Finite-size scaling of the Higgs-Yukawa model near the mean-field fixed point

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The theory

\[
S_{\text{cont}}^{(c)}[\psi(c), \phi(c)] = \int d^4 x \left\{ \frac{1}{2} \left( \partial_\mu \phi(c) \right)^\dagger \left( \partial^\mu \phi(c) \right) + \frac{1}{2} m_0^2 \phi(c)^\dagger \phi(c) + \frac{\lambda_0}{4} \left( \phi(c)^\dagger \phi(c) \right)^2 \right\} \\
+ \int d^4 x \left\{ t(c) \phi(c) + b(c) \phi(c) + yv_0 \psi_L(c) \phi(c) b_R(c) + yt_0 \psi_L(c) \psi(c) t_R(c) + \text{h.c.} \right\}
\]

\[
a \phi^{(\text{latt})} = \begin{pmatrix} \Phi^2 + i \Phi^1 \\ \Phi^4 - i \Phi^3 \end{pmatrix}
\]

\[
S_{\Phi}^{(\text{latt})} = \sum_{\alpha=1}^{4} \left\{ - \sum_{x, \mu} \Phi^\alpha \Phi^\alpha_{x+\mu} + \sum_{x} \left[ \frac{1}{2} \left( 8 + m_0^2 \right) \Phi^\alpha \Phi^\alpha_x + \frac{\lambda_0}{4} \left( \Phi^\alpha \Phi^\alpha_x \right)^2 \right] \right\}
\]

\[
\Phi^\alpha = \sqrt{2} \kappa \phi^\alpha, \quad \lambda_0 = \frac{\hat{\lambda}}{\kappa^2}, \quad m_0^2 = \frac{1 - 2 \lambda - 8 \kappa}{\kappa}
\]

\[
S_{\phi}^{(\text{latt})} = \sum_{\alpha=1}^{4} \left\{ - 2 \kappa \sum_{x, \mu} \phi^\alpha x \phi^\alpha_{x+\mu} + \sum_{x} \left[ \phi^\alpha x \phi^\alpha_x + \hat{\lambda} \left( \phi^\alpha x \phi^\alpha_x - 1 \right)^2 \right] \right\}
\]

\[
S_{f}^{(\text{latt})} = \sum_{x} \bar{\psi}_x \left[ D^{ov} + P_+ \phi^\alpha_x \phi^\dagger_x \right. \text{diag}(\hat{y}_t, \hat{y}_b) \hat{P}_+ + P_- \text{diag}(\hat{y}_t, \hat{y}_b) \phi^\alpha_x \theta_\alpha \hat{P}_- \left. \right] \psi_x
\]

\[
\theta_{1,2,3} = -i \tau_{1,2,3}, \quad \theta_4 = 1_{2 \times 2} \quad \text{Also introduce } Y = y^2
\]
Outline

• Motivation.
• FSS, general consideration, and our previous work.
• FSS, 4-d Higgs-Yukawa model near the MF FP.
• Test of method in 4-d O(4) pure scalar model.
• Outlook.
Motivation

- Very little doubt that the pure-scalar sector of the SM is trivial.

  M. Aizenman, PRL. 47 (1981)

  J. Fröhlich, NPB 200 (1982)


  M. Hoogervorst and U. Wolff, NPB 855 (2012)

  J. Sievert and U. Wolff, PLB 733 (2014)  (High-precision study with large volumes)

  ...

- How about the Higgs-Yukawa sector?
Two-loop perturbation theory

\[ a_y \equiv \frac{y^2}{(4\pi)^2}, \quad a_\lambda \equiv \frac{\lambda}{(4\pi)^2} \]
Early work on the quantum phase structure

The continuum limit

\[ a \to 0 \quad \text{and} \quad \Lambda \to \infty \]

- The key point is the separation of the scales.
- It can be achieved at 2nd-order bulk phase transitions:
  \[ \frac{\xi}{a} \to \infty. \]
- Condensed matter physics:
  At fixed \( a \), take \( \xi \to \infty \).
- For our purpose:
  At fixed \( \xi \), take \( a \to 0 \).
Scanning for the continuum limit

Step one: locate the possible continuum limit.

$$\lambda_0 \rightarrow \infty$$
Scanning for the continuum limit

Step two: study the scaling behaviour for confirmation.

\[
\lambda_0 \rightarrow \infty
\]
Next question: which continuum limit?  
We resort to the FSS techniques for this.  
(Suitable for the search of strongly-coupled FP)

For observables containing only the scalar fields:

\[ \hat{M}_b \left[ m_b^2, \lambda_b, Y_b; a, L \right] Z_\phi^{-DM/2}(a, l) = \hat{M} \left[ m^2(\hat{l}), \lambda(\hat{l}), Y(\hat{l}); l, L \right], \]

\[ = \xi_M(l, L) \hat{L}^{-DM} \hat{M} \left[ \hat{m}^2(\hat{L}) \hat{L}^2, \lambda(\hat{L}), Y(\hat{L}); 1, 1 \right] \]

\[ \xi_M(l, L) = \exp \left( \int_l^L \gamma_M(\rho) d\log \rho \right) \]

Fixed point: \( \lambda(\hat{L}) \approx \lambda_* \), \( Y(\hat{L}) \approx Y_* \), \( \gamma_M \approx \gamma \). \( \rightarrow \) Power law in \( \xi_M \).

Similar RG argument also results in the power-law scaling:

\[ \hat{m}^2(\hat{L}) \sim \hat{L}^{1/\nu - 2} \quad \text{where} \quad 1/\nu = 2 + \gamma_{m^2} \]

\( \text{Naive expectation for the mean-field FP} \)  \( \text{mass-square anomalous dimension} \)
Scanning for the continuum limit

Step three: which continuum limit?

\[ \lambda_0 \to \infty \]

\[ t \sim \hat{m}^2_0 - \hat{m}^2_{\text{crit}} \]

J. Bulava et al., AHEP 2013
Scanning for the continuum limit
Step three: which continuum limit?

\[ \lambda_0 \rightarrow \infty \]

<table>
<thead>
<tr>
<th>( T_c^{(L=\infty)} )</th>
<th>( \nu )</th>
<th>interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa = 0.06 )</td>
<td>18.147(24)</td>
<td>0.550(1)</td>
</tr>
<tr>
<td>( \kappa = 0.00 )</td>
<td>16.667(27)</td>
<td>0.525(6)</td>
</tr>
<tr>
<td>O(4)</td>
<td>0.3005(34)</td>
<td>0.50000(3)</td>
</tr>
</tbody>
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Results from Binder’s cumulant with a curve-collapse method

J. Bulava et al., AHEP 2013

We need to better understand the mean-field scaling law.
Mean-field FSS for the Higgs-Yukawa model
Sequel of E. Brezin and J. Zinn-Justin, NPB 257 (1985)

$O(N)$ scalar sector coupled to fermions via the Yukawa coupling

$\Phi^T = (\phi_1, \cdots, \phi_N), \Psi^T = (\psi_1, \cdots, \psi_{N_f})$

$Z = \int D\Phi \, D\bar{\Psi} \, D\Psi \exp \left( -S[\Phi, \bar{\Psi}, \Psi] \right)$

$\phi_a = \varphi_a + \chi_a$

$Z = \int_{-\infty}^{\infty} d^N \varphi_a \, \mathcal{N} \exp(-S_{eff}[\varphi_a]) = \Omega_{N-1} \int_0^{\infty} d\varphi \, \varphi^{N-1} \mathcal{N} \exp(-S_{eff}[\varphi])$

$\exp(-S_{eff}[\varphi]) = \det(M_F[\varphi]) \det(M_B[\varphi])^{-1} \exp \left( -sL^4 \frac{1}{2} M_b^2 \varphi^2 - sL^4 \lambda_b \varphi^4 \right)$

renormalises the couplings, and generates higher-dimensional operators.

dropped in perturbation theory

anisotropy

non-Gaussian modes of $\chi_a$ decouples at one-loop
Mean-field FSS for the Higgs-Yukawa model

Sequel of E. Brezin and J. Zinn-Justin, NPB 257 (1985)

\[ \star \text{Rescale: } \varphi \rightarrow (sL^4 \lambda (L^{-1}))^{-1/4} \varphi \equiv S^{-1/4} \varphi \star \]

\[ Z = \mathcal{N} \Omega_{N-1} S^{-N/4} \int_0^\infty d\varphi \varphi^{N-1} \exp \left( -\frac{1}{2} z \varphi^2 - \varphi^4 \right) \]

\[ \equiv \mathcal{N} \Omega_{N-1} S^{-N/4} \bar{\varphi}_{N-1}(z), \]

Logarithms

\[ \text{The scaling variable: } z = \sqrt{sM^2 (L^{-1}) \hat{L}^2 \lambda (L^{-1})}^{-1/2} \]

\[ \bar{\varphi}_0 = \frac{\pi}{8} \exp \left( \frac{z^2}{32} \right) \sqrt{z} \left[ I_{-1/4} \left( \frac{z^2}{32} \right) - \text{Sgn}(z) I_{1/4} \left( \frac{z^2}{32} \right) \right], \]

\[ \bar{\varphi}_1 = \frac{\sqrt{\pi}}{8} \exp \left( \frac{z^2}{16} \right) \left[ 1 - \text{Sgn}(z) \text{Erf} \left( \frac{|z|}{4} \right) \right], \bar{\varphi}_{n+2} = -2 \frac{d}{dz} \bar{\varphi}_n \]

\[ \text{Can compute } \langle \varphi^k \rangle. \]
Mean-field FSS for the Higgs-Yukawa model

\[
\hat{M}_b \left[ m_b^2, \lambda_b, Y_b; a, L \right] Z_{\phi}^{-D_M/2}(a,l) = \hat{M} \left[ m^2(l), \lambda(l), Y(l); l, L \right], \\
= \xi_M(l, L) \hat{L}^{-D_M} \hat{M} \left[ \hat{m}^2(\hat{L}) \hat{L}^2, \lambda(\hat{L}), Y(\hat{L}); 1, 1 \right]
\]

One-loop RGE in perturbation theory

\[-\rho \frac{d}{d\rho} Y(\rho) = \beta_{YY^2} Y(\rho)^2, \quad -\rho \frac{d}{d\rho} \phi(\rho) = 2\delta_Y Y(\rho) \phi(\rho),\]

\[-\rho \frac{d}{d\rho} \lambda(\rho) = \beta_{\lambda \lambda^2} \lambda(\rho)^2 + \beta_{\lambda \lambda Y} \lambda(\rho) Y(\rho) + \beta_{\lambda Y^2} Y(\rho)^2,\]

\[-\rho \frac{d}{d\rho} m^2(\rho) = 2 \left[ \gamma_Y Y(\rho) + \gamma_\lambda \lambda(\rho) \right] m^2(\rho),\]

Integrate this simple one first from \( l \) to \( L \).

Four integration constants, plus the critical mass.

Also need to specify \( l \).
Mean-field FSS for the Higgs-Yukawa model

\[ \hat{M}_b \left[ m_b^2, \lambda_b, Y_b; a, L \right] Z_\phi^{D_M/2}(a, l) = \hat{M} \left[ m^2(\hat{l}), \lambda(\hat{l}), Y(\hat{l}); l, L \right], \]
\[ = \zeta_M(l, L) L^{-D_M} \hat{M} \left[ \hat{m}^2(\hat{L}) \hat{L}^2, \lambda(\hat{L}), Y(\hat{L}); 1, 1 \right] \]

Identify the scalar (Higgs) pole mass, \( m_p \), as \( 1/l \).

Extrapolate away the volume effects from light-mode-around-the-world.
Identify as the renormalised mass at the pole mass (on-shell scheme).
Run to \( 1/L \) using perturbation theory.

Reduces the number of fit parameters by two.
Numerical test in $O(4)$ pure scalar model

Using the cluster algorithm

All results are exploratory hitherto, and only at $\lambda_0 = 0.15$. Refer to David Chu’s Lattice 2016 talk for updates.

Improved (analytic and numerical aspects) version of M. Gockeler et al., NPB 404 (1993)
First scanning, the magnetisation (VEV)
Susceptibility and Binder’s cumulant

\begin{align*}
\chi_L & \quad \text{vs.} \quad m_0^2 \\
Q_L & \quad \text{vs.} \quad m_0^2
\end{align*}

Preliminary
Scalar wavefunction renormalisation

Note: one-loop perturbation theory gives unity.
The pole mass

\[ m_p \]

\[ m_0^2 \]
Extrapolating away, in pole mass, the around-the-world volume effects

\[ m_0^2 = -0.53252 \]
Extrapolating away, in pole mass, the around-the-world volume effects

\[ m_0^2 = -0.54141 \]

near the critical point
Extrapolating away, in pole mass, the around-the-world volume effects

\[ m_0^2 = -0.54585 \]

broken phase
The extrapolated pole mass

Extrapolation performed with data at the five largest volumes

Extrapolation performed with data at the five largest volumes
Scaling of Binder’s cumulant

Fit performed with the five largest volumes.

$\lambda_b = 0.0947$
$\lambda_s = 0.0808$
Scaling of magnetisation and susceptibility

Fit performed with the five largest volumes.
Conclusion and outlook

- The triviality (or alternative scenarios) of the Higgs-Yukawa system still needs to be investigated.

- We worked out a FSS strategy for this task.

- Detailed understanding of the MF FSS scaling of the model is needed.

- We will complete the test of our method in the scalar model.

- Test of our method in the weak-coupling regime of the Higgs-Yukawa model is needed.

- Finally, a thorough and careful analysis of the Higgs-Yukawa model phase structure should be carried out.

Of course, alternative methods are also welcome.