A lattice Higgs-Yukawa Model with Overlap Fermions

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Outline

Motivation/Introduction

Higgs Boson Mass bounds

Finite T phase transition

Collaborators:

- DESY-Zeuthen, Germany: Karl Jansen, Attila Nagy, Philipp Gerhold, Jim Kallarackal - Higgs mass bounds, Finite T phase transition at large y
- CERN: JB - Higgs mass bounds, Finite T phase transition at large y
- National C.T. University, Taiwan: George Hou, David Lin, Bastian Knippschild, Brian Smigielski - bulk phase transition at very large y

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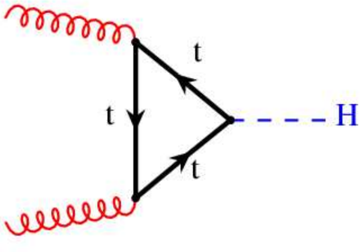
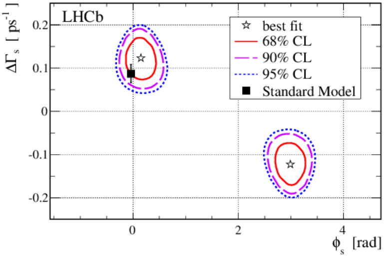
Motivation

- A heavy 4th gen. of quarks $\begin{pmatrix} t' \\ b' \end{pmatrix}$ has not been (completely) ruled out.
- Large $m_{t',b'}$ means large $y_{t',b'}$. Perturbative methods may not work.
- Lattice field theory enables study of the Higgs-Yukawa sector (NO gauge fields) at large y.
- Effects on the Higgs boson mass bounds from a possible 4G are timely.
- Effects on the thermal phase transition may be relevant for electroweak baryogenisis.

The 4G is dead:

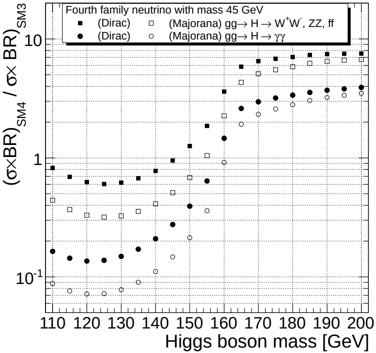
- Direct searches: ATLAS, CMS most stringent bound $\sim 450\mathrm{GeV}$ in $t' \to bW$, $b' \to tW$.
- Higgs boson production: Enhancement of σ_H
- Precision flavor observables: e.g $\sin 2\beta_s$ (LHCb). In 4G $\sin 2\beta_s \sim -0.33$ (Hou, Ma '10)

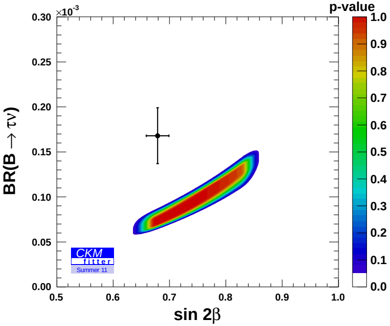




The 4G is not dead:

- Most stringent direct searches assume e.g. $B(t' \to bW) = 100\%$.
- (Cetin, et al '11): Really should consider $B(pp \to H) \times B(H \to X)$
- (Lenz, et al '11): $V^{CKM}_{4\times4} \neq 1$, l_4 really complicate things! Limits as a function of the mixing angles. (CKMFitter)





What can the 4G do for me? (Holdom et al, '08)

- Alleviate tension btw. EW fits of M_H and exclusion bounds
- Dark matter candidate
- More CKM phases → more CP violation
- Enforces R-parity in MSSM (Wise, et al)
- Possibilities of a UV fixed point (P.Q. Hung, et al)

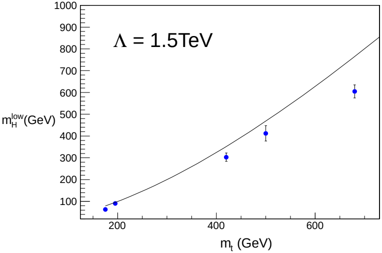
Two questions we will ask:

- How does the 4G affect Higgs boson mass bounds?
  - (Holdom, et al '08): Allowed range decreases with m_f
  - Does this hold up non-perturbatively?
- How does the 4G affect the EW-phase transition?
  - (Laine, et al '96): Gauge-Higgs models unsuitabe for EW baryogenisis
  - Can large yukawas induce a 1st order transition?

Main Results

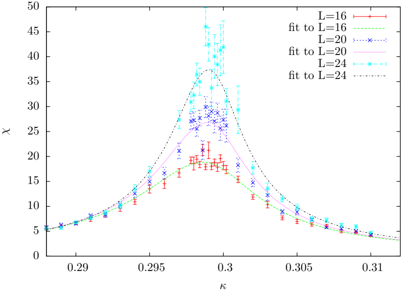
Higgs mass lower bound:

- Numerical data increases w/ m_q (blue points)
- Trend suggested by minimizing the LO eff. potential. (black line)



Thermal phase transition (preliminary):

m_q	T_c
0GeV	350GeV
175GeV	500GeV
650GeV	$\gtrsim$ 1000GeV



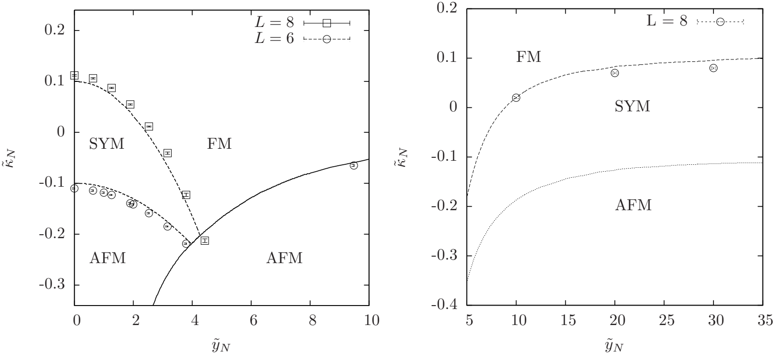
Introduction

Examine the Higgs-Yukawa sector of the standard model:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_\phi + \mathcal{L}_F \\
\mathcal{L}_\phi &= \frac{1}{2}\sum_\mu |\partial_\mu \phi|^2 + \frac{1}{2} m_0^2 |\phi|^2 + \lambda |\phi|^4 \\
\mathcal{L}_F &= \sum_{i=1}^{N_f} \bar{\psi}^i \partial\!\!\!/ \psi^i + y_b \bar{\psi}_L^i \phi b_R^i + y_t \bar{\psi}_L^i \tilde{\phi} t_R^i + h.c. \\
&= \sum_i \bar{\psi}^i M[\phi] \psi^i
\end{aligned}$$

- Complex scalar ϕ is a **2** of $SU(2)_L$
- Fermion doublet $\psi = \left(\begin{smallmatrix} t \\ b \end{smallmatrix}\right)$. ψ_L is a **2** of $SU(2)_L$
- NO GAUGE FIELDS!! $\Rightarrow$ GLOBAL $SU(2)_L$ symmetry

Phase structure (e.g. Gerhold, Jansen '07): two order parameters (m, s).



LO Large-N_f and $N_f = 10$ MC results for fixed λ. Dashed (2nd order) and solid (1st order) lines are phase transtions.

We study the model in the (spontaenously broken) $m = v \neq 0, s = 0$ FM phase.

- Fermions aquire a mass : $m_f \sim yv$
- Spectrum: fermion, goldstones, and a massive scalar (Higgs)

UV fixed point is trivial: cutoff cannot be removed w/ non-zero interactions.

Triviality (upper) bound on m_H (Daschen, Neuberger '83) (Luscher, Weisz '87):

$$\ln\left(\frac{\Lambda}{m}\right) \le \frac{A}{g_r} + B \ln g_r + \mathrm{O}(1)$$

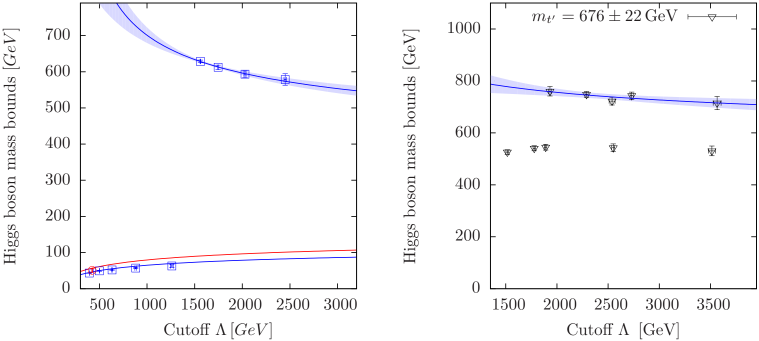
Large-N_f evidence that yukawa theory is also trivial (Kuti, et al '07).

Non-universal! Value of the bounds can vary by $\sim 20 - 30\%$ with the lattice discretization. (Heller, et al. '93)

No evidence for non-trivial f.p.'s at large coupling. (e.g. Weisz, Wolff '11).

Vacuum instability (lower) bound on m_H (e.g. Elias-Miro, *et al.* '11): Demand that Eff. potential 'curves up'.

For ϕ^4, bounds occur at $\lambda = 0, \infty$.



(Gerhold, Kallarackal, Jansen '10)

History of Higgs-Yukawa models on the lattice

- (Quenched) Naive fermions (Bock, et al. ‘89) : Doublers!!!
- Wilson fermions (Smit ’86)(Swift ’84): Wilson term breaks chiral symmetry.
- Mirror fermions (e.g. Lin et al. ‘92): Tuning required to decouple mirror fermions.
- Overlap! (Lusher ‘98): Exact chiral symmetry at finite a.

Lattice Discretization

Integrate out fermion fields

$$\langle 0|O[\hat{\phi}]|0\rangle_c = \frac{\int \prod_x d\phi_x \; O[\phi] \det M[\phi] \, \mathrm{e}^{-S[\phi]}}{\int \prod_x d\phi_x \det M[\phi] \, \mathrm{e}^{-S[\phi]}}$$

Limit $L_s \to \infty$ must be taken. Finite T $\Leftrightarrow$ Finite $L_t = \frac{1}{T}$

Continuum Limit ($a \to 0$) cannot be taken. Instead (provided $am << 1$):

- Vary a, dependence of results should be mild
- Vary regularization (discretization), dependence of results should be mild

Examine the free naive Fermion Propagator for $m = 0$

$$D(p)\Big|_{m=0} = \frac{-ia\sum_\mu \gamma_\mu \sin(ap_\mu)}{\sum_\mu \sin^2(ap_\mu)}$$

Poles at $p_\mu = 0, \pi/a \Rightarrow 2d$ Fermion states!

Theorem (Nielsen, Ninomiya '81): You CAN'T have all of the following

- Chiral Symmetry of the form

$$\{D, \gamma_5\} = 0\ ,\ \ D = \sum_\mu \gamma_\mu D_\mu$$

- No fermion doublers
- A real, local action.

NN thm workaround: Settle for the chiral symmetry (Ginsparg, Wilson ‘82) (Luscher, ‘98)

$$\{D, \gamma_5\} = aD\gamma_5 D$$

The Overlap operator (Neuberger ‘97)

$$\mathcal{D}^{(ov)} = \rho\left[1 + \frac{A}{\sqrt{A^\dagger A}}\right], \; A = \mathcal{D}^W - \rho$$

$$\mathcal{D}^W = \gamma_\mu \nabla_\mu - \frac{ar}{2}\nabla^2$$

No gauge fields $\Rightarrow \mathcal{D}^{ov}_{pp'} = d(p)\delta_{pp'}$

At finite a, action obeys a (modified) $SU(2)_L$

$$\phi \to U\phi, \qquad U \in SU(2)$$

$$\psi_L = \hat{P}_-\psi \to U\psi_L, \qquad \bar{\psi}_L = \bar{\psi}P_+ \to \bar{\psi}_L U^\dagger$$

$$\hat{P}_\pm = \frac{1 \pm \hat{\gamma}_5}{2}, \qquad \hat{\gamma}_5 = \gamma_5\left(1 - \frac{a}{\rho}\mathcal{D}^{ov}\right)$$

Introduce pseudo-fermion fields ω.

$$\left(\det M[\phi]\right)^{N_f} = \mathrm{e}^{-\omega^\dagger (M^\dagger M)^{-\frac{N_f}{2}}\omega}$$

Computational cost dominated by $\mathcal{M}[\phi]\omega$

$$\mathcal{M}[\phi] = \mathcal{D}^{ov} + P_+ y \Phi \hat{P}_+ + h.c.$$

N_f odd → PHMC (Frezzotti, et al)

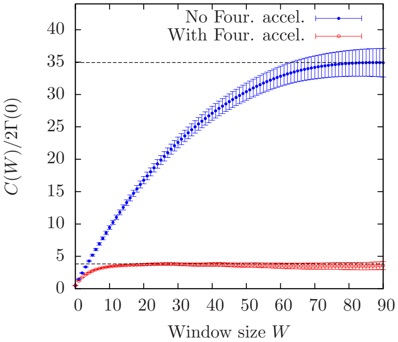
Simulation in momentum space: FFT algorithms are $\mathrm{O}(V \log V)$

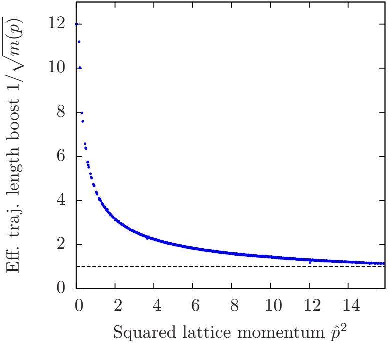
No expansion in $\mathcal{D}^{ov}$ is needed!

Additional tricks

Fourier Accleration (Batrouni et al. ‘85) : longer MC time for low p modes

$$\frac{\pi^\dagger\pi}{2} \to \sum_p \frac{\pi^\dagger\pi}{2m(p)}, \qquad m(p) = \frac{1}{4}\langle|F_p[\phi,\omega]|^2\rangle, \qquad \tau_p = \frac{\tau}{\sqrt{m(p)}}$$





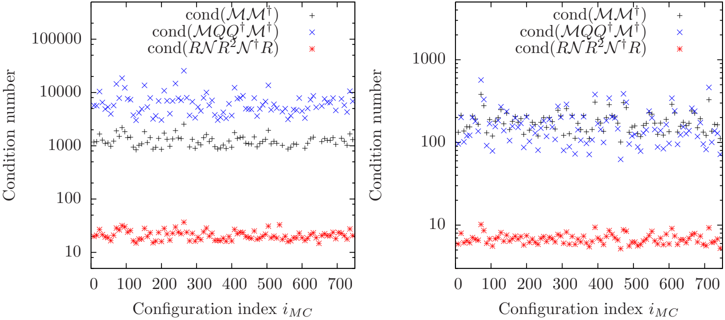
(Gerhold ‘10)

Preconditioning : Assume SSB ansatz

$$\phi_v = v\hat{\phi}, \qquad Q = M[\phi_v]^{-1}, \qquad \det M^\dagger M = \det M^\dagger Q^\dagger Q M$$

Cond. number of the product is not the produt of the cond. nums.!

Condition number reduced by $\sim 100 - 1000$

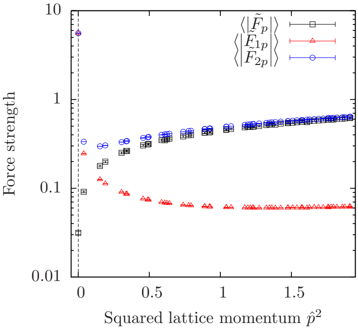


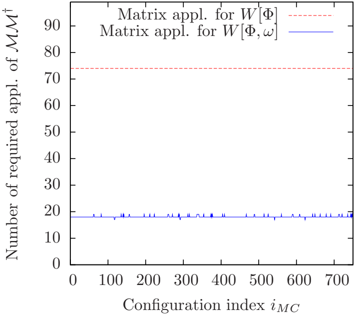
(Gerhold, '10)

Krylov reweighting: correct for finite expansion.

Multiple-timescale Integration:

- F_p: Total force
- F_{1p}: Psuedo-fermion
- F_{2p}: Scalar





(Gerhold, '10)

Set the scale using $v^{phys} = 246\mathrm{GeV}$. $\Lambda = \frac{1}{a} = \frac{v^{phys}}{v_R}$

Obtain the v.e.v from the rotated field (no ext. 'trigger' J)

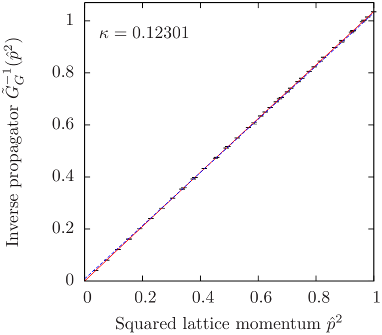
$$\phi^{rot} = \frac{1}{V}\begin{pmatrix} 0 \\ |\sum_x \phi(x)| \end{pmatrix}$$

$$\lim_{V\to\infty} \langle \phi^{rot} \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

bare v is renormalized

$$v_R = Z_G^{-1/2} v$$

$$Z_G^{-1} = \frac{\partial}{\partial p^2} \mathrm{Re}\ G_G^{-1}(p^2)\Big|_{p^2=-m_G^2}$$



Z_G from fits to the $G_G^{-1}(p^2)$:

$L_s/a = 32 = L_t/a$

$m_t = 175\mathrm{GeV}$, $\Lambda = 1.2\mathrm{TeV}$

Z_G (Pert. Fit) : 0.9641(37)

Z_G (Linear Fit) : 0.9655(4)

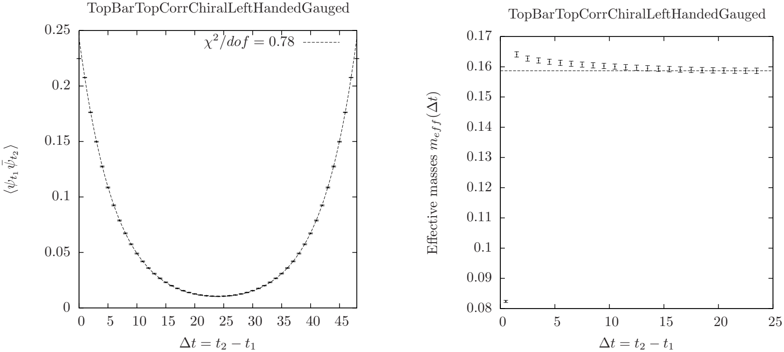
Two Fit ansätze:

- Perturbative. Fit:

$$G_G^{-1}(p^2) = \frac{1}{Z_G}\left(p^2 + m_G^2 + A\left[\mathcal{I}(p^2, m_H^2, m_G^2) - \mathcal{I}(0, m_H^2, m_G^2)\right]\right)$$

- Linear Fit:

$$G_G^{-1}(p^2) = \frac{p^2 + m_G^2}{Z_G}$$



Top quark mass determination: $L_s/a = 24$, $\Lambda = 1.5\mathrm{TeV}$, $m_t = 195\mathrm{GeV}$, $\lambda = 0$

- Fits to temporal correlation function(Left):

$$C(t) = A \cosh\left[m(t - L_t/2)\right]$$

- Fit value and the effective mass (Right).

$$m_{eff}(t) = m + \mathrm{O}(e^{-\Delta t})$$

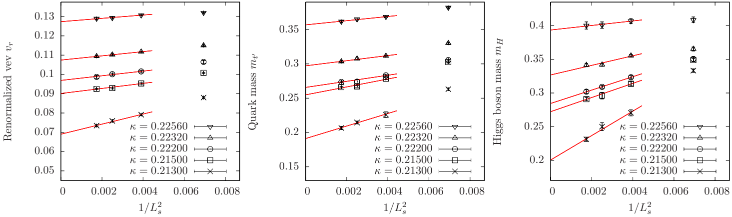
Finite Volume Effects

Finite voume effects in $O(4)$ symmetric EFT:

- p-regime $(m_G >> 1/L)$: F.V. effects are $\mathcal{O}(e^{-ML})$
- ϵ-regime $(m_G << 1/L)$: F.V effects are $\mathcal{O}(1/L^2)$

Lattice Sizes shown here: $L_s/a = \{12, 16, 20, 24\}$

Left : v, Middle: m_t, Right : M_H^{up}



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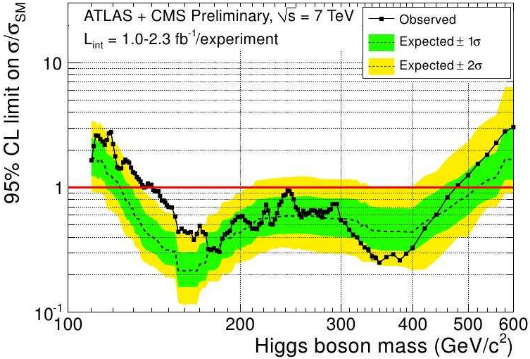
Finite T phase transition

Experimental situation



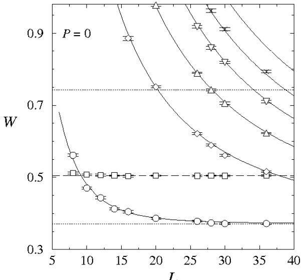
Unstable states in F.V.

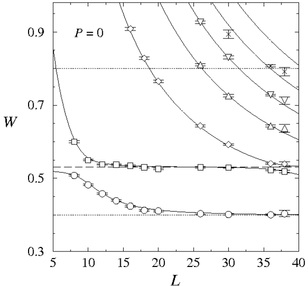
If ‘alone’, F.V energy corresponds to I.V. pole up to (weak) F.V. effects.

Near threshold, F.V energies are distorted. Avoided level crossing occurs.

Example: taken from (Gottlieb, Rummakianen ’95)

- Two scalars: $4m_\phi > m_\rho > 2m_\phi$, $\mathcal{L}_{int} = \frac{\lambda_1}{4!}\rho^4 + \frac{\lambda_2}{4!}\phi^4 + \frac{g}{2}\rho\phi^2$
- Spectrum from GEVP with single and multi-particle ops.
- Left: $g = 0$, Right: $g = 0.008$





Three ways to determine the higgs boson mass:

Pole Mass:

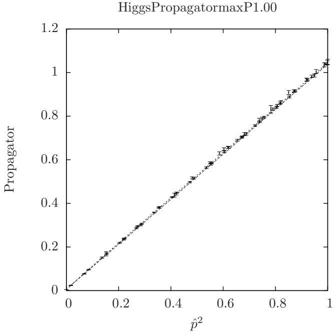
- Fits to the Higgs propagator
- Describe the unstable nature perturbatively or ignore it

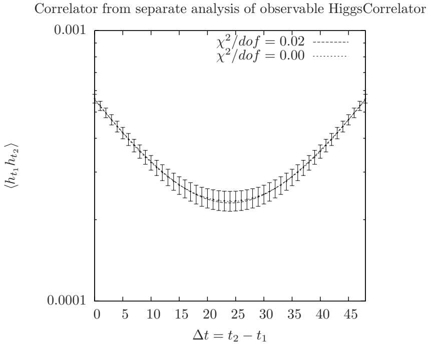
Correlator Mass:

- fits to the temporal correlation function at large t
- at very large t, dominated by two goldstones

Resonance analysis:

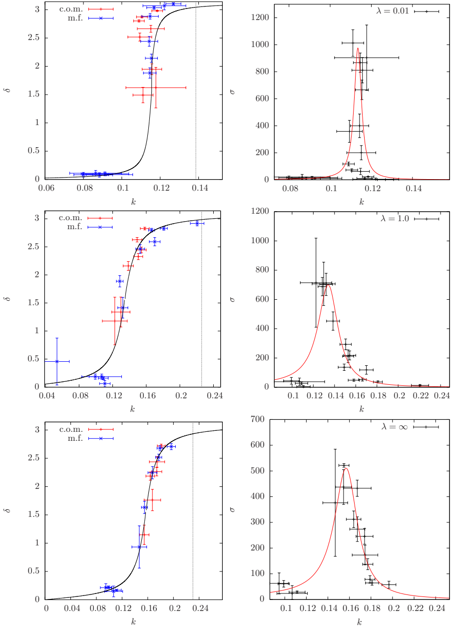
- (Rigorously) calculate $\delta(k)$ below inelastic threshold
- Need to explictly break the symmetry to get $m_G \sim m_H/3$
- Much more difficult! However, can obtain $\Gamma(H \to GG)$





Masses from the propagator and correlator:

- Example: $L_s/a = 24 = L_t/(2a)$, $m_t = 195\mathrm{GeV}$, $\Lambda = 1.5\mathrm{TeV}$
- Fits to the higgs propagator (left): $m_H = 96.0(4.3)\mathrm{GeV}$
- Fits to the higgs temporal correlation function(right): $m_H = 96.4(6.9)\mathrm{GeV}$



$\delta(k)$ and $\sigma(k)$ (Kallarackal '10):

- $m_t = 175\mathrm{GeV}$
- $L_s/a = \{12, 16, 18, 20, 24, 32, 40\}$
- Moving frame to increase resolution (blue points)

λ	0.0	1.0	∞
am_{res}	0.278(2)	0.386(28)	0.405(4)
am_{prop}	0.2811(6)	0.374(4)	0.411(3)
$a\Gamma$	0.007(1)	0.033(4)	0.040(4)

Perturbative Calculations

Use the same regulator: Lattice perturbation theory w/ $\mathcal{D}^{ov}$

Finite volume: $\int \mathrm{d}^4 p \;\to\; \sum_{\hat{p}^\mu}$, Large-$N_f$ limit.

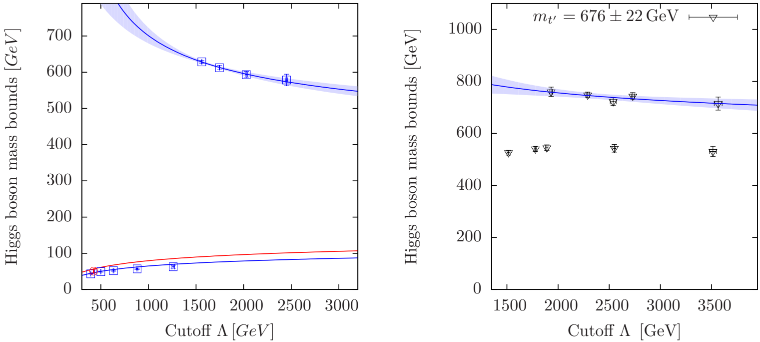
Constraint Effective Potential(Fukuda, Kryiakopoulos '75):

$$VU(v) = \left[S_\phi[v] - N_f \log \det \mathcal{M}[v]\right]\left[1 + \mathrm{O}(N_f^{-1})\right]$$

M_H from curvature at the minimum.

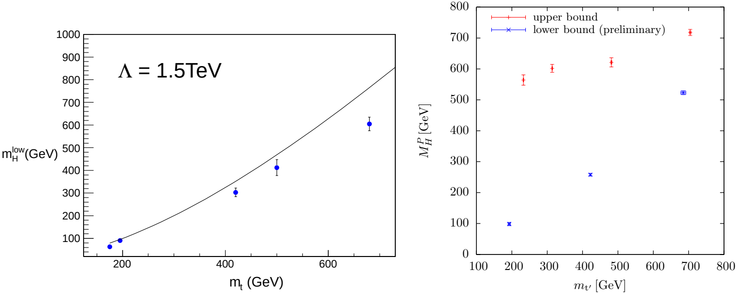
$$m_H^2 = 8\lambda v^2 - \frac{1}{v}U_F'(v) + U_F''(v)$$

$$U_F(v) = \frac{-2N_f}{VT}\sum_p \log\left|\nu^+(p) + y_t v\big(1 - \frac{\nu^+(p)}{2\rho}\big)\right|^2 + \log\left|\nu^+(p) + y_b v\big(1 - \frac{\nu^+(p)}{2\rho}\big)\right|^2$$



Dependence of mass bounds on cutoff (Gerhold, Kallarackal, Jansen '10):

- Lower bound has weak cutoff dep.
- Splitting seems to be a small effect at $m_t = 175\mathrm{GeV}$
- Lower bound increases w/ m_t.



Dependence of the Higgs Mass bounds on m_q at $\Lambda = 1.5\mathrm{TeV}$ (JB, Kallarackal, Jansen, Nagy, in prep):

- TODO: Include corrections to Z_G
- (Denner, *et al.* '11): For $m_{t'} \sim 600\mathrm{GeV}$, EW rad. corrs. in $H \to WW, ZZ$ are -60%.
- 'Window' narrors w/ m_t.

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Electroweak baryongenisis

We live in a matter dominated universe!

Sakharov Requirements for generated a matter/antimatter asymmetry

- Baryon number violation
- CP violation (and C violation)
- Out-of-equilibrim dynamics

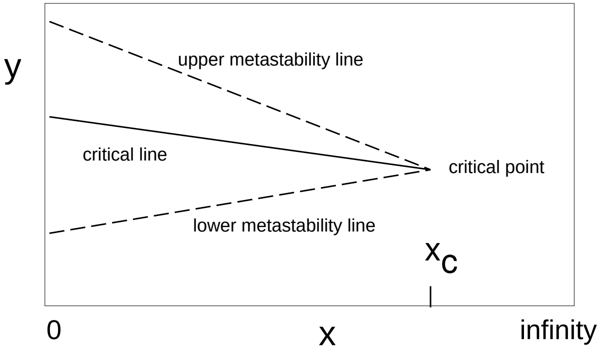
Standard Model (in principle) can generate all of these.

First order phase transitions require bosonic degrees of freedom.

scalar vev. is no longer order param. (Fradkin, Shenker ‘76)

In gauge-higgs models, line of 1st order, tricritical point (Laine, et al ‘96)

Critical point occurs at an m_H below LEP bound!



Finite T transition

(Kikukawa, *et al.* ‘09): 4th generation may cause strongly 1st order EW phase trans.

$L_s^3 \times L_t$ lattice $\Leftrightarrow$ grand-canonical ensemble w/ $T = \frac{1}{L_t}$.

In pure ϕ^4 phase transition is second order (e.g. Jansen, Seuferling ‘90)

With scalars and $SU(2)$ gauge fields, transition is first order (e.g. Fodor, et al. ‘95)

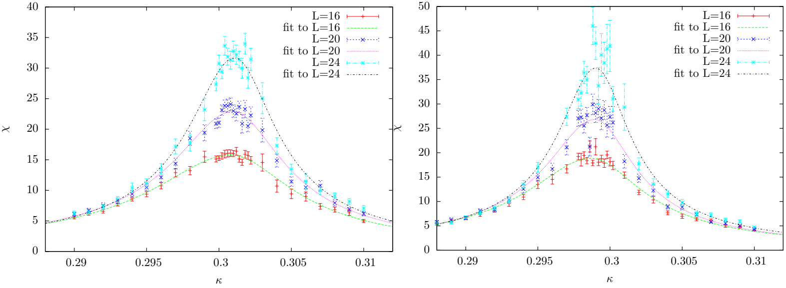
Strategy:

- Fix L_t/a small
- Vary κ at fixed λ and y. Find κ_c.
- Do an $L_t \approx \infty$ simulation using κ_c, λ, y.
- Set the scale in the usual way. Use a to determine L_t from L_t/a.

Susceptibility Fits(JB, Jansen, Nagy, in prep.):

$$\chi = A\left(L_s^{-2/\nu} + B(\kappa-\kappa_c)^2\right)^{-\gamma/2},\ \nu = 0.68,\ \gamma = 1.38$$

Left: $L_t/a = 4$, Right: $L_t/a = 6$ (at $m_t = 175\mathrm{GeV}$).



L_t/a	$T_c(\mathrm{GeV})$	$m_t(\mathrm{GeV})$
4	517.5(2.8)	175
6	509(11)	175
4	$\gtrsim 1000$	650

compare to $T_c = 350\mathrm{GeV}$ in the pure ϕ^4.

Conclusions

Effects of a Heavy Fermion doublet can be studied for arbitrary y using lattice techniques.

- Higgs Boson mass bounds
- Higgs Boson Resonance parameters (w/ $m_G \sim m_H/3$)
- Finite Temp. phase transition

The Higgs mass lower bound seems to depend strongly on m_q, similiar to LO eff. potential.

Finite T phase transition studies are underway

Future Plans:

- Examine the effects of adding $D > 4$ terms to the action.
- Complete Finite Temp. study
- Bound states?
- Examine the effect of gauge fields.