Universal non-perturbative features in the thermal behavior of non-Abelian gauge theories

Marco Panero

University of Turin and INFN, Turin, Italy

NIC, DESY Zeuthen
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Outline

Based on:

- ★ M. Caselle, A. Nada and M. P., JHEP **07** (2015) 143, arXiv:1505.01106
- ★ M. Bruno, M. Caselle, M. P. and R. Pellegrini, JHEP **03** (2015) 057, arXiv:1409.8305

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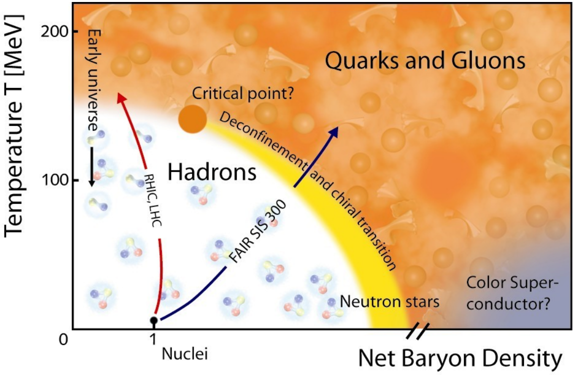
Strong nuclear interactions under extreme conditions

Quantum Chromodynamics is believed to possess a rich phase structure



[N. Cabibbo and G. Parisi, 1975], **[J. C. Collins and M. J. Perry, 1975]**

Why is it important?

Adapted from **[Z. Weiner, 2010]**

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- An important test of QCD, one of the building blocks of the Standard Model
- Temperatures $\gtrsim 200$ MeV realized in nature until about 1 μs after the Big Bang; the early Universe cooling rate depends on the QCD equation of state (EoS)
- Cold and dense QCD matter probably exists in compact stars
- The quark-gluon plasma (QGP) has very peculiar properties **[B. Müller, 2013]**
- Connections to seemingly distant physical systems: superfluids, ultracold atoms, fermionic condensed matter systems, black holes, . . . **[E. Shuryak, 2009]**
- Very rich physics, involving several non-trivial theoretical problems
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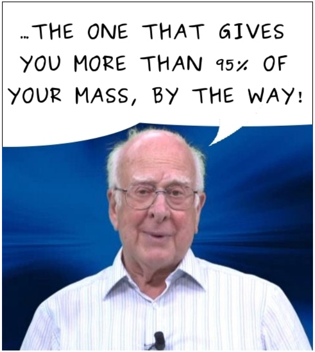

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  - ★ "The most perfect" liquid—shear viscosity close to its quantum limit **[G. Policastro, D. T. Son and A. O. Starinets, 2001]**
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Motivation to study finite-temperature QCD and QCD-like theories on the lattice

- Robust *ab initio* theoretical predictions for strong interactions under the conditions probed in heavy-ion collisions
- Weak-coupling expansions in thermal QCD become accurate only at high temperatures, fail to capture the physics of long-wavelength modes in the deconfined phase **[A. Linde, 1980]** and the dynamics in the hadronic phase
- The lattice regularization **[K. G. Wilson, 1974]** provides *the* only known mathematically well-defined, non-perturbative formulation of QCD
- Thanks to steady theoretical, algorithmic and computer-power progress, lattice computations are now producing accurate numerical predictions for QCD at finite temperature
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Outline

Glueball gas thermodynamics

- The physical states in the confining, low-temperature phase are color-singlet hadrons
- For non-supersymmetric, pure-glue $\mathrm{SU}(N)$ theories, these hadrons are glueballs
- Glueballs are classified according to their J^{PC} quantum numbers, have a finite mass gap, and are weakly coupled
- The equation of state can be described in terms of a gas of massive, free bosons; for example, for the pressure:

$$p(T) = -\frac{\partial F}{\partial V} = \lim_{V\to\infty} \frac{T}{V} \ln Z = \sum_i (2J_i + 1) p(m_i; T)$$

where $p(m_i; T)$ denotes the contribution from each degree of freedom of a state of mass m_i:

$$p(m_i; T) = \frac{T^4}{2} \left(\frac{m_i}{T\pi}\right)^2 \sum_{n=1}^{\infty} \frac{1}{n^2} K_2\left(n \frac{m_i}{T}\right)$$

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A string model for the spectrum

- How many glueball states should be included in the sum?
  - ★ At temperatures much lower than the deconfinement transition temperature T_c, the sum is completely dominated by the lightest glueballs
  - ★ At temperatures closer to T_c, the contribution of heavier states becomes non-negligible
- While the lattice determination of heavy states in the spectrum is challenging **[M. Teper, 1998]**, the spectral density $\hat{\rho}(m)$ can be predicted by modelling glueballs as "rings of glue" **[N. Isgur and J. E. Paton, 1984]**
- The effective description of confining flux tubes as open strings **[M. Lüscher, 1980]** suggests to model the glueballs as closed bosonic strings, leading to:

$$\hat{\rho}(m) = \frac{1}{m}\left(\frac{2\pi T_{\mathrm{H}}}{3m}\right)^3 \exp\left(m/T_{\mathrm{H}}\right)$$

where T_{H} is the Hagedorn temperature **[R. Hagedorn, 1965]**

- The total contribution to the pressure, including contributions from the lightest states and from a continuous spectrum above a threshold $m_{\mathrm{threshold}} = 2m_{0^{++}}$, reads

$$p(T) = \sum_{m_i < m_{\mathrm{threshold}}} (2J_i+1)p(m_i;T) + n_C \int_{m_{\mathrm{threshold}}}^{\infty} \mathrm{d}m' \hat{\rho}(m') p(m';T)$$

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$\mathrm{SU}(2)$ gauge theory

- $\mathrm{SU}(2)$ Yang-Mills theory is the simplest confining gauge theory based on a non-Abelian Lie group
- Due to its small dimension, it has been studied as a QCD prototype since the early days of lattice QCD **[M. Creutz, 1980]**
- Larger Lie groups have (several) $\mathrm{SU}(2)$ subgroups—a feature which is also used in simulation algorithms **[N. Cabibbo and E. Marinari, 1982]**
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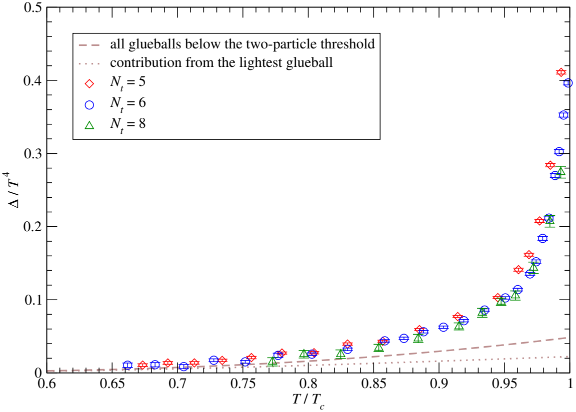
Lattice results: Impact of heavy states

Our results for the trace anomaly in units of the fourth power of the temperature

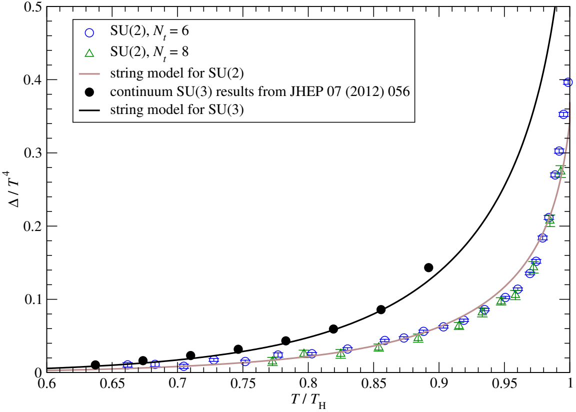
$$\frac{\Delta}{T^4} = T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right)$$

show that heavy glueballs give a large contribution to the equation of state close to T_c



Lattice results: Comparison with the bosonic string model

The predictions of a gas of massive glueballs, including contributions from heavy states modelled by a closed bosonic string model, are in excellent agreement with lattice data for both $\mathrm{SU}(2)$ and $\mathrm{SU}(3)$ Yang-Mills theories **[S. Borsányi et al., 2012]**—see also **[H. B. Meyer, 2009]**, **[M. Caselle et al., 2011]**, **[F. Buisseret and G. Lacroix, 2011]**



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G_2 Yang-Mills theory

- The exceptional group G_2 (with dimension 14 and rank 2) is the smallest compact, simply connected Lie group with a trivial center
- "Quarks" in the fundamental representation **7**, "gluons" in the adjoint **14**
- Due to the absence of center vortices, which are important rôle for confinement **[L. Del Debbio et al., 1996]**, **[P. de Forcrand and M. D'Elia, 1999]**, G_2 Yang-Mills theory has triggered interest in the lattice community **[K. Holland et al., 2003]**
- The theory is actually "screening" at large distances: a fundamental color source can be screened by three gluons

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- The absence of N-ality also implies that the theory admits both bosonic and fermionic "baryons"

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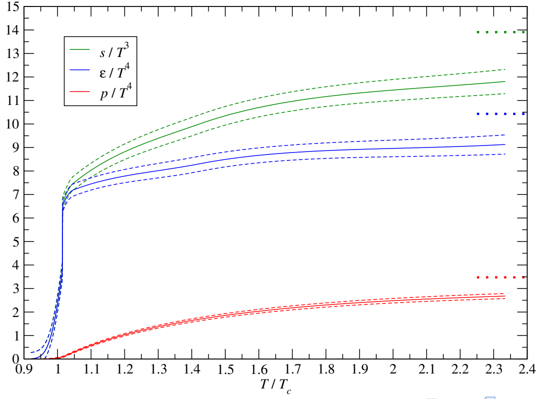

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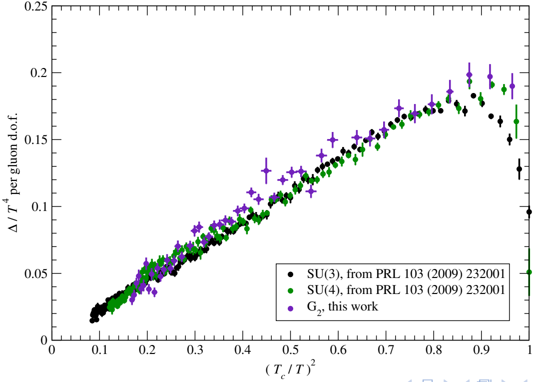

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- Lattice simulations also reveal interesting universal, non-perturbative aspects in the thermodynamics of different, QCD-like gauge theories
  - ★ The confining phase can be described as a gas of free glueballs, provided contributions from a Hagedorn-like excitation spectrum are included
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