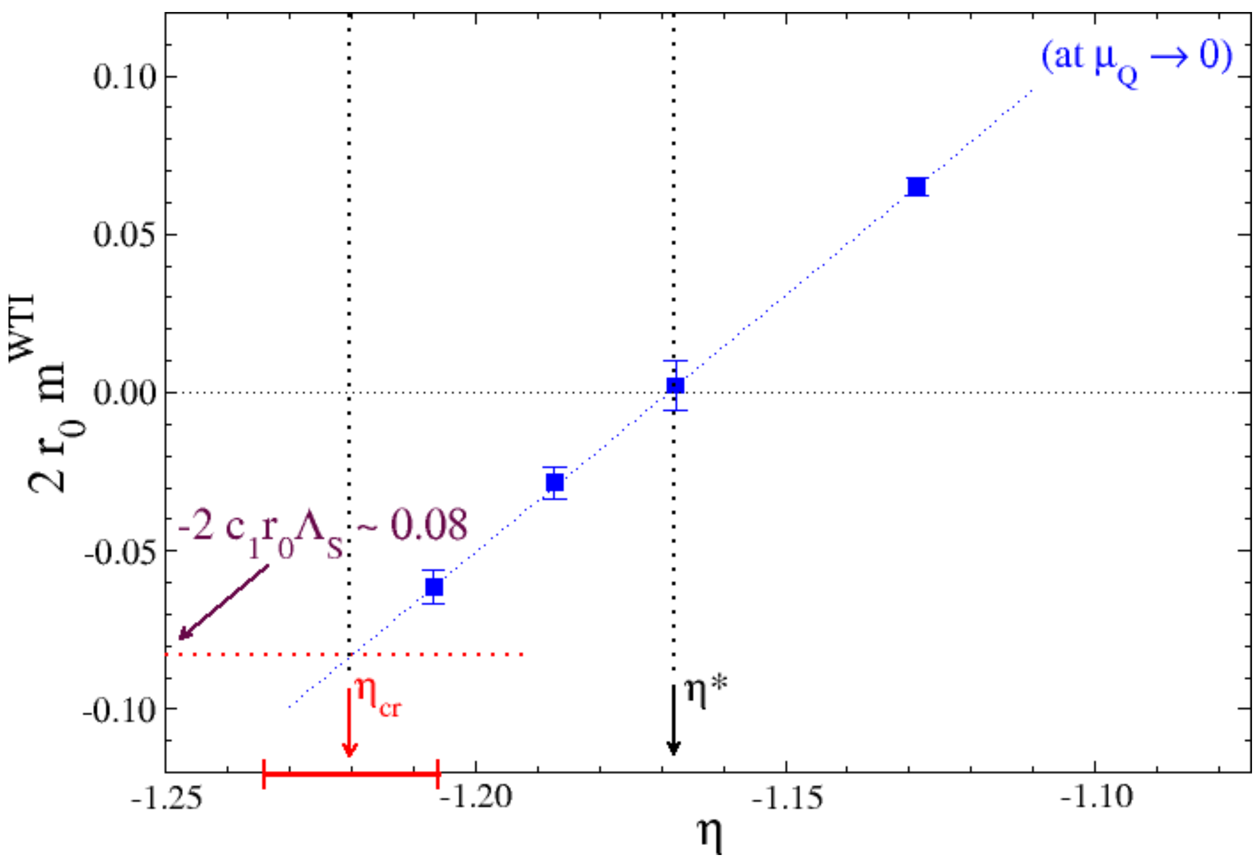
Neither M_{PS} nor m^{WTI} vanish at η_{cr}

The $\widetilde{\chi}$ breaking effects are miminal at a special value

$$\eta^* = \eta_{cr} - c_1 \Lambda_S \frac{1}{v}$$

We find strong indications that indeed $\eta^* \neq \eta_{cr}$

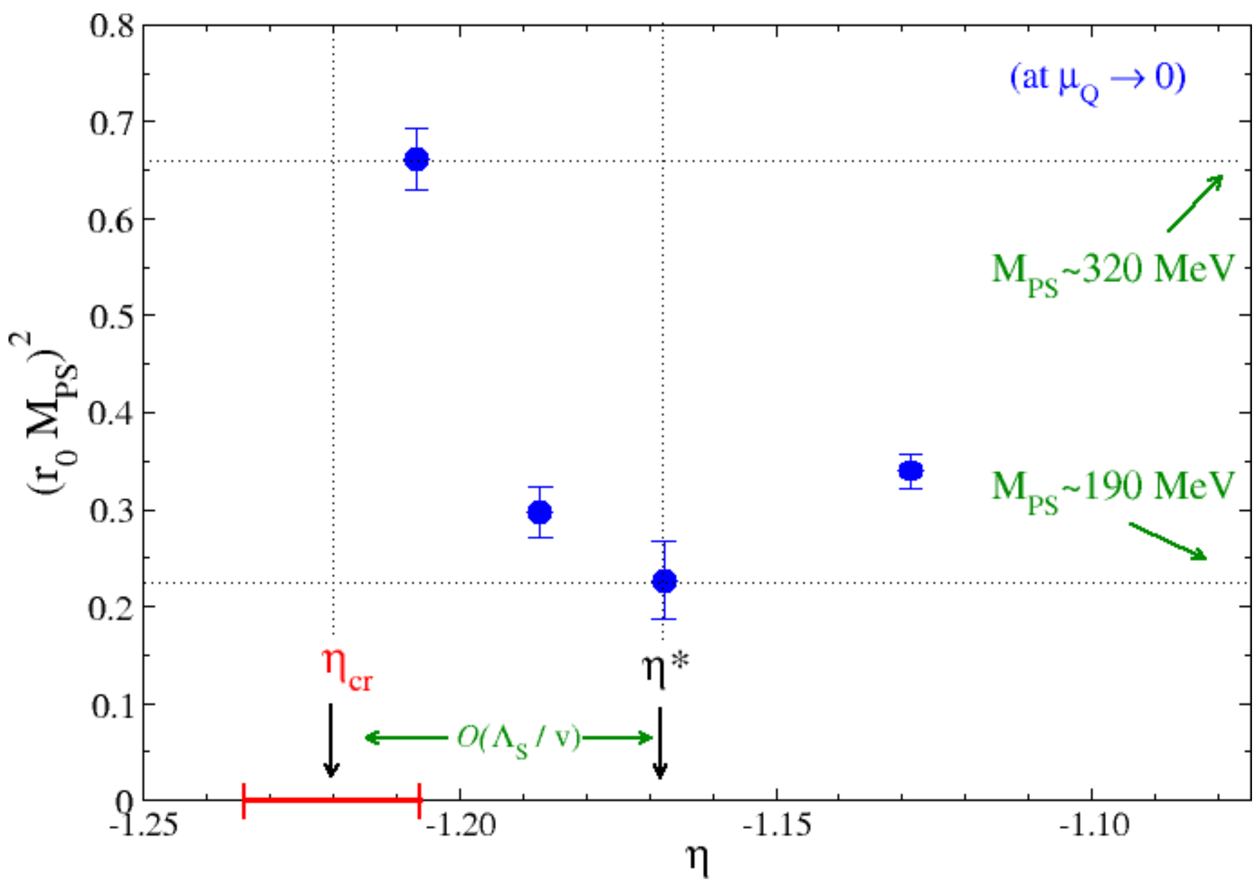
This means $c_1 \neq 0$: **we have observed mass generation!**
(at the moment, only at this value of β)



η^* minimizes the pion mass, but still this mass does not vanish

loop effects coming from the effective action, or lattice artefacts

Preliminary results at $\eta = \eta_{cr}$ for $\beta = 5.85$: $M_{PS} \sim 400\ MeV$ and $m_{bare}^{WTI} \sim 16\ MeV$



A hierarchy of masses: superstrong interactions

Extend the toy model by introducing in a gauge invariant way superstrongly interacting particles with a RGI scale $\Lambda_T \gg \Lambda_{QCD}$

Besides ordinary quarks, an extra family of fermions subjected to ordinary Yang-Mills forces with gauge coupling g_s as well as superstrong vector gauge interactions with gauge coupling g_T

Under the exact $\chi_L \times \chi_R$ symmetries scalars, quarks and superstrongly interacting fermions are simultaneously transformed

Then an interesting ordering of fermion masses emerges

Both quarks and superstrongly interacting fermions get a mass of the order of Λ_T, multiplied by powers of the strong (g_s) and superstrong (g_T) gauge coupling respectively

The difference in the strength of the two interactions is then reflected in the fact that the top quark mass is a fraction of the large scale Λ_T

From the experimental values of the masses, Λ_T must be of about a few TeV

Also the weak bosons acquire a mass proportional to Λ_T

Weak gauge bosons and charged leptons masses are scaled by powers of the electroweak gauge coupling constants

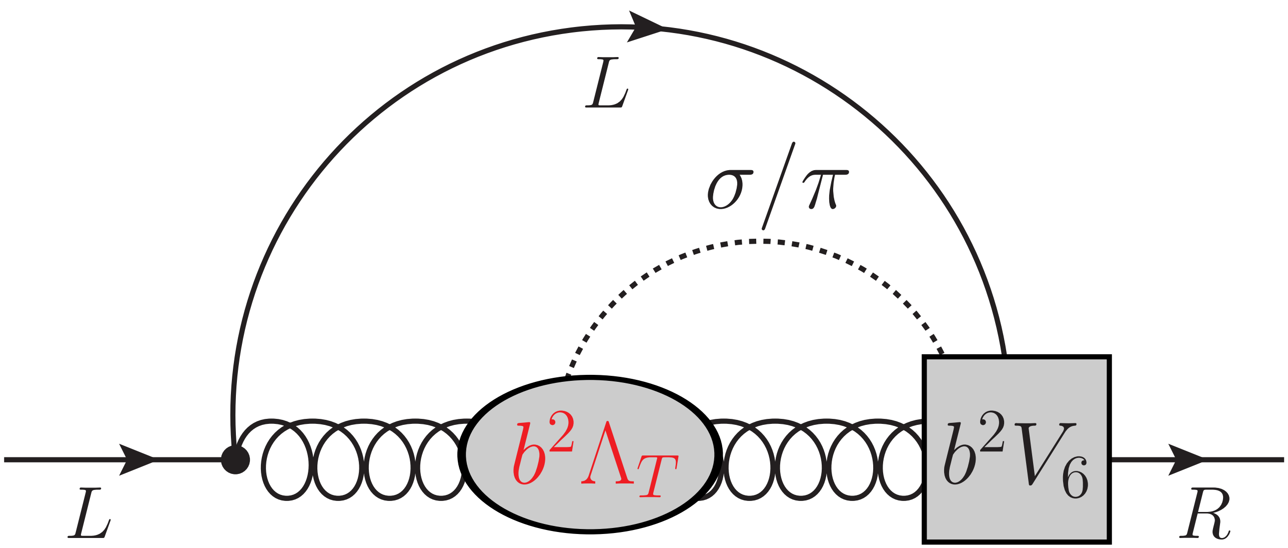
There are now separate transformations: $\tilde{\chi}_L^q \times \tilde{\chi}_R^q$ acting only on quarks, and $\tilde{\chi}_L^T \times \tilde{\chi}_R^T$ acting only on superstrongly interacting fermions

These transformations are promoted to symmetries of the action at the critical values η_{cr}^q (quarks) and η_{cr}^T (superstrongly interacting fermions)

In the Nambu-Goldstone phase we expect dynamical spontaneous breaking of both $\tilde{\chi}_L^q \times \tilde{\chi}_R^q$ (driven by strong forces) and $\tilde{\chi}_L^T \times \tilde{\chi}_R^T$ (driven by superstrong forces)

Similarly to what happens in the toy model, superstrongly interacting fermions acquire then a nonperturbatively generated mass of the order $g_T^2 \alpha_T \Lambda_T$

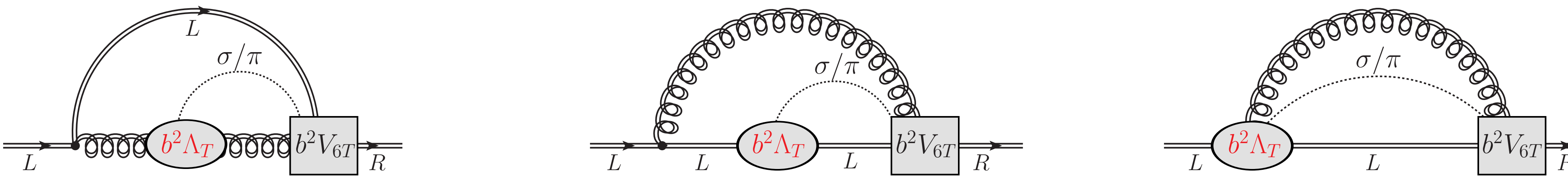
For ordinary quarks, what now happens is that there are diagrams where superstrongly interacting fermions contribute to the nonperturbative correction of the gluon-gluon-scalar vertex, and they generate nonperturbative mass terms of order $g_s^2 \alpha_s \Lambda_T$



Since $\Lambda_T \gg \Lambda_{QCD}$, these self-energy contributions are much larger than the ones discussed in the toy model, and completely dominate the effective value of the quark mass

At this order in g_s^2 the quark-quark-scalar vertex receives no analogous correction

Typical nonperturbative superstrong self-energy diagrams:



$b^2 V_{6T}$ is the insertion of the superstrong Wilson-like vertex

Double lines stand for superstrong fermions and gluons

These diagrams give a nonperturbative mass to the superstrongly interacting fermions

So, the leading contributions to the masses of the quark and of the superstrongly interacting fermions are both proportional to Λ_T

But multiplied by the fourth power of the coupling constant of the strongest of the vector gauge interactions the particle is subjected to

With similar considerations as in the toy model, one obtains the estimates

$$m_q^{dyn} = k_{LO}^{(q)}\, g_s^2 \alpha_s\, \Lambda_T\,, \qquad k_{LO}^{(q)} = O(1)$$
$$m_T^{dyn} = k_{LO}^{(T)}\, g_T^2 \alpha_T\, \Lambda_T\,, \qquad k_{LO}^{(T)} = O(1)$$

Since their masses are scaled down by $(g_s/g_T)^4$, quarks acquire an effective mass substantially smaller than the one of superstrongly interacting fermions

Must go beyond these leading order formulae, by using at least leading-log improved perturbative expressions, and decide at what scale the effective fermion masses should be evaluated

By using running effective masses, one can roughly estimate the ratio m_T^{dyn}/m_q^{dyn} of superstrongly interacting fermions to quark masses at a convenient scale μ_T, where $\alpha_T(\mu_T) \sim 1/2$

One chooses the scale μ_T rather than Λ_T itself (with $\alpha_T(\Lambda_T) = O(1)$) in order to have better control over higher-order perturbative corrections

In the $\overline{MS}$ scheme we can expect μ_T to be only 2–3 times larger than Λ_T

As $\alpha_T(\mu_T) \gg \alpha_s(\mu_T)$ we get for the mass ratio

$$\frac{m_T^{dyn}(\mu_T)}{m_q^{dyn}(\mu_T)} \simeq \frac{k_{LO}^{(T)}}{k_{LO}^{(q)}} \frac{\alpha_T^2(\mu_T)}{\alpha_s^2(\mu_T)} \Big(1 + O(\alpha_T(\mu_T))\Big)$$

If the pattern of $\tilde{\chi}$ breaking is similar for superstrongly interacting fermions and (the third generation of) quarks (which implies $k_{LO}^{(T)}/k_{LO}^{(q)} \simeq 1$) one gets

$$\frac{m_T^{dyn}(\mu_T)}{m_q^{dyn}(\mu_T)} \simeq 25 \times (1 \pm 0.5)$$

If q is the top quark, we get $m_T^{dyn}(\mu_T) \simeq 4 \times (1 \pm 0.5)$ TeV

The top mass at 170 GeV implies that there is a superstrong interaction at the scale of a few TeV

For tighter preditions, one would need lattice simulations

Outlook

- We have presented and explained a nonperturbative mechanism for the generation of fermionic mass
- A toy model should be able to confirm or disprove this mechanism explicitly
- Numerical studies of this model are underway
- The numerical results at the first lattice spacing that we have explored give support for the existence of this mass generation mechanism
- Need to confirm this finding at different lattice spacings – does the mass term survive the continuum limit?
- New simulations at other lattice spacings are currently in progress
- Extensions to include electroweak interactions
- Extension with superstrong forces: natural hierarchy of masses
- An alternative to the Standard Model?
- Incidentally, if to the particles of the Standard Model, minimal strongly interacting fermions are added to complete the model, unification of gauge couplings is automatically achieved (without invoking SUSY)

Literature

- R. Frezzotti and G.C. Rossi
  "Nonperturbative mechanism for elementary particle mass generation"
  PRD 92 (2015) 054505
- R. Frezzotti, M. Garofalo and G.C. Rossi
  "Nonsupersymmetric model with unification of electroweak and strong interactions"
  PRD 93 (2016) 105030

- Lattice 2013: parallel talk by R. Frezzotti
- Lattice 2016: parallel talk by M. Garofalo
- Lattice 2017: parallel talks by M. Garofalo, P. Dimopoulos and F. Pittler