Universal non-perturbative features in the thermal behavior of non-Abelian gauge theories

Marco Panero

University of Turin and INFN, Turin, Italy

NIC, DESY Zeuthen
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Outline

Introduction

Thermodynamics in the confining phase: comparing SU(2) and SU(3) Yang-Mills theories

Thermodynamics in the deconfined phase: comparing SU(N) and G_2 theories

Conclusions

Based on:

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Conclusions
**Strong nuclear interactions under extreme conditions**

Quantum Chromodynamics is believed to possess a rich phase structure

[N. Cabibbo and G. Parisi, 1975], [J. C. Collins and M. J. Perry, 1975]
Why is it important?

Adapted from [Z. Weiner, 2010]
Why is it important?

- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about $1 \mu$s after the Big Bang; the early Universe cooling rate depends on the QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties [B. Müller, 2013]
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, ... [E. Shuryak, 2009]
- Very rich physics, involving several non-trivial theoretical problems
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...THE ONE THAT GIVES YOU MORE THAN 95% OF YOUR MASS, BY THE WAY!
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  - A liquid that cools into a gas
  - “The most perfect” liquid—shear viscosity close to its quantum limit [G. Policastro, D. T. Son and A. O. Starinets, 2001]
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Motivation to study finite-temperature QCD and QCD-like theories on the lattice

- Robust *ab initio* theoretical predictions for strong interactions under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, fail to capture the physics of long-wavelength modes in the deconfined phase [A. Linde, 1980] and the dynamics in the hadronic phase
- The lattice regularization [K. G. Wilson, 1974] provides the only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice computations are now producing accurate numerical predictions for QCD at finite temperature
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Glueball gas thermodynamics

- The physical states in the confining, low-temperature phase are color-singlet hadrons
- For non-supersymmetric, pure-glue SU(N) theories, these hadrons are glueballs
- Glueballs are classified according to their $J^{PC}$ quantum numbers, have a finite mass gap, and are weakly coupled
- The equation of state can be described in terms of a gas of massive, free bosons; for example, for the pressure:

$$p(T) = -\frac{\partial F}{\partial V} = \lim_{V \to \infty} \frac{T}{V} \ln Z = \sum_i (2J_i + 1)p(m_i; T)$$

where $p(m_i; T)$ denotes the contribution from each degree of freedom of a state of mass $m_i$:

$$p(m_i; T) = \frac{T^4}{2} \left( \frac{m_i}{T} \right)^2 \sum_{n=1}^{\infty} \frac{1}{n^2} K_2 \left( \frac{n m_i}{T} \right)$$
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A string model for the spectrum

- How many glueball states should be included in the sum?
  - At temperatures much lower than the deconfinement transition temperature $T_c$, the sum is completely dominated by the lightest glueballs
  - At temperatures closer to $T_c$, the contribution of heavier states becomes non-negligible

- While the lattice determination of heavy states in the spectrum is challenging [M. Teper, 1998], the spectral density $\hat{\rho}(m)$ can be predicted by modelling glueballs as “rings of glue” [N. Isgur and J. E. Paton, 1984]

- The effective description of confining flux tubes as open strings [M. Lüscher, 1980] suggests to model the glueballs as closed bosonic strings, leading to:

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  \hat{\rho}(m) = \frac{1}{m} \left( \frac{2\pi T_H}{3m} \right)^3 \exp \left( m / T_H \right)
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  where $T_H$ is the Hagedorn temperature [R. Hagedorn, 1965]

- The total contribution to the pressure, including contributions from the lightest states and from a continuous spectrum above a threshold $m_{\text{threshold}} = 2m_0^{++}$, reads

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SU(2) gauge theory

- SU(2) Yang-Mills theory is the simplest confining gauge theory based on a non-Abelian Lie group
  - Due to its small dimension, it has been studied as a QCD prototype since the early days of lattice QCD [M. Creutz, 1980]
  - Larger Lie groups have (several) SU(2) subgroups—a feature which is also used in simulation algorithms [N. Cabibbo and E. Marinari, 1982]
  - The theory undergoes a second-order deconfining phase transition at a critical temperature $T_c$ [J. Engels, F. Karsch and K. Redlich, 1994]
  - When (a sufficiently large number of) dynamical fermions (in a suitable representation) are included, the theory could provide a potentially viable walking technicolor model for dynamical electro-weak symmetry breaking (DEWSB) [Del Debbio, 2010]
  - When a small number of fundamental fermions are included, the (pseudo-)real nature of the SU(2) representations allows one to simulate the theory at finite quark chemical potential [S. Hands et al., 1999]
  - The (pseudo-)real nature of the gauge group representations implies that in SU(2) Yang-Mills theory there exist no glueballs with $C = -1$ charge-conjugation eigenvalue
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Lattice results: Impact of heavy states

Our results for the trace anomaly in units of the fourth power of the temperature

\[
\frac{\Delta}{T^4} = T \frac{\partial}{\partial T} \left( \frac{p}{T^4} \right)
\]

show that heavy glueballs give a large contribution to the equation of state close to \( T_c \)
Lattice results: Comparison with the bosonic string model

The predictions of a gas of massive glueballs, including contributions from heavy states modelled by a closed bosonic string model, are in excellent agreement with lattice data for both SU(2) and SU(3) Yang-Mills theories [S. Borsányi et al., 2012]—see also [H. B. Meyer, 2009], [M. Caselle et al., 2011], [F. Buisseret and G. Lacroix, 2011]
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Introduction

Thermodynamics in the confining phase: comparing SU(2) and SU(3) Yang-Mills theories

Thermodynamics in the deconfined phase: comparing SU(N) and G2 theories

Conclusions
G₂ Yang-Mills theory

- The exceptional group G₂ (with dimension 14 and rank 2) is the smallest compact, simply connected Lie group with a trivial center
- “Quarks” in the fundamental representation 7, “gluons” in the adjoint 14
- Due to the absence of center vortices, which are important rôle for confinement [L. Del Debbio et al., 1996], [P. de Forcrand and M. D'Elia, 1999], G₂ Yang-Mills theory has triggered interest in the lattice community [K. Holland et al., 2003]
- The theory is actually “screening” at large distances: a fundamental color source can be screened by three gluons
  \[
  14 \otimes 14 \otimes 14 = 1 \oplus 7 \oplus 14 \oplus 14 \oplus 14 \oplus 14 \oplus 27 \oplus 27 \oplus 27 \oplus 64 \oplus 64 \\
  \oplus 77 \oplus 77 \oplus 77 \oplus 77 \oplus 77' \oplus 77' \oplus 77' \oplus 182 \oplus 189 \oplus 189 \oplus 189 \\
  \oplus 273 \oplus 448 \oplus 448
  \]
- The absence of N-ality also implies that the theory admits both bosonic and fermionic “baryons”
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- As an SO(7) subgroup, G₂ has real representations: this allows one to simulate the theory with dynamical quarks at finite chemical potential [A. Maas et al., 2012]
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⭐ First-order thermal deconfinement transition [G. Cossu et al., 2007], as predicted by semiclassical computations [E. Poppitz, T. Schäfer and M. Ünsal, 2012] and by phenomenological models [A. Dumitru et al., 2012]

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★ In the deconfined phase, the thermodynamic observables normalized per gluon d.o.f. exhibit remarkable gauge-group independence—see also [M. P., 2009], [A. Mykkänen, M. P. and K. Rummukainen, 2012], [B. Lucini and M. P., 2012]

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- Lattice simulations also reveal interesting universal, non-perturbative aspects in the thermodynamics of different, QCD-like gauge theories.
  - The confining phase can be described as a gas of free glueballs, provided contributions from a Hagedorn-like excitation spectrum are included.
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Thanks for your attention!