QCD with magnetic fields

Falk Bruckmann
(Univ. Regensburg)

HU Berlin, January 2012

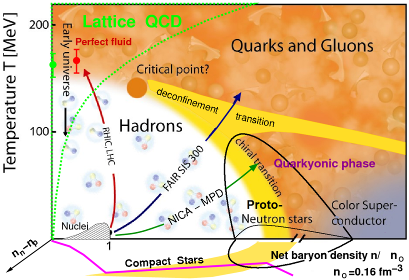
FB, G. Endrődi
1104.5664 [PRD 84]

G. Bali, FB, G. Endrődi, Z. Fodor, S. Katz, S. Krieg, A. Schäfer, K. Szabó
1111.4956 [JHEP accepted]

SFB/
TR 55

Introduction

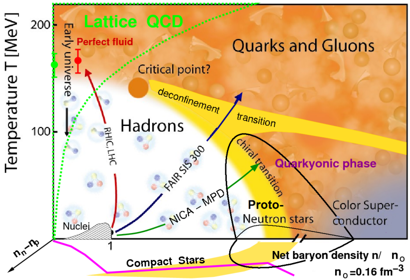
- need to understand quark confinement and chiral symm. breaking
- but also deconfinement and chiral symmetry restoration at finite temperature and/or density ⇒ new phases of matter



© NICA JINR Dubna

Introduction

- need to understand quark confinement and chiral symm. breaking
- but also deconfinement and chiral symmetry restoration at finite temperature and/or density $\Rightarrow$ new phases of matter

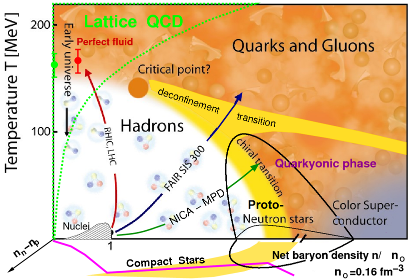


© NICA JINR Dubna

heavy ion colliders: two beams of pos. charges in opposite direction

Introduction

- need to understand quark confinement and chiral symm. breaking
- but also deconfinement and chiral symmetry restoration
  at finite temperature and/or density ⇒ new phases of matter



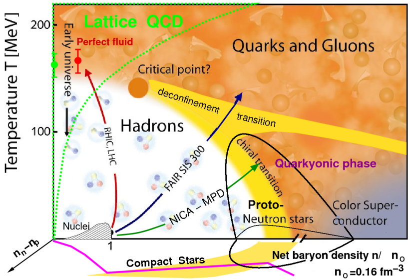
© NICA JINR Dubna

heavy ion colliders: two beams of pos. charges in opposite direction

⇒ ext. magnetic field: short-time, large magnitude; QCD out of equil.!?

Introduction

- need to understand quark confinement and chiral symm. breaking
- but also deconfinement and chiral symmetry restoration
  at finite temperature and/or density ⇒ new phases of matter



© NICA JINR Dubna

heavy ion colliders: two beams of pos. charges in opposite direction

⇒ ext. magnetic field: short-time, large magnitude; QCD out of equil.!?

- ‘chiral magnetic effect’ in particle correlations Kharzeev
- magnetic field as another axis of the QCD phase diagram

Amplitudes of magn. fields

earths magn. field	0.6 G	
magnet	100 G	
laboratory: stable .. unstable	$10^5..10^7$ G	
neutron stars, magnetars	$10^{13}..10^{15}$ G	$\sqrt{eB} \simeq 1$ MeV
RHIC/LHC		$\sqrt{eB} \simeq 0.1..0.5$ GeV
early universe		$\sqrt{eB} \simeq 2$ GeV

Outline

(0) magn. fields on the lattice

(1) dressed Wilson loops — FB, Endrődi 11

magn. fields as tool

(2) QCD transition (at zero density) — Bali et al 11

staggered quarks at realistic masses

magn. field external, Euclidean space = thermal equilibrium

Magnetic fields on the lattice

B along z-direction and constant

- recall gauge field for continuum and infinite volume:

$$A_x = 0\,, \qquad A_y = Bx$$

- naively: $SU(3)$ links to be multiplied by

$$u_x = 1\,, \qquad u_y = \exp(ia \cdot qB \cdot an_x)$$

  in Dirac operator and its determinant
- recall quarks' electric charges $(q_u, q_d, q_s) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})e$

both not periodic in x, make periodic:

Magnetic fields on the lattice

- magn. fields on finite area quantized 'T Hooft 79

$$qB = 2\pi \frac{N_B}{L_x L_y}\,, \quad L_x L_y = N_x N_y a^2\,, \quad N_B \in \mathbb{Z} \qquad (q = \frac{e}{3})$$

  like momentum (quantized in terms of extension L)

- magn. fields on discrete space bounded

$$N_B \leq N_x N_y$$

  like momentum (bounded in UV = Brillouin zone)
  at max. B flux through a plaquette $exp(i2\pi \frac{N_x N_y}{A} a^2) = 1$: no effect

- correct link factors slightly more complicated than above
- state-of-the-art lattice configurations at finite $T = 1/N_t a$:

$$\sqrt{qB} \simeq 0.1 \ldots 1\,\text{GeV}$$

Dressed Wilson loops: idea

‘Wilson Xloop 800’

quark-antiquark pair:

creation, life, annihilation of inf. heavy quarks

confinement = area law:

$\langle \mathrm{tr}\, W \rangle \stackrel{!}{\to} e^{-T\sigma R} = e^{-\sigma S}$

real. quarks $\to$ fuzzy lines

how to get Wilson loops of fixed area, but arbitrary geometry?

Dressed Wilson loops: idea

'Wilson Xloop 800'

quark-antiquark pair:

creation, life, annihilation of inf. heavy quarks

confinement = area law:

$\langle \text{tr}\, W \rangle \stackrel{!}{\to} e^{-T\sigma R} = e^{-\sigma S}$

real. quarks $\to$ fuzzy lines

how to get Wilson loops of fixed area, but arbitrary geometry?

constant magnetic field:

$\Rightarrow$ flux prop. to area S

Dressed Wilson loops: construction

- abelian magn. field $B_z \Rightarrow$ Wilson loops in (x, y)-plane:

$$W \;\rightarrow\; W\, e^{i \oint_C A} = W\, e^{i \iint B} = W\, e^{iBS} \quad \text{[commutes, Stokes]}$$

Dressed Wilson loops: construction

- abelian magn. field $B_z \Rightarrow$ Wilson loops in (x, y)-plane:

$$W \;\rightarrow\; W\, e^{i \oint_C A} = W\, e^{i \iint B} = W\, e^{iBS} \quad \text{[commutes, Stokes]}$$

- in quark condensate:

$$\frac{1}{V}\left\langle \mathrm{tr}\, \frac{1}{D_B + m} \right\rangle \equiv \Sigma_B = \ldots \mathrm{tr}1 \cdot e^0 + \ldots \mathrm{tr}\, W_{1\times 1} \cdot e^{iB} + \ldots$$

  want planar loops only: D_B restricted to (x, y)-plane

- dual condensate = Fourier components:

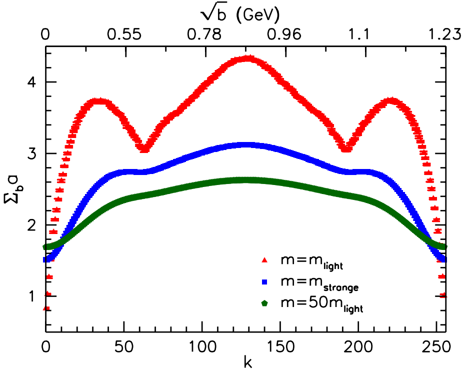
$$\tilde{\Sigma}_S \equiv \sum\!\!\!\!\!\!\int_B \; e^{-iSB} \Sigma_B$$

  picks out all Wilson loops of area $S \equiv$ dressed Wilson loops

Dressed Wilson loops: lattice results I

$2+1$ staggered fermions at $B=0$ ($16^3 \cdot 4$ lattice, $T=146$ MeV)

- condensate Σ_B over magn. field $B = 2\pi \frac{N_B}{256a^2}$, $N_B = 0..256$ [called k]



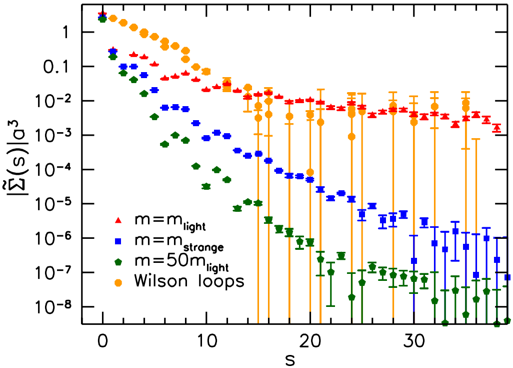
grows with magn. field, shoulders?

heavy quarks: effect washed out ✓

not flat $\overset{\text{Fourier components}}{\Rightarrow}$ dual condensate = dressed Wilson loop

Dressed Wilson loops: lattice results II

- dressed Wilson loop $\tilde{\Sigma}_S$ ($S_{\max} = 256a^2$)



together with conventional Wilson loops

decays with area ✓

better signal/noise (more loops)

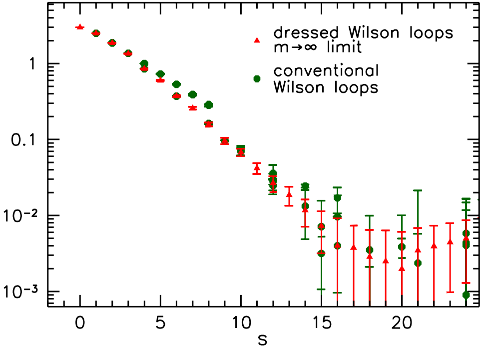
pattern at small sizes? geometry of lattice loops …

geometric series (on the lattice):

$$\tilde{\Sigma}_S \equiv \sum_B e^{-iSB} \frac{1}{V_2} \left\langle \mathrm{tr} \frac{1}{D_B + m} \right\rangle \rightsquigarrow \mathrm{tr} \sum_{n=0}^{\infty} (-1)^n \frac{(D_B)^n}{m^n} \rightsquigarrow \mathrm{tr} \left(\frac{U - U^\dagger}{2am} \right)^n$$

n = length ⇒ large mass suppresses long loops and hence fuzziness

- dressed Wilson loop $\tilde{\Sigma}_S$ for large m (incl. combinatorical factors)



agrees with Wilson loops ✓ ← used for scale setting

Dressed Wilson loops: features

- links chiral observable Σ_B with confinement criterion $\tilde{\Sigma}_S \to e^{-\sigma S}$
- dressed Wilson loops inherit renormalization from condensates, latter rather simple (additive renorm. gone after Fourier transform)
- very similar to dressed Polyakov loops   Bilgici, FB, Gattringer, Hagen 08

  Fourier: boundary phase (imag. μ) vs. winding number in temporal dir.
- physical intuition:

  abelian magn. field $\Rightarrow$ spatial Wilson loops

  abelian el. field $\Rightarrow$ conv. space-time Wilson loops

  confinement as response to el. fields, which pull quark-antiquark pair apart and thus probe QCD forces

  all el. fields $\sim$ inverse scattering
- approaches beyond lattice: nonlocal Wilson loops maybe hard, magn. fields simpler to incorporate

QCD transition with magnetic fields

- QCD transition at $B = 0$ and $\mu = 0$ is a crossover Aoki et al 06, hotQCD
  pseudo-critical temperatures:
  $T_c^{\chi} = 151(3)(3)$ MeV, $T_c^{\text{deconf}} = 176(3)(4)$ MeV
- now external magn. field coupled to sea quarks
  = in quark determinant of each flavour

“magnetic catalysis”:

quark condensate increases = spontaneous mass generation(?!)

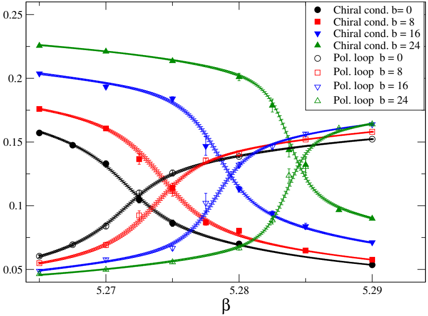
Results from effective models and lattice simulations

- T_c^χ increases with B:
  sigma model and other approx.s — Mizher et al 10 and others
  - splits from T_c^{deconf}, transition stronger

- T_c^χ decreases with B:
  χPT for $N_f = 2$ — Agasian, Fedorov 08

lattice simulations of staggered $N_f = 2$: — D'Elia, Mukherjee, Sanfilippo 10

- magn. catalysis
- T_c^χ increases with B
- T_c^{deconf} increases, too
- transition stronger



Simulation details

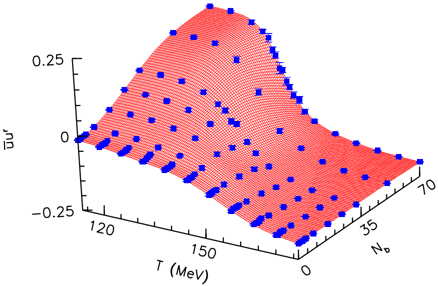
- as for transition at $B = 0$ Budapest-Wuppertal
  - Symanzik improved gauge action
  - $N_f = 2 + 1$ stout smeared staggered quarks
  - physical (lowest pion) mass
- scale set at $T = 0$, $B = 0$
- various T and N_B, fit all points by a 2D spline
- exp. values: random estimator with 40 vectors
- continuum limit with $N_t = 6, 8, 10$ (at physical B fixed)
- infinite volume with $N_s = 16, 24, 32$ (at physical B fixed)

Observables

- chiral condensate:

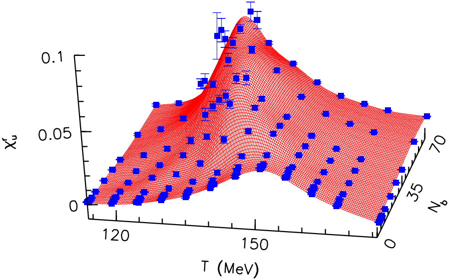
$$\bar{u}u = \frac{T}{V}\frac{\partial \log Z}{\partial m_u}$$

plots for $N_t = 6$



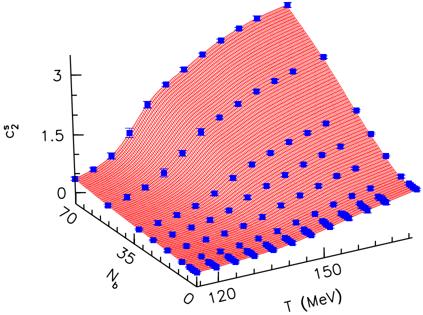
- chiral susceptibility:

$$\chi_u = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m_u^2}$$



- strange quark number susceptibility:

$$c_2^s = \frac{T}{V} \left. \frac{\partial^2 \log Z}{\partial \mu_s^2} \right|_{\mu_s = 0}$$



Renormalization

additive and multiplicative:

$$\langle\bar{\psi}\psi\rangle^r(B,T) = m\big[\langle\bar{\psi}\psi\rangle(B,T) - \langle\bar{\psi}\psi\rangle(0,0)\big]\frac{1}{m_\pi^4}$$

$$\chi^r(B,T) = m\big[\chi(B,T) - \chi(0,0)\big]\frac{1}{m_\pi^4}$$

none for c_2^s (connected to conserved current)

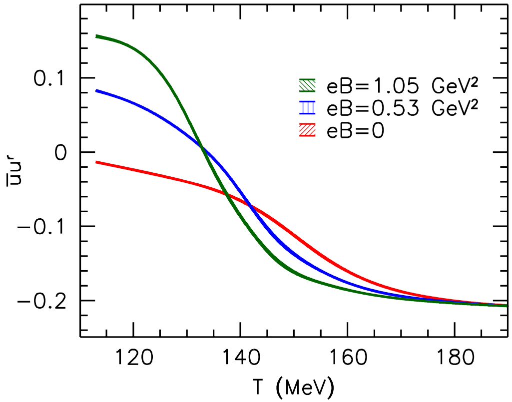
external magn. field B does not introduce new divergences:

- no internal photon lines $\Rightarrow$ no additional divergent diagrams
- divergent part of vacuum energy B-independent   Salam, Strathdee 75
- $(eB)^r = (eB)$ due to WT identity $Z_e Z_{A_\mu} = 1$ in QED
- quantization of magn. field $\Rightarrow B^r$ would imply shift in a, but we find the expected behavior of m_{π^+} and r_0 in cont. limit   ▸ data

Magnetic catalysis?

(renorm.) condensate as function of T for various B's:

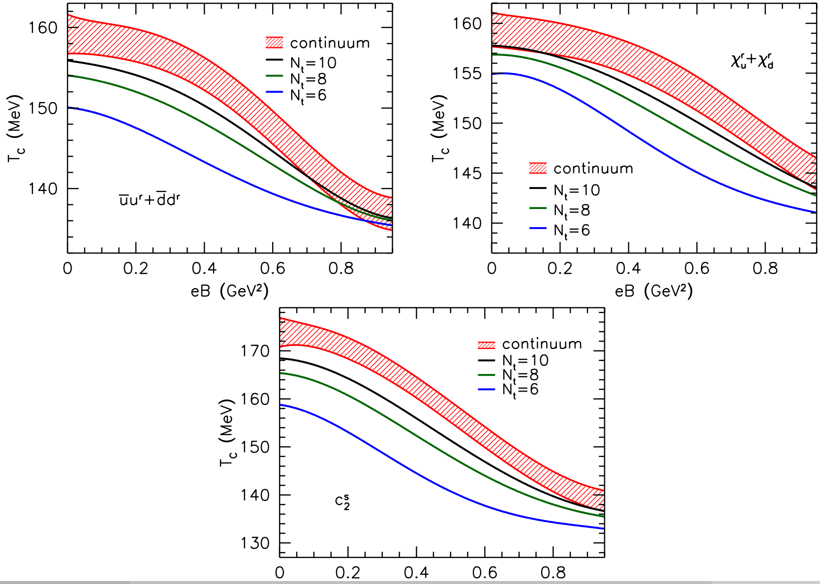
▸ of B as funct. of T



- condensate increases at low T, but decreases at high T!
- inflection point (T_c) moves towards lower T!

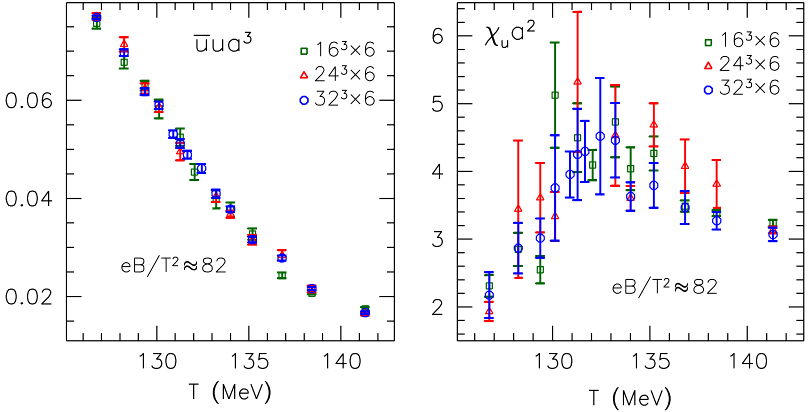
Phase diagram

from chiral condensate, chiral susc. and strange number susc.:



Finite size effects – Transition strength

analysis of $N_s = 16, 24, 32$ on $N_t = 6$ (largest volume $\sim 7\ \text{fm}^3$) for chiral condensate and chiral susceptibility:

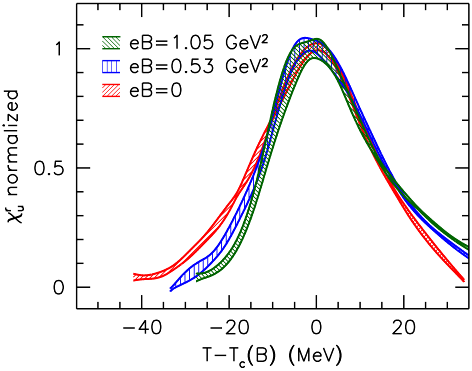


- crossover persists up to $\sqrt{eB} = 1$ GeV

Width of the transition

χ_u peak grows with B, like in D'Elia et al 10,

but if normalized by peak height:

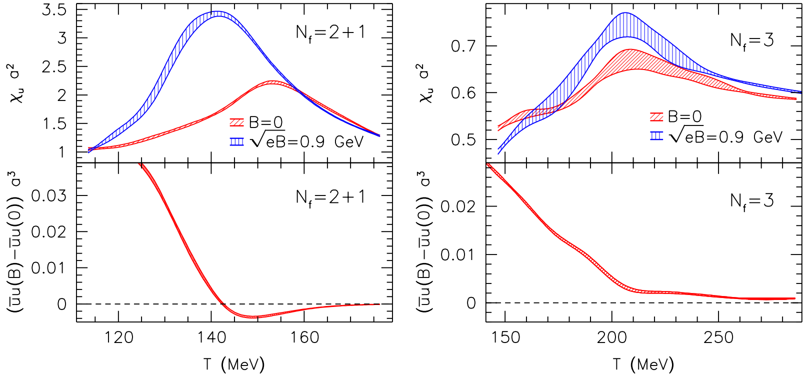


- width of transition reduces only mildly with B

Role of quark mass

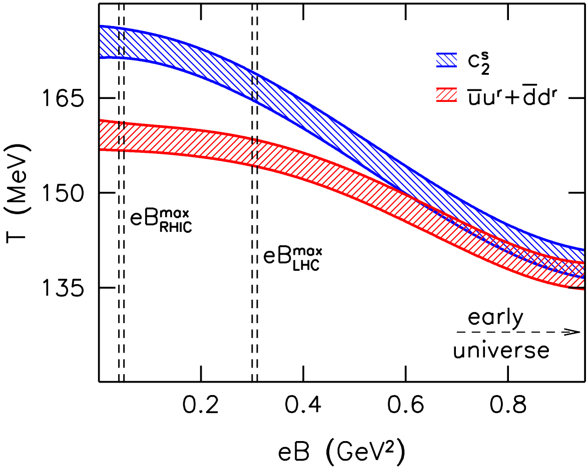
motivation: D'Elia et al. got different results using (beside $N_f = 2$) coarser lattices and higher quark masses ($m_\pi = 195$ MeV)

change to $N_f = 3$ where all quarks have masses m_s:



- no clear change in T_c anymore
- magn. catalysis always

Phase diagram and heavy ion colliders



- decrease of T_c with B negligible for RHIC
- 5..10 MeV for LHC
- may be significant for early universe ($>$ 20%)

Summary

magnetic fields in lattice QCD:

- dressed Wilson loops

  B as a tool

  - links condensate and Wilson loop (confinement)
  - learn more about renormalization
  - beyond lattice QCD

- phase diagram in (B, T) plane for staggered $N_f = 2 + 1$

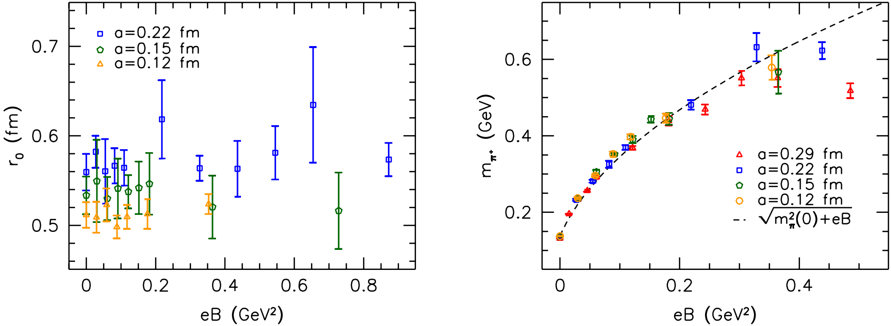
  phys. masses, discretisation and finite volume errors under control

  - T_c decreases
  - crossover persists
  - complicated dependence $\langle \bar{u}u \rangle (B, T)$, very sensitive to quark mass
    understand!

Back up: Renormalization at finite B

assumption: scale $a(\beta)$ and line of constant physics $m(\beta)$ from $B = 0$

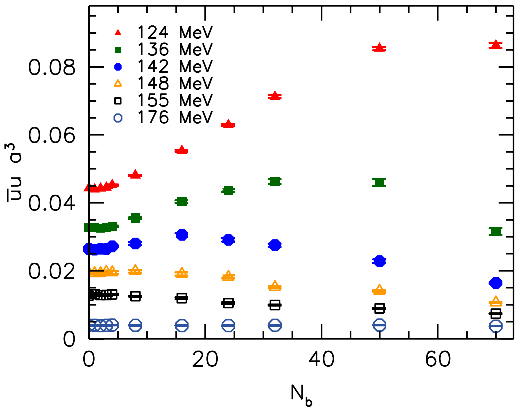
measure Sommer parameter and mass of charged pion:



- compare to $r_0(B) = r_0$ and $m_{\pi^+}(B) = \sqrt{m_{\pi^+}^2 + |eB|}$
- no additional a-dependence in continuum limit
  $\Rightarrow$ a not shifted by B $\Rightarrow$ B not renormalized

Back up: Dependence of condensate

(in lattice units) on flux quantum N_B for various temperatures :



- non-monotonic!