

Infrared behavior of SU(2) gauge theory

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Outline

- Motivation
- Conformal window
- Lattice setup
- Measurement of the coupling
- Mass anomalous dimension

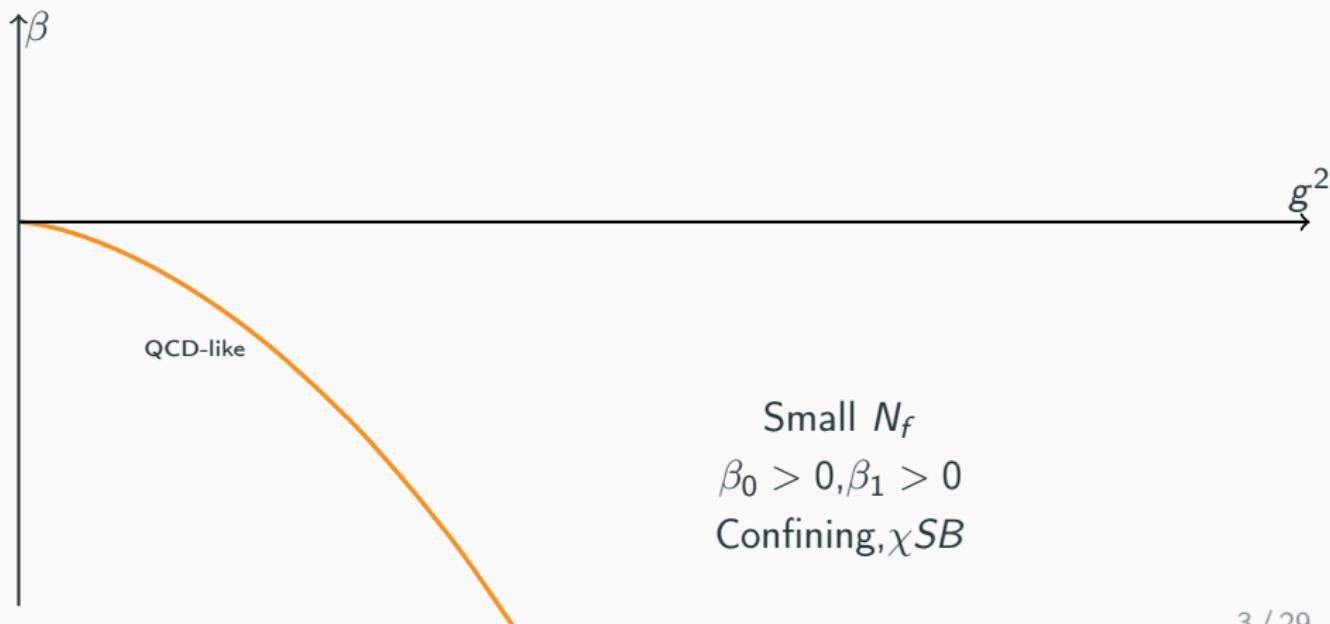
Motivation

- Understanding strongly coupled theories
- Phase structure of the gauge field theories
- Possible applications for particle phenomenology
- There is still room for BSM physics
 - Composite Higgs models e.g. (extended) technicolor
- Depending on number of fermions we can have χSB /Walking/IRFP
- Walking coupling nearly freezes between two energy scales
 - Could enable light composite Higgs
 - Can connect scales e.g. Λ_{EW} and Λ_{ETC}

β -function

$$\beta(g) = \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}, \quad \left\{ \begin{array}{l} \beta_0 = \frac{11}{3} N_c - \frac{4}{3} T_r N_f \\ \beta_1 = \frac{34}{3} N_c^2 - \left(\frac{20}{3} N_c - 4 C_r\right) T_r N_f \end{array} \right.$$

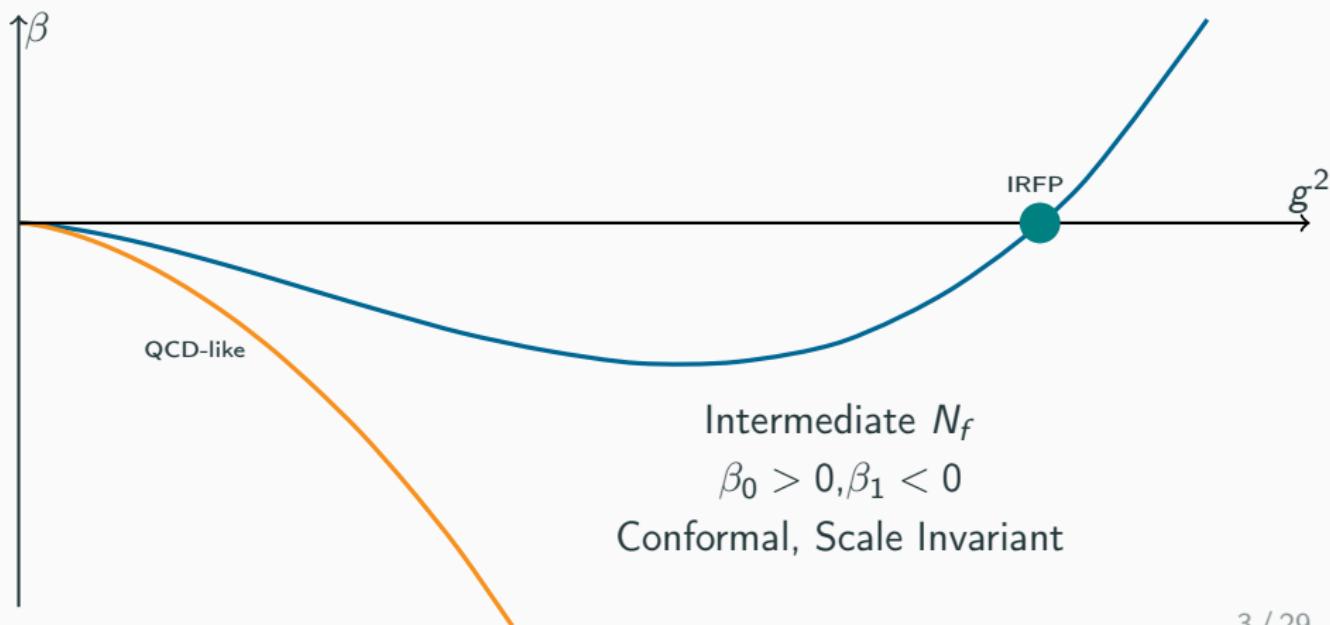
- Behavior depends on **group**, **representation** and **number of fermions**



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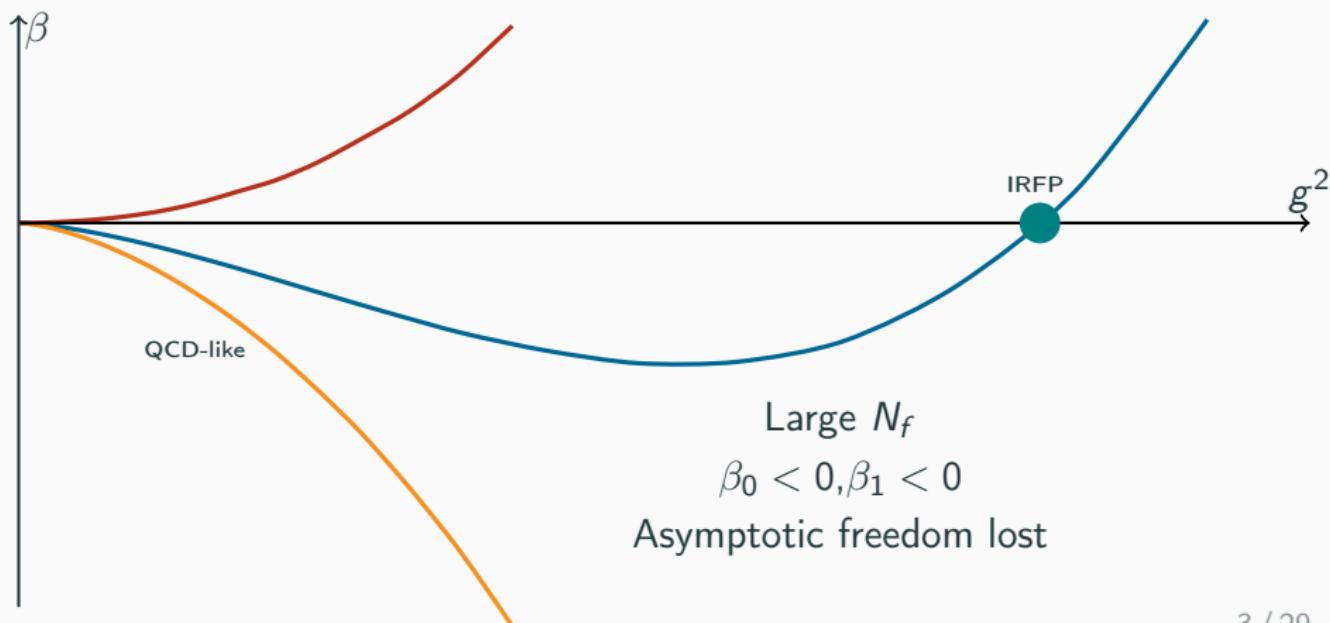
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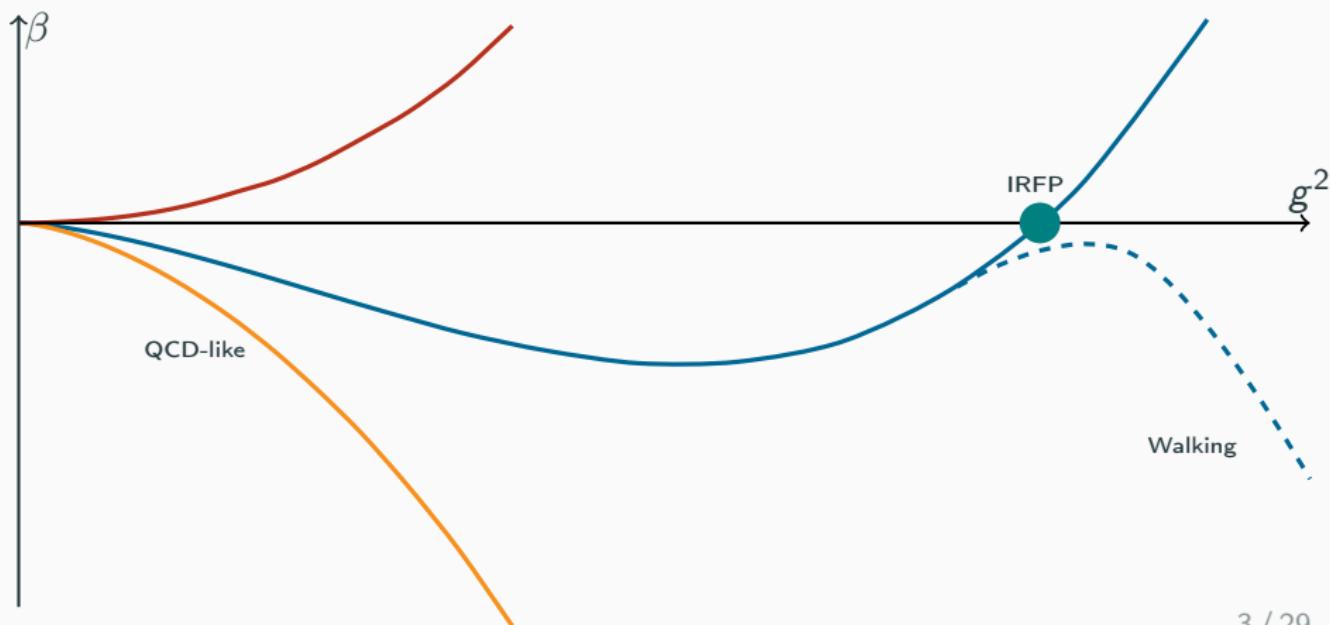
- Behavior depends on group, representation and number of fermions



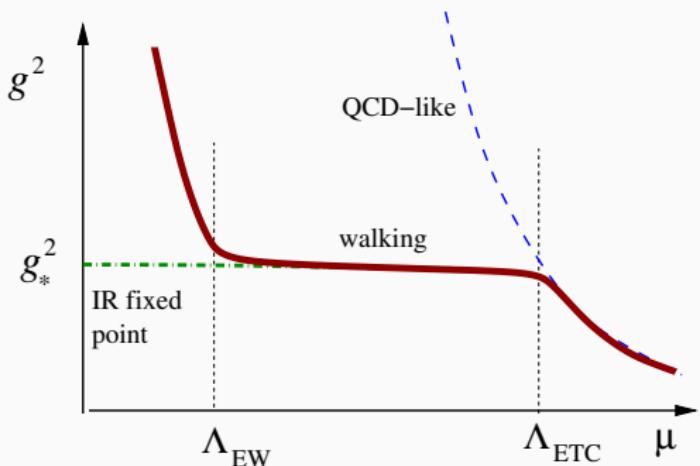
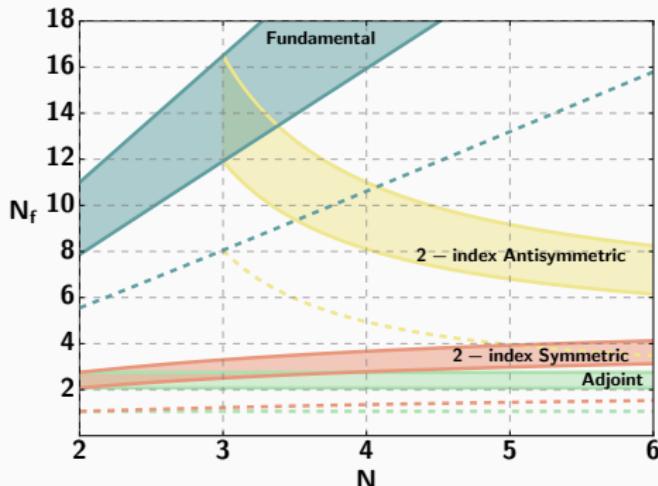
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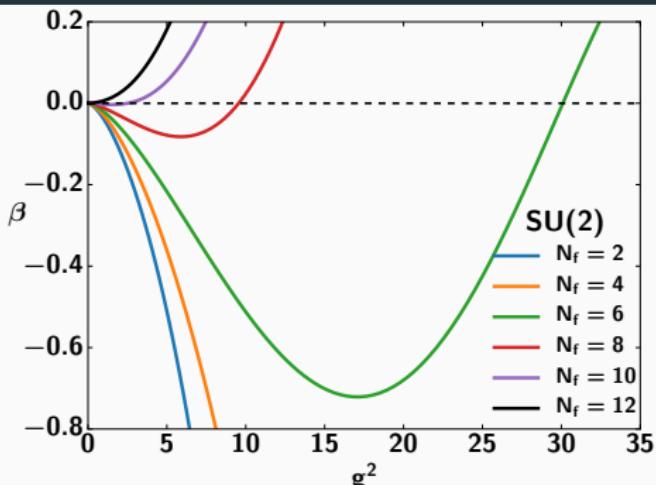
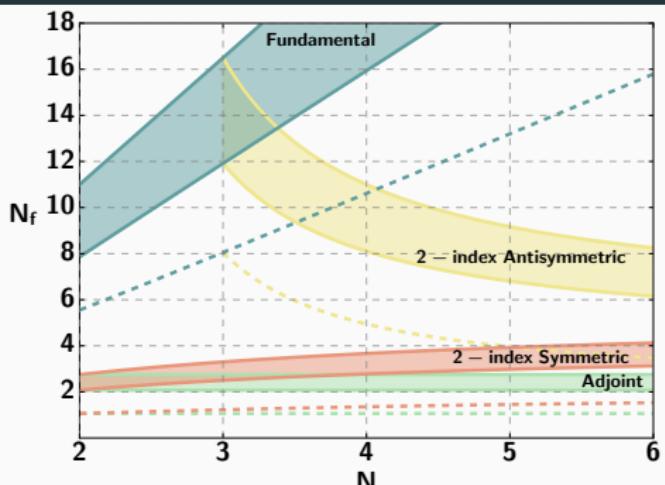


Conformal Window

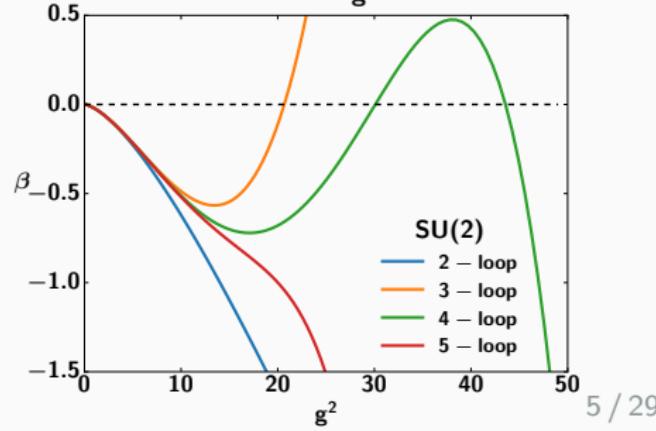


- Location of lower edge of conformal window unknown.
- What happens when we approach the conformal window from below
- Walking?
- Commonly studied: SU(3) with 8-12 fundamental or 2 sextet, SU(2) with 6-8 fundamental or 2 adjoint

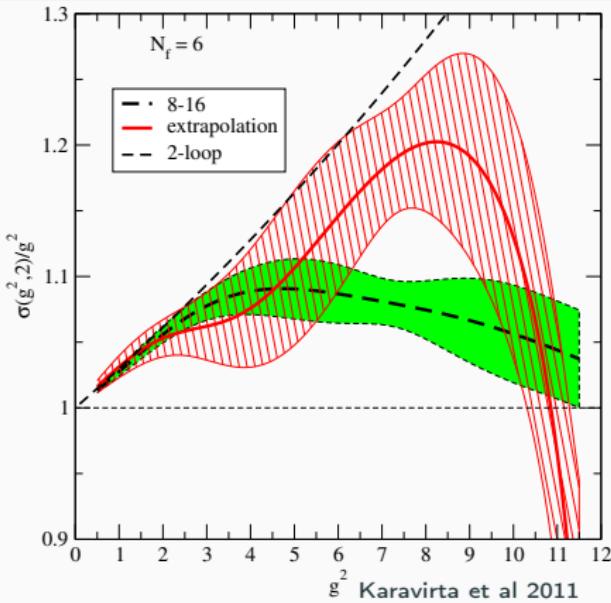
SU(2) Conformal Window



- Lower edge $N_f \sim 6, 8$
- $N_f = 6$ IRFP at high coupling
- Different loop orders disagree



SU(2) Conformal Window



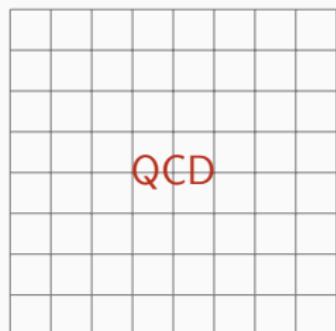
SU(2) previous studies

$N_f = 4$	χSB	Karavirta et al. 2011
$N_f = 6$	Inconclusive	Bursa et al. 2010
$N_f = 8$	Inconclusive	Karavirta et al. 2011
$N_f = 10$	IRFP	Hayakawa et al. 2013
		Appelquist et al. 2014
		Ohki et al. 2010
		Karavirta et al. 2011
$N_f = 6, 8$	IRFP	
		This talk

- Previous studies inconclusive
- Mostly done with Schrödinger functional coupling which can be noisy especially on larger lattices
- We use gradient flow coupling with HEX-smeared Wilson-Clover action

What makes the simulations difficult

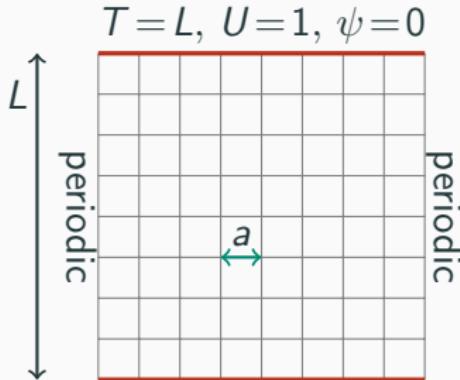
Coupling is large here



\leftrightarrow , But small here

- In QCD, coupling depends on lattice scale
- On a walking/conformal theory, coupling large and mostly independent on scale
 - Small β required, regardless of L
 - Cannot take continuum limit at weak bare coupling

Lattice setup: Boundary conditions



$T = 0, U = 1, \psi = 0$

$$S = (1 - c_g) S_G(U) + c_g S_G(V) + S_F(V)$$

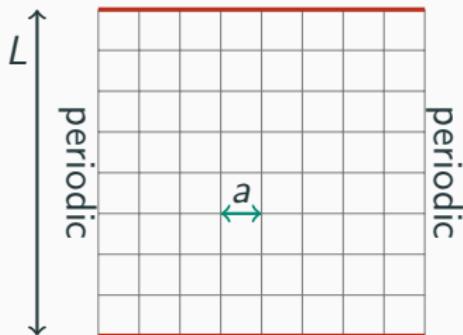
$$S_G = \beta \sum_{\square} \left(1 - \frac{1}{2} \text{Tr} \square(x) \right)$$

$$S_F = a^4 \sum_{N_f} \sum_x (\bar{\psi} (iD + m_0) \psi + \frac{ai}{4} c_{sw} \bar{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi)$$

- Schrödinger functional boundaries $c_t = 1$
 - Periodic boundary conditions on spatial boundaries
 - Dirichlet boundary conditions on time boundaries
- Easier to tune fermion masses to zero
- Enables measurement of mass anomalous dimension

Lattice setup: Action

$$T=L, U=1, \psi=0$$



$$T=0, U=1, \psi=0$$

$$S = (1 - c_g) S_G(U) + c_g S_G(V) + S_F(V)$$

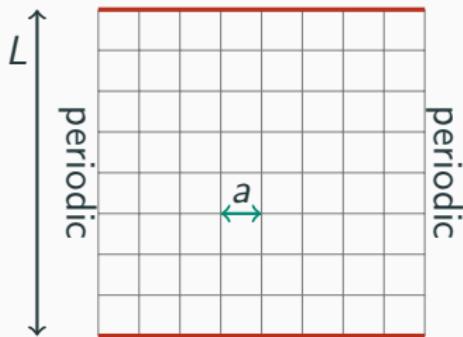
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- Use HEX-smearing
 - Moves bulk phase and allows reaching zero mass at higher couplings
- Mix smeared (V) and unsmeared (U) gauge fields with c_g
- Clover improved Wilson action $c_{sw} = 1$

Lattice setup: General

$$T = L, U = 1, \psi = 0$$



$$T = 0, U = 1, \psi = 0$$

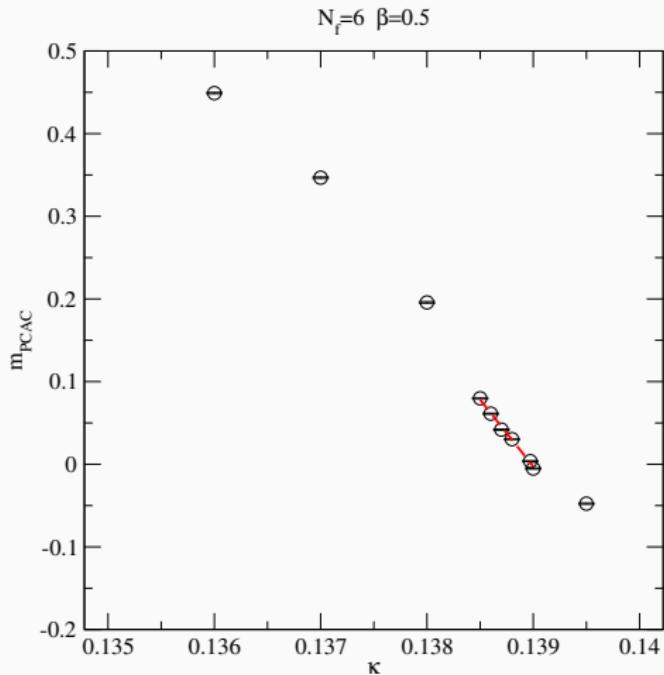
- $N_f = 6$
 - $L = 8^4 - 30^4$ and $\beta = 8 - 0.5$
- $N_f = 8$
 - $L = 6^4 - 32^4$ and $\beta = 8 - 0.4$
- 10 000 - 100 000 well thermalized trajectories
- HMC step length tuned to have acceptance > 80%

$$S = (1 - c_g) S_G(U) + c_g S_G(V) + S_F(V)$$

$$S_G = \beta \sum_{\square} \left(1 - \frac{1}{2} \text{Tr} \square(x) \right)$$

$$S_F = a^4 \sum_{N_f} \sum_x (\bar{\psi} (iD + m_0) \psi + \frac{ai}{4} c_{sw} \bar{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi)$$

Tuning of κ



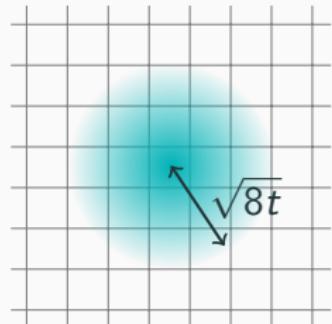
- $\kappa = (8 + 2am_0)^{-1}$
- $M(x_0) = \frac{1}{4} \frac{(\partial_0^* + \partial_0)f_A(x_0)}{f_P(x_0)}$
- Tune κ (m_0) in $L = 24$
- $m_{\text{PCAC}} < 10^{-5}$
- Use same κ for each L
- Bulk phase at small β

Gradient flow

$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu},$$

$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}].$$

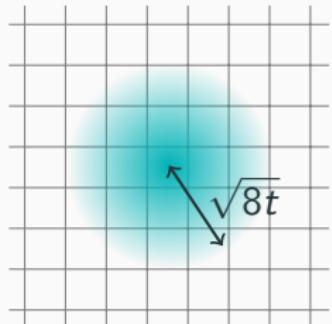
$B_{0,\mu} = A_\mu$ ← the original gauge field



- No fermions in the equation
- Introduce fictitious time coordinate t and evolve the gauge field
- Drives B_μ towards minima of S_{YM}
- Diffuses the initial gauge field with radius $\sqrt{8t}$
- Removes the UV divergences
- Automatically renormalizes gauge invariant observables
- One has to choose discretization for the S_{YM}

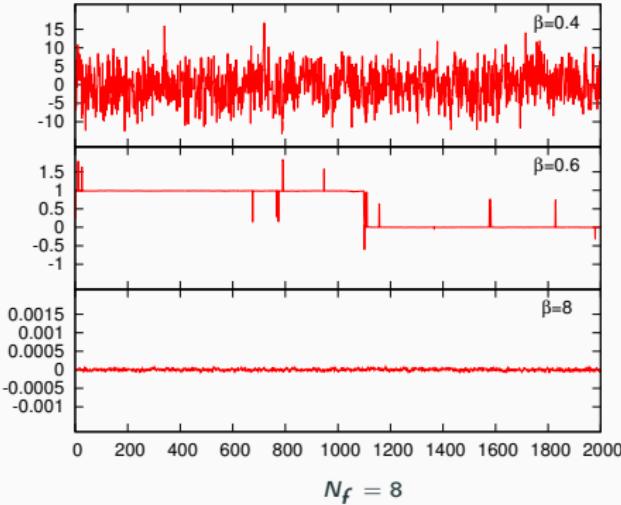
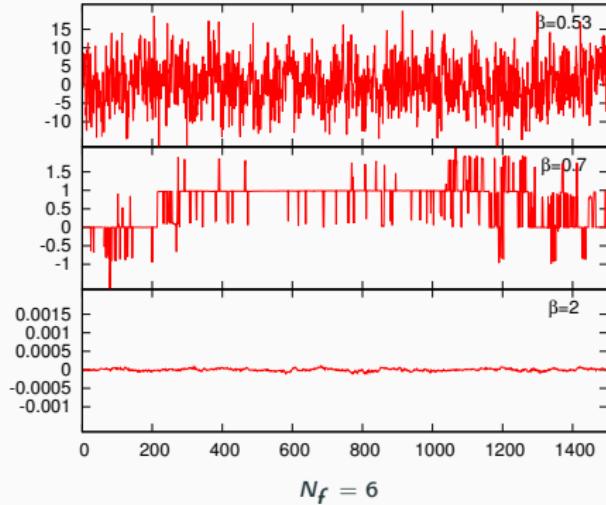
Gradient flow

$$\begin{aligned}\langle E(t) \rangle &= \frac{1}{4} \langle G_{\mu\nu}(t) G_{\mu\nu}(t) \rangle \\ &= \frac{3(N^2 - 1)g_0^2}{128\pi^2 t^2} + \mathcal{O}(g_0^4), \\ g_{\text{GF}}^2(\mu) &= \mathcal{N}^{-1} t^2 \langle E(t) \rangle |_{x_0=L/2, t=1/8\mu^2},\end{aligned}$$



- Evolve the flow equation to time t
- Each t defines different scheme, coupling is at scale $\mu^{-1} = \sqrt{8t}$
- To make scale free of lattice artifacts and finite volume effects:
 $\mu^{-1} = cL$
- We use $c = 0.4$ for $N_f = 8$, and $c = 0.3$ for $N_f = 6$
- For consistency compare to different c 's
- One has to choose discretization for the energy density
- Boundary conditions break time translation, only use $x_0 = L/2$

Topology

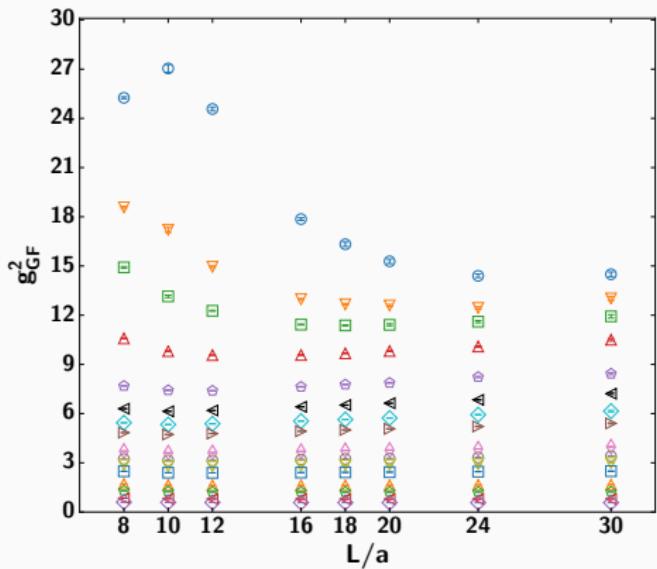


- Gradient flow allows measurement of topological charge

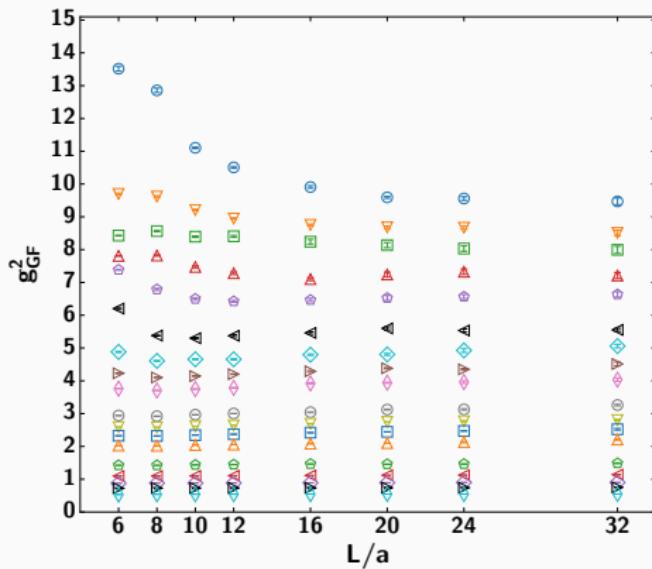
$$Q = \frac{1}{32\pi^2} \sum_x \epsilon_{\mu\nu\alpha\beta} G_{\mu\nu}^a(x; t) G_{\alpha\beta}^a(x; t)$$

- Frozen at small coupling, unfrozen at large couplings
- On intermediate couplings frozen to nonzero

Raw couplings



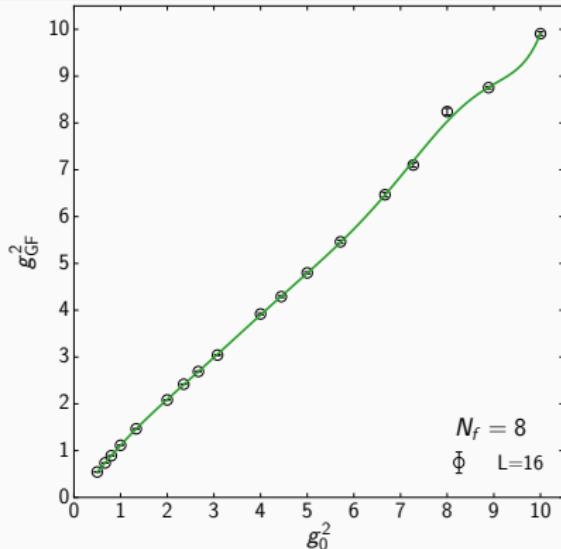
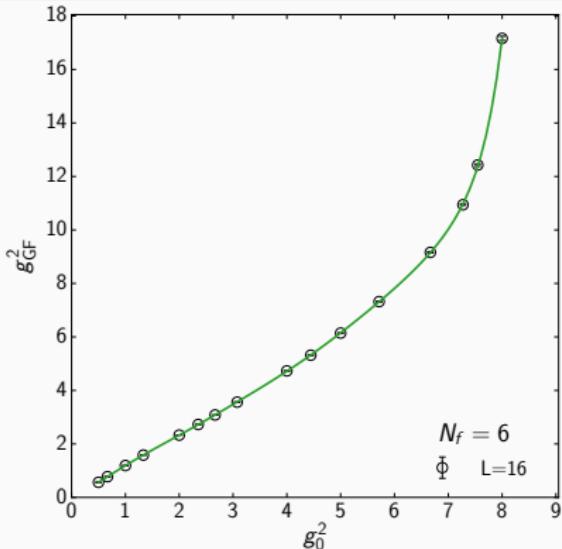
$$N_f = 6$$



$$N_f = 8$$

- Strong finite size effects on small lattices
→ Only use lattices of size 10 or bigger

Interpolation

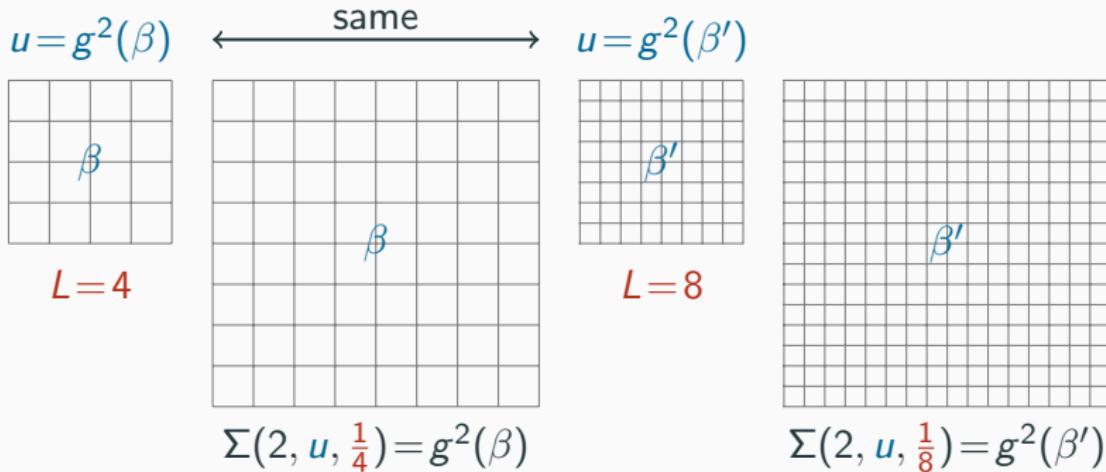


$$g_{GF}^2 = g_0^2 \left(1 + \sum_{i=1}^n a_i g_0^{2i} \right),$$

$$g_{GF}^2 = g_0^2 \frac{1 + \sum_{i=1}^n a_i g_0^{2i}}{1 + \sum_{j=1}^m b_j g_0^{2j}}$$

- For $N_f = 6$ $s = 1.5$, Interpolate using polynomial function $n = 9$
- For $N_f = 8$ $s = 2$, Interpolate using rational function $n = 7$ $m = 1$
- Estimate systematic errors by changing n, m by 1

Step scaling idea



- Choose $\beta = 4/g_0^2$ and L , measure $u = g_{GF}^2(L)$
- Choose stepsize $s = 2$
- Double the lattice size, and measure $\Sigma(u, 1/L) = g_{GF}^2(sL)$
- Choose new bigger lattice size L'
- Tune β' such that $g_{GF}^2(L') = u$
- Double the lattice and measure $\Sigma(u, 1/L') = g_{GF}^2(sL')$
- Do for all lattice sizes, change u and repeat

Step scaling theory

- Step scaling function in the lattice and continuum:

$$\Sigma(s, u, a/L) = g_{GF}^2(g_0, s \frac{L}{a}) \Big|_{g_{GF}^2(g_0, \frac{L}{a})=u}, \quad \sigma(u, s) = \lim_{a/L \rightarrow 0} \Sigma(u, s, a/L)$$

- Use interpolated couplings for consistent u and do the limit as:

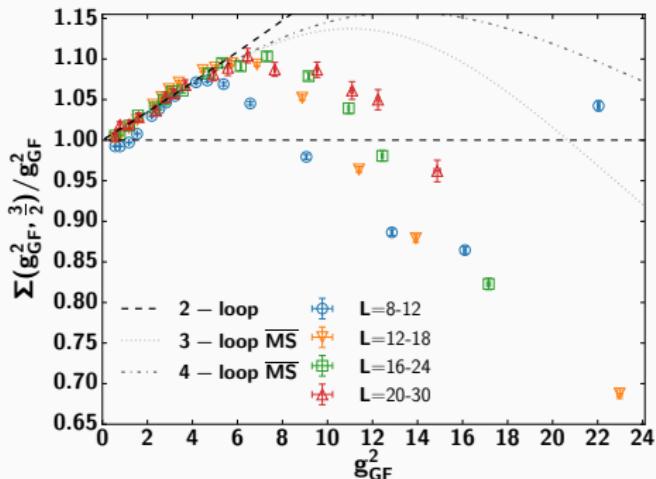
$$\Sigma(u, s, a/L) = \sigma(u, s) + c(u) \left(\frac{L}{a} \right)^{-2}$$

- At fixed point $\sigma(u)/u = 1$
- Related to beta function:

$$-2 \ln(s) = \int_{\sqrt{u}}^{\sqrt{\sigma(u,s)}} \frac{dx}{\beta(x)}, \quad \beta(g) = \frac{g}{2 \ln(s)} \left(1 - \frac{\sigma(g^2, s)}{g^2} \right)$$

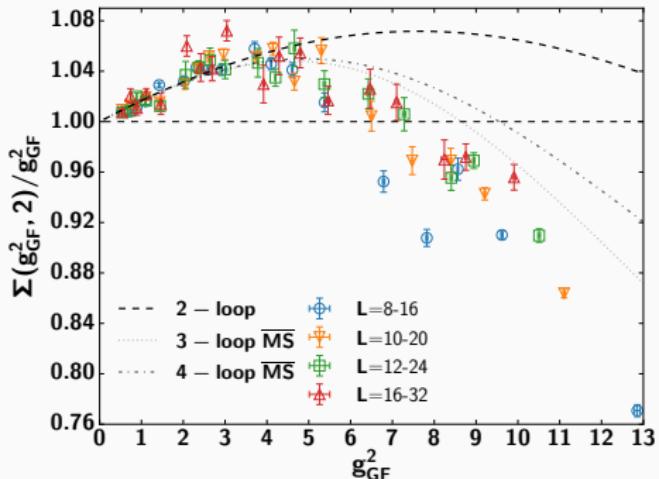
- For $N_f = 8$ we use $s = 2$, Pairs: 8 – 16, 10 – 20, 12 – 24, 16 – 32
- For $N_f = 6$ we use $s = 1.5$, Pairs: 8 – 12, 12 – 18, 16 – 24, 20 – 30

Raw step scaling function



$$N_f = 6$$

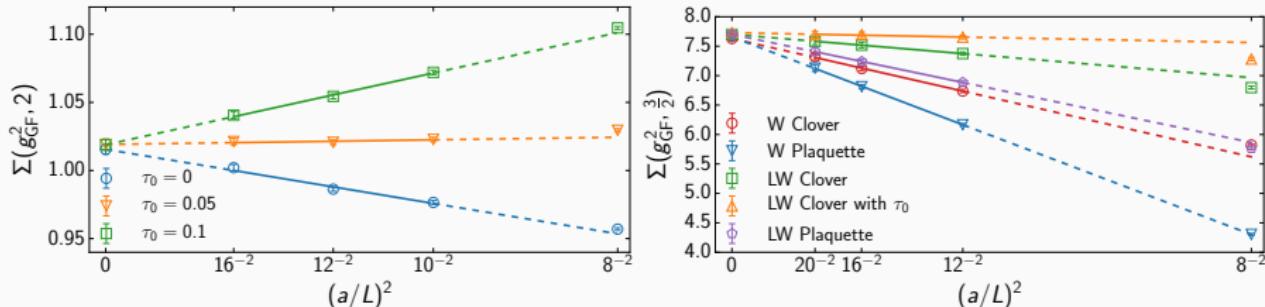
$$s = 3/2, c = 0.3$$



$$N_f = 8$$

$$s = 2, c = 0.4$$

Discretizations and τ -correction

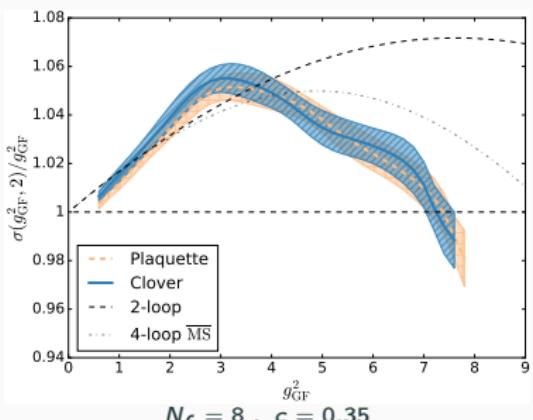
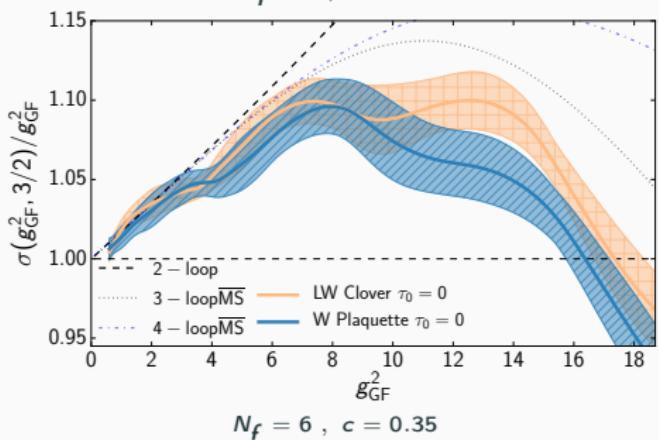
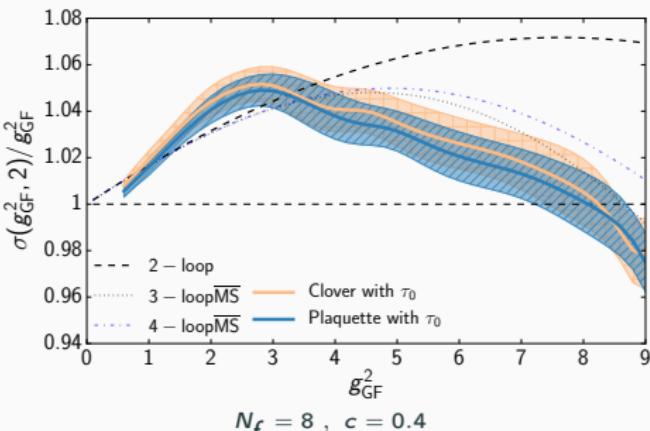
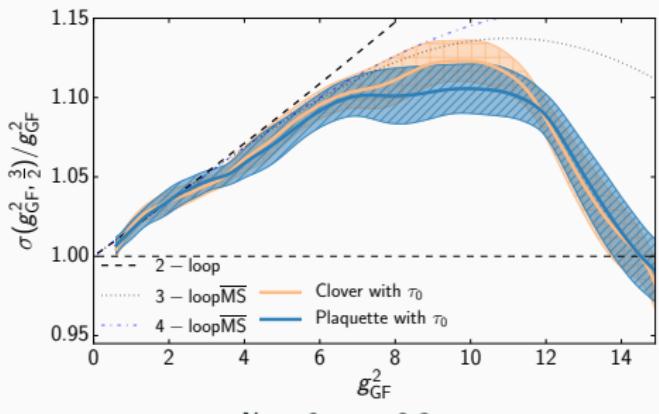


- Different actions of simulation, flow and energy density give different discretization effects
- Reduce effects by τ -shift:

$$g_{GF}^2 = \mathcal{N}^{-1} t^2 \langle E(t + \tau_0 a^2) \rangle = \mathcal{N}^{-1} t^2 \langle E(t) \rangle + \mathcal{N}^{-1} t^2 \langle \frac{\partial E(t)}{\partial t} \rangle \tau_0 a^2$$

- $N_f = 8$: $\tau = 0.06 \log(1 + g_{GF}^2)$, $N_f = 6$: $\tau = 0.025 \log(1 + 2 * g_{GF}^2)$
- Chosen discretization: LW evolved flow with Clover measurement

Continuum Step scaling function



Slope of β -function: γ_g^*

- Scheme independent observable at fixed point
- β -function related to step scaling function as:

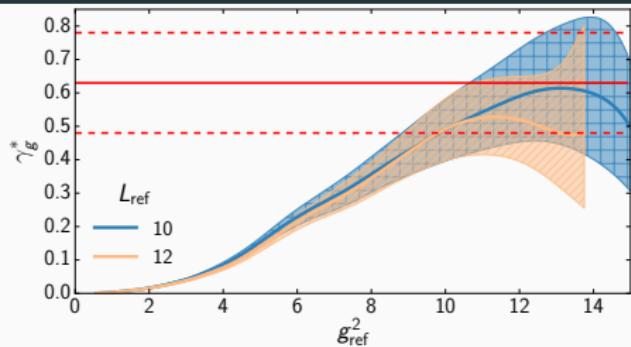
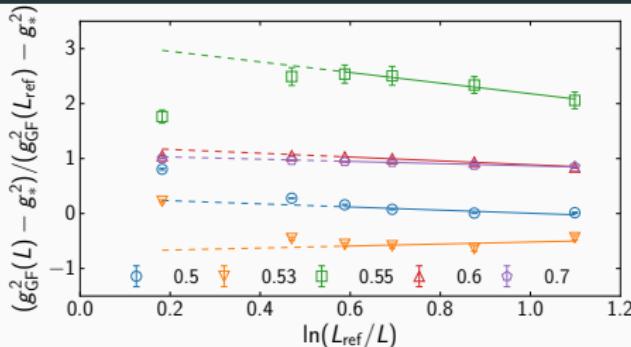
$$\beta(g) = \frac{g}{2 \ln(s)} \left(1 - \frac{\sigma(g^2, s)}{g^2} \right)$$

- We can fit a line to points around IRFP
- Possible with $N_f = 6$ ($N_f = 8$ errors too big)

	$c_t = 0.3$	0.35	0.4	0.45
g_*^2	$14.5(3)^{+0.41}_{-1.38}$	$17.3(5)^{+0.77}_{-1.73}$	$22.6(7)^{+1.14}_{-2.89}$	$31(1)^{+1.8}_{-21.1}$
γ_g^*	$0.63(15)^{+0.28}_{-0.27}$	$0.67(11)^{+0.21}_{-0.11}$	$0.69(11)^{+0.11}_{-0.26}$	$0.67(12)^{+0.15}_{-0.55}$

- γ_g^* constant w.r.t c_t (though errors are big)
- Agrees with the recent scheme-independent theoretical estimate:
 $\gamma_g^* = 0.6515$

Slope of β -function: γ_g^*



- Previous slide relies on reliability of continuum limit at IRFP
- Alternative method:

$$\beta(g_{\text{GF}}^2) = -\mu \frac{dg_{\text{GF}}^2}{d\mu} = \gamma_g^*(g_{\text{GF}}^2 - g_*^2)$$

- Integrate from L_{ref} to L

$$g_{\text{GF}}^2(\beta, L) - g_*^2 = [g_{\text{GF}}^2(\beta, L_{\text{ref}}) - g_*^2] \left(\frac{L_{\text{ref}}}{L} \right)^{\gamma_g^*}.$$

Mass anomalous dimension: γ_m

- Another scheme independent observable at fixed point
- Running of the quark mass is determined by:

$$\mu \frac{dm(\mu)}{d\mu} = -\gamma(g^2)m(\mu)$$

- In conformal theories all masses follow the power-law:

$$m_q^{1/(1+\gamma)}$$

- Combines the two scales needed by ETC theories

$$\langle \bar{\psi}\psi \rangle_{ETC} \sim \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^\gamma \langle \bar{\psi}\psi \rangle_{TC}$$

- Many technicolor theories require γ of order 1

Mass anomalous dimension: Step scaling method

- Measure the pseudoscalar density renormalization constant:

$$Z_P(g_0, \frac{L}{a}) = \frac{\sqrt{Nf_1}}{f_p(\frac{1}{2} \frac{L}{a})}$$

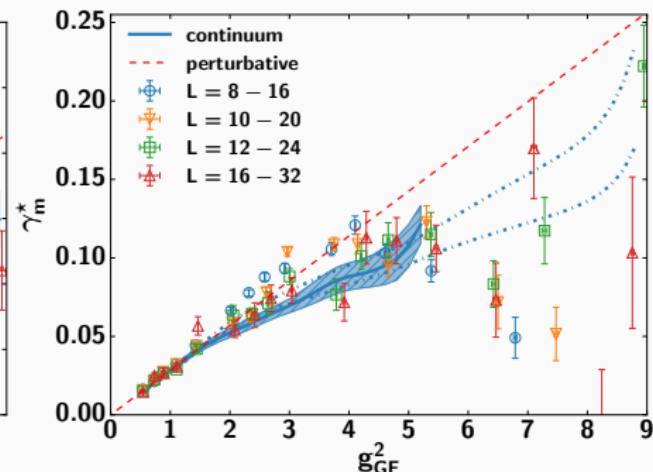
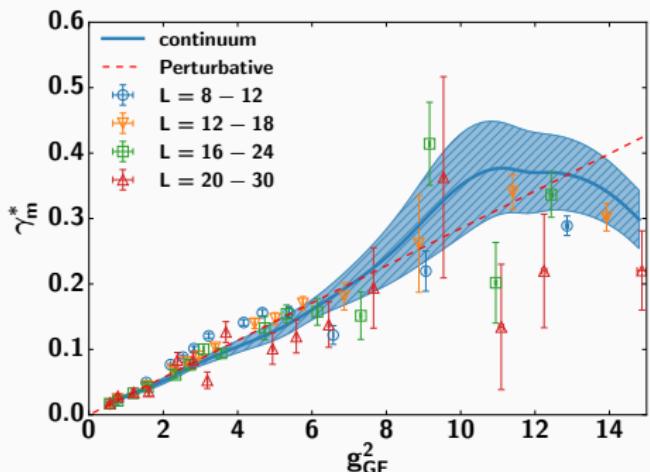
- Step scaling procedure similar to coupling

$$\Sigma_P(u, \frac{a}{L}) = \left. \frac{Z_P(g_0, \frac{sL}{a})}{Z_P(g_0, \frac{L}{a})} \right|_{u=g_{GF}^2}, \quad \begin{aligned} \Sigma_P(u) &= \sigma_P(u, s) + c(u)(\frac{L}{a})^{-2} \\ \sigma_P(g^2) &= \lim_{a \rightarrow 0} \Sigma_P(g^2, \frac{a}{L}) \end{aligned}$$

- We choose $s = 2$ for both N_f 's
- Interpolate Z_P with $Z_P = 1 + \sum_{i=1}^5 a_i g_0^{2i}$
- Related to mass anomalous dimension γ by:

$$\gamma^* = -\frac{\log \sigma_P(g^2)}{\log s}$$

Mass anomalous dimension: Step scaling method



- Gives results comparable to perturbation theory
- Method breaks at large coupling
- γ is quite small

Mass anomalous dimension: Spectral method

- The mode number of Dirac operator defined from eigenvalue density:

$$\nu(\Lambda) \equiv 2 \int_0^{\sqrt{\Lambda^2 - m^2}} \rho(\lambda) d\lambda$$

- For massless theory at IRFP should follow the scaling:

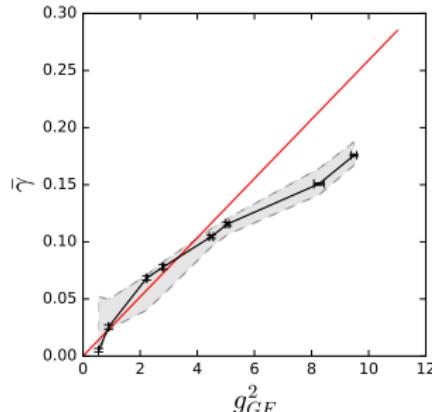
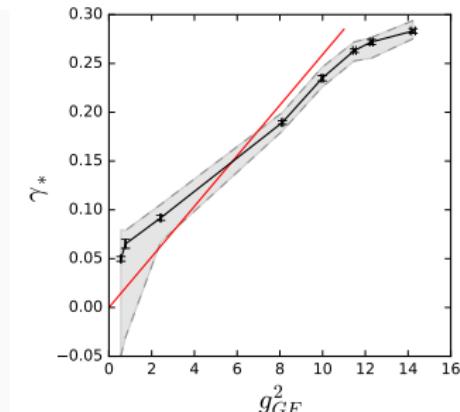
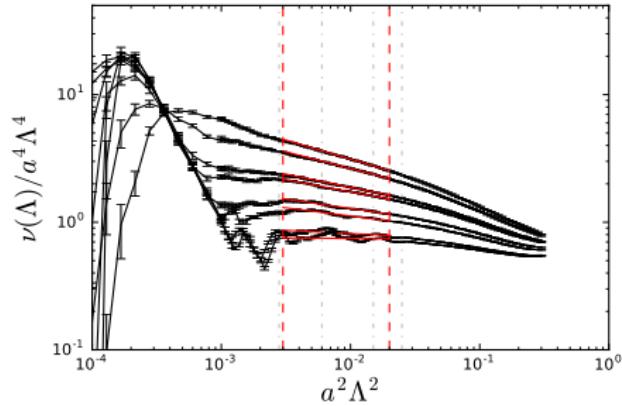
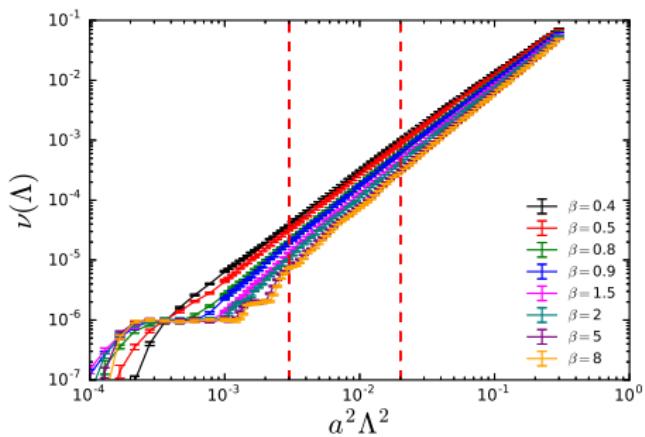
$$\nu(\Lambda) \simeq C \Lambda^{4/(1+\gamma_*)}$$

- Measure mode number from the lattice configurations:

$$\nu(\Lambda) = \lim_{V \rightarrow \infty} \frac{1}{V} \langle \text{tr } \mathbb{P}(\Lambda) \rangle,$$

- \mathbb{P} projects full eigenspace of $M = m^2 - \not{D}^2$ to the eigenspace of eigenvalues smaller than Λ^2 ; Stochastically do the trace

Mass anomalous dimension: Spectral method



Conclusions

- We can accurately reach strong coupling with GF
- Clear indication for fixed point
 - $N_f = 6$: $g_*^2 = 14.5(3)^{+0.41}_{-1.38}$
 - $N_f = 8$: $g_*^2 = 8.24(59)^{+0.97}_{-1.64}$

⇒ Conformal window edge between $N_f = 4 - 6$
- Mass anomalous dimension:
 - Mass step scaling works for small couplings
 - Spectral method works for large couplings
 - γ^* has relatively small value
 - $N_f = 6$: $\gamma_m^* = 0.283(2)^{+0.01}_{-0.01}$
 - $N_f = 8$: $\gamma_m^* = 0.15(2)$
- In $N_f = 6$ also measure $\gamma_g^* = 0.63(15)^{+0.28}_{-0.27}$, close to theoretical estimate $\gamma_g^* = 0.6515$