A model-independent parametrization of semileptonic $B \to \pi$ decays for determining $|V_{ub}|$

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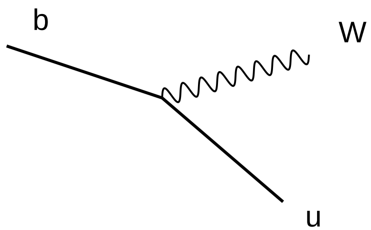
Flavor physics

Test SM paradigm of quark flavor mixing and CP violation and look for new physics

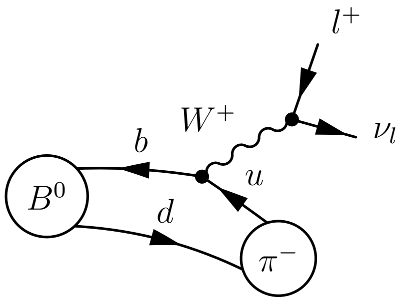
Unitary CKM matrix



$$\sim V_{ub} \rightarrow V = \begin{matrix} & d & s & b \\ u & 1-\frac{\lambda}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ c & -\lambda & 1-\frac{\lambda}{2} & A\lambda^2 \\ t & A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{matrix} + \mathcal{O}(\lambda^4)$$

$\rightarrow$ scalar product of $d-b$ columns $=0$
$\Rightarrow$ unitarity triangle

In experiment, must account for confining QCD interactions

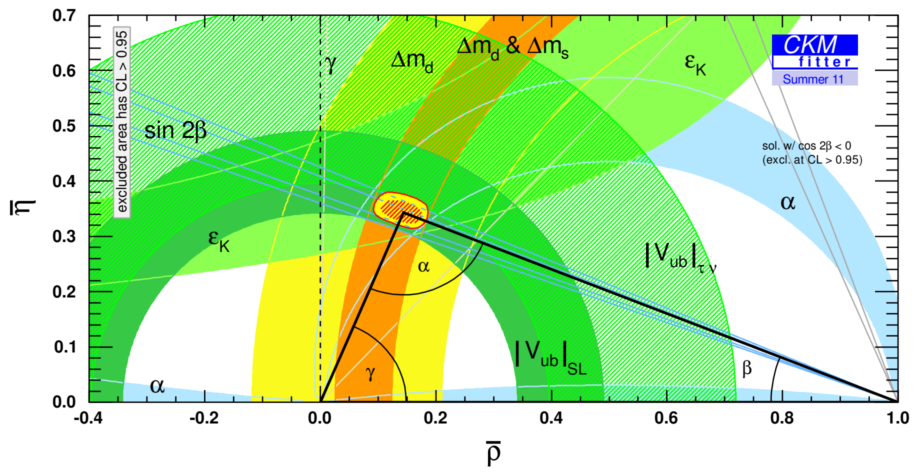


$$\sim \quad |V_{ub}| \times \langle \pi^- | \bar{b}\gamma_\mu u | B^0 \rangle$$

$\rightarrow$ lattice QCD (or LCSR)

Motivation

- $|V_{ub}|$ determines side of UT opposite $\sin 2\beta$



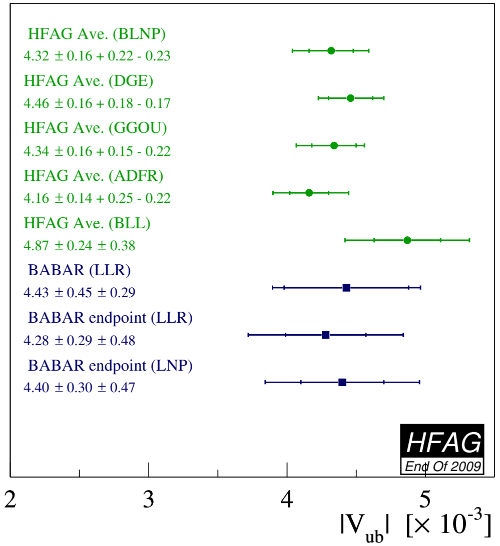
Input for plot:

$$|V_{ub}| = (3.92 \pm 0.09 \pm 0.45) \times 10^{-3}$$

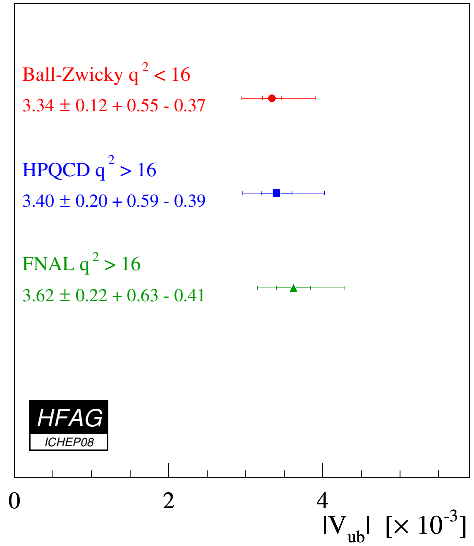
Global fit output (w/out being input):

$$|V_{ub}| = (3.42^{+0.20}_{-0.10}) \times 10^{-3}$$

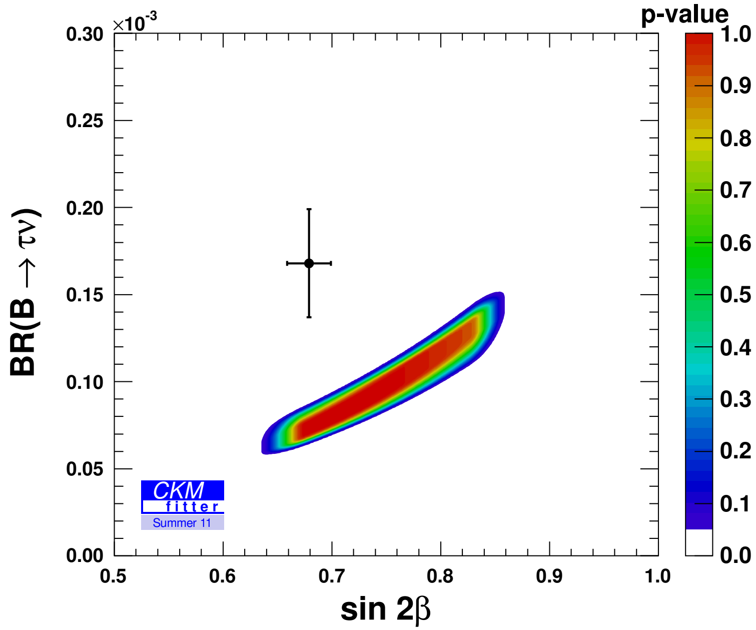
- Inclusive vs exclusive $b \to u\ell\nu$ determination



- Inclusive (BLNP) is 1.5σ above exclusive (HPQCD)
- SM strongly favors lower exclusive values: 2.8σ away from BLNP!

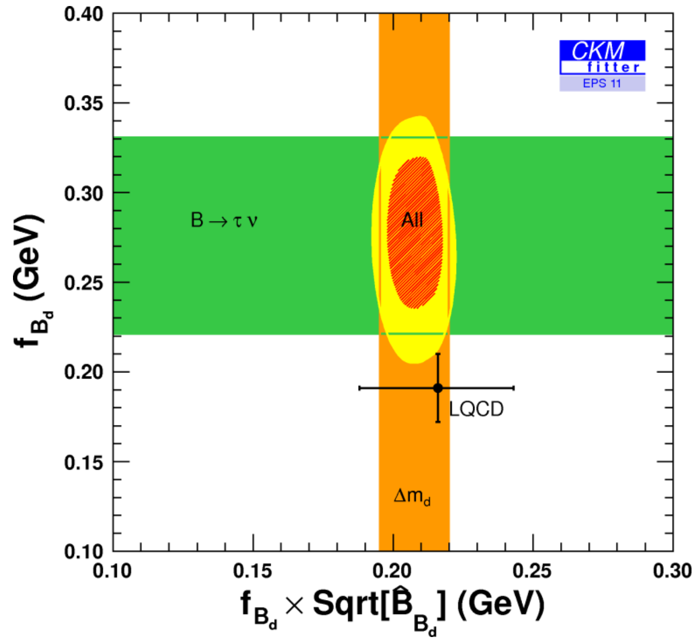


Motivation



$$B(B \to \tau\nu) = \begin{cases} (0.81 \pm 0.15) \times 10^{-4} & \text{SM} \\ (1.68 \pm 0.31) \times 10^{-4} & \text{Expt} \end{cases}$$

Would require 44% larger $|V_{ub}| \sim 5.6 \times 10^{-3}$ (60% larger than exclusive value) or $f_{B_d} \sim 270\,\mathrm{MeV}$!



- f_{B_d} can only only be increased by 44% if B_{B_d} is reduced by $\sim$ 90%!

→ Can $|V_{ub}|$ really be $\sim 5.6 \times 10^{-3}$?

→ New physics? 2HDM w/ small M_{H^+} and large $\tan\beta$

...

$B \to \pi$ form factors on the lattice

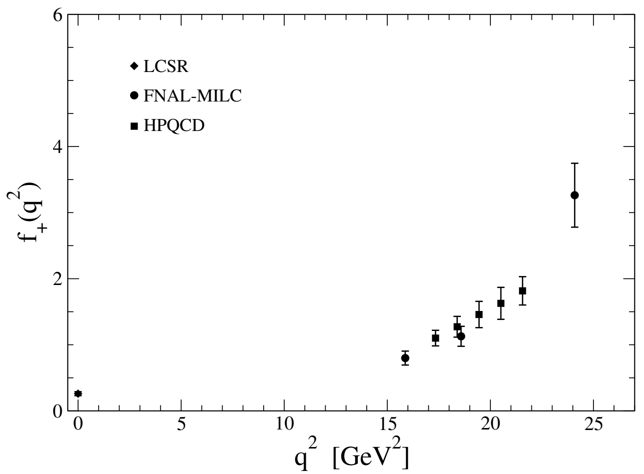
Have

$$\langle \pi^-(p_\pi)|\bar{b}\gamma_\mu u|B^0(p_B)\rangle \quad \longrightarrow \quad f_+(q^2) \quad \& \quad f_0(q^2) \qquad \text{w/} \quad q_\mu = (p_B - p_\pi)_\mu$$

and for $\ell = e,\ \mu$, contribution of $f_0(q^2)$ negligible

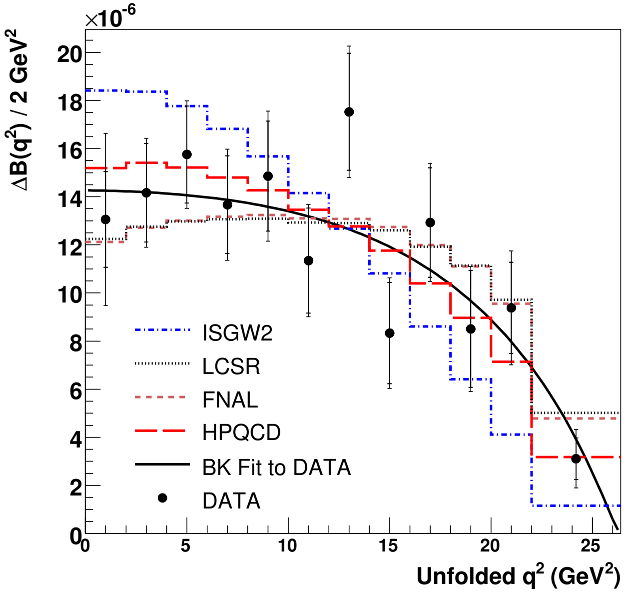
Semileptonic domain, $q^2 : 0 \to t_- \equiv (m_B - m_\pi)^2 = 26.4\,\mathrm{GeV}^2$

$\longrightarrow$ in B^0 rest frame, $|\vec{p}_\pi| \ : 0 \to 2.7\,\mathrm{GeV}$



LQCD limited by discretization errors (and chiral extrapolation) to $q^2 \gtrsim 15\,\mathrm{GeV}^2$ and LCSR $q^2 \sim 0$

Want to use all available data



$\Rightarrow$ need model-independent parametrization to cover whole range

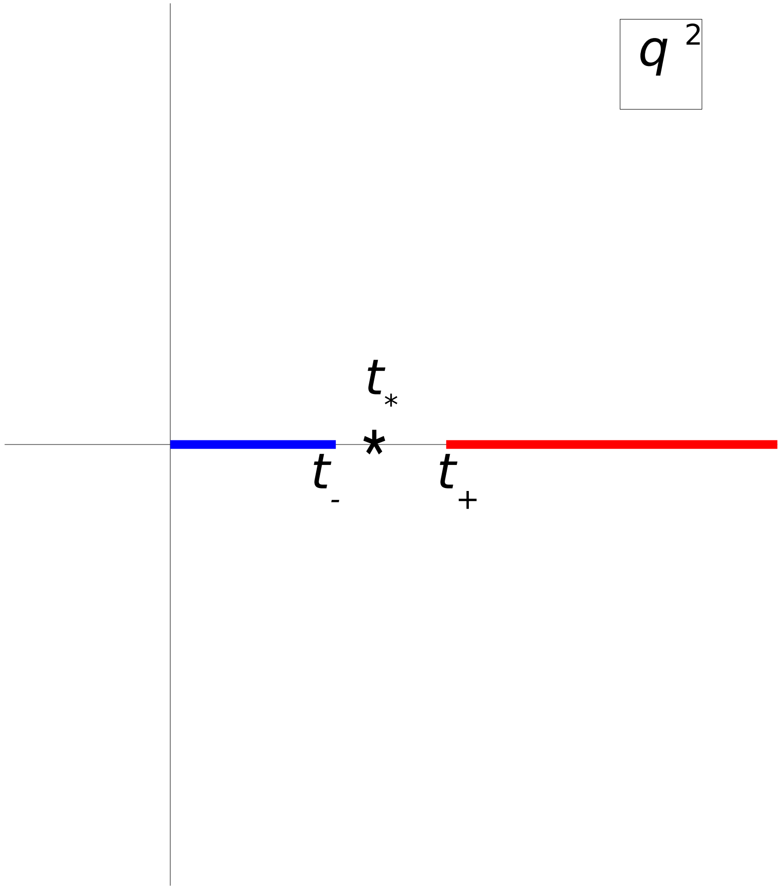
Properties of $f_+(q^2)$

$$(p+p')_\mu \,\mathrm{Im} f_+(q^2) + \cdots = \frac{1}{2}\sum_\Gamma (2\pi)^4 \delta^{(4)}(p_\Gamma - q)\langle 0|V_\mu|\Gamma\rangle\langle\Gamma|\bar{B}^0(p)\pi^-(-p')\rangle$$

- Real, analytic fn in q^2-plane cut for $q^2 \geq t_+ = (m_B + m_\pi)^2 = 29.4\,\mathrm{GeV}^2$
  ...
- ... up to pole below cut at $q^2 = m_{B*}^2 = 28.4\,\mathrm{GeV}^2$
- Angular momentum conservation

  $\Rightarrow\ \mathrm{Im} f_+(q^2) \sim (q^2 - t_+)^{3/2}$ for $q^2 \gtrsim t_+$
- Brodsky-Lepage (Lepage et al '80, Akhoury et al '94)

  $\Rightarrow f_+(q^2) \sim 1/q^2$ for $|q^2| \to \infty$, up to logs



Recent parametrizations: Omnès representation

Flynn et al '07 use Omnès representation (Omnès '58)

$$f_+(q^2) = f_+(q_1^2)\,\frac{m_{B^*}^2 - q_1^2}{m_{B^*}^2 - q^2}\exp\left[\frac{q^2 - q_1^2}{\pi}\int_{t_+}^{\infty}\frac{\delta(t)\,dt}{(t-q_1^2)(t-q^2)}\right]$$

where q_1^2 is arbitrary subtraction point and $\delta(t)$ is $J=1$, $I=1/2$, $B\pi \to B\pi$ elastic phase below inelastic threshold

Little is known about $\delta(t)$

$\to$ neglect integral after suppressing it w/ multiply-subtracted dispersion relation

$$f_+(q^2) = \frac{1}{m_{B^*}^2 - q^2}\prod_{j=1}^{n}[f_+(q_j^2)(m_{B^*}^2 - q_j^2)]^{\alpha_j(q^2)}, \qquad \text{w/} \quad \alpha_j(q^2) = \prod_{i=0,i\neq j}^{n}\frac{q^2 - q_i^2}{q_j^2 - q_i^2}$$

where the $f_+(q_j^2)$ are the parameters

Recent parametrizations: Omnès rep. (cont'd)

Problems

- Defines an entire function in the complex q^2-plane, apart from pole at $q^2 = m_{B*}^2$
  $\Rightarrow$ does not have correct analytic properties, in particular cut for $q^2 \geq t_+$
- Yields $f_+(q^2) \sim \exp[C(q^2)^{n-1}]$ when $|q^2| \to \infty$!
- Behavior arises because $f_+(q_j^2)$ are considered independent while they are related through $f_+(q_1^2)$ and dispersive integral

Recent parametrizations: unitarity bound inspired

Based on positivity and analyticity properties of (Okubo et al '71, Bourrely et al '81)

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x}\langle 0|T\{V_\mu(x)V_\nu^\dagger(0)\}|0\rangle \qquad \text{w/} \quad V_\mu = \bar{u}\gamma^\mu b$$

$$= (q^\mu q^\nu - g^{\mu\nu}q^2)\Pi_{1^-}(q^2) + q^\mu q^\nu \Pi_{0^+}(q^2)$$

Use twice-subtracted dispersion relation

$$\chi_{1^-}(q^2) = \left(\frac{\partial}{\partial q^2}\right)^2 \left(q^2\Pi_{1^-}(q^2)\right) = \frac{1}{\pi}\int_0^\infty dt\, \frac{t\,\mathrm{Im}\Pi_{1^-}(t+i\epsilon)}{(t-q^2)^3}$$

where

$$(q^\mu q^\nu - g^{\mu\nu}q^2)\mathrm{Im}\Pi_{1^-}(q^2+i\epsilon) + \cdots = \frac{1}{2}\sum_\Gamma (2\pi)^4\delta^{(4)}(q-p_\Gamma)\,\langle 0|V^\mu|\Gamma\rangle\langle\Gamma|V^{\nu\dagger}|0\rangle$$

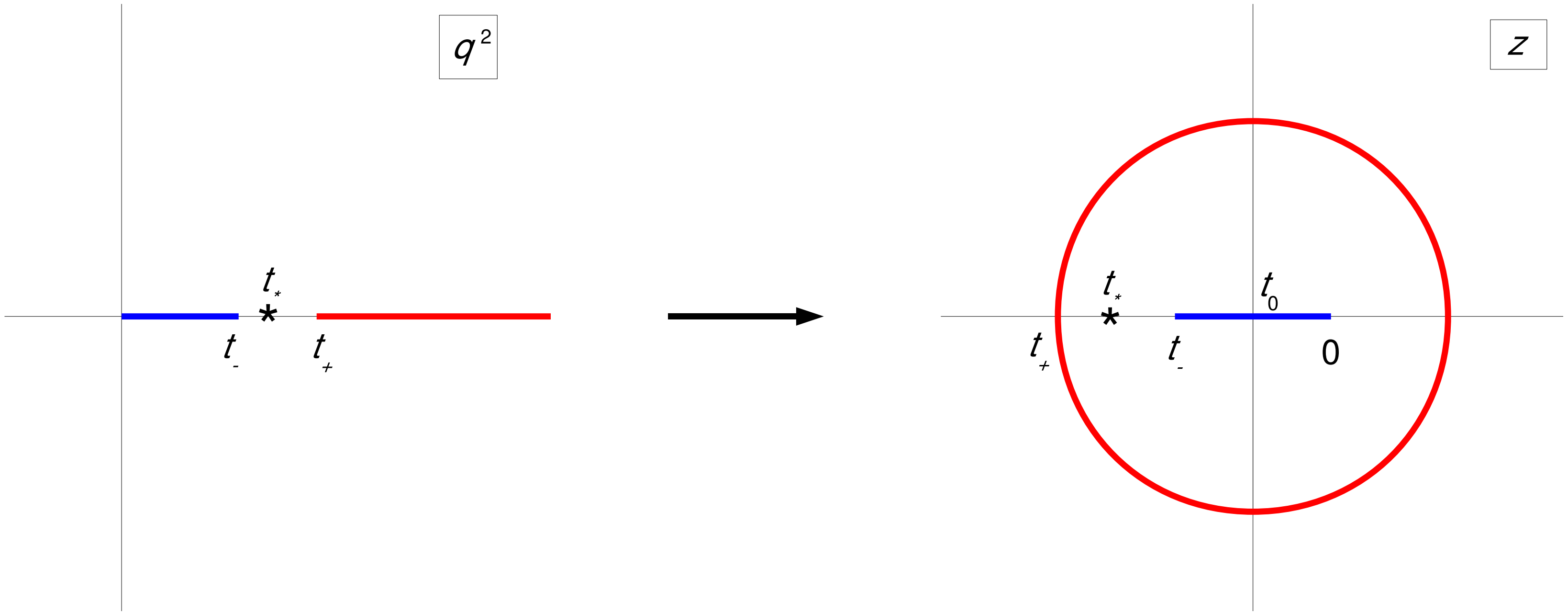
Saturate RHS w/ $|\Gamma\rangle = |\bar{B}^0\pi^-\rangle$ and use positivity

$$\Rightarrow \chi_{1^-}(q^2) \geq \int_{t_+}^\infty dt\, k(t,q^2)\,|f_+(t)|^2$$

For $q^2 \ll m_b^2$, compute LHS using OPE: $\chi_{1^-}(0) = \frac{3[1+1.14\,\alpha_s(\bar{m}_b)]}{32\pi^2 m_b^2} - \frac{\bar{m}_b\langle\bar{u}u\rangle}{m_b^6} - \frac{\langle\alpha_s G^2\rangle}{12\pi m_b^6}$

Perform conformal mapping

$$z \equiv z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$



Recent parametrizations: unitarity . . . (cont'd)

Obtain constraint

$$\oint_{|z|=1} \frac{dz}{2\pi iz} \left|\phi(q^2(z,t_0),t_0)B(q^2(z,t_0))f_+(q^2(z,t_0))\right|^2 \leq 1$$

where

- $B(q^2) = z(q^2, m_{B^*})^2$ cancels B^* pole and $|B(q^2)| = 1$ for $q^2 \geq t_+$ by construction
- $\phi(q^2, t_0) \sim (t_+ - q^2)\cdots$ is constructed to match kinematical factors in dispersive integral along cut and to be analytic in $|z| < 1$

Thus, $\phi\, B\, f_+(z)$ is analytic in cut plane and form factor can be expanded as (Boyd et al '95, Arnesen et al '05)

$$f_+(q^2) = \frac{1}{B(q^2)\phi(q^2,t_0)} \sum_{n\geq 0} a_n(t_0) z(q^2,t_0)^n$$

where $a_n(t_0) \in \mathbb{R}$ and $\sum_{n=0}^{\infty} a_n(t_0)^2 \leq 1$

Recent parametrizations: unitarity bounds (cont'd)

Above expression used as parametrization by truncating sum at a finite order

$\Rightarrow$ Problems

- Yields $f_+(q^2) \sim (q^2)^{1/4}$ for $|q^2| \to \infty$ (Becher et al '06)
- Gives $f_+(q^2)$ an unphysical pole at $q^2 = t_+$
- More generally, $\phi(q^2, t_0)$ comes from kinematics of dispersive integral
  $\to$ it's q^2 dependence has nothing to do with that of $f_+(q^2)$

Our proposal

We note that $(1 - q^2/m_{B^*}^2) f_+(q^2)$ is:

- analytic in the complex q^2-plane cut along real axis $q^2 \geq t_+$
- finite as $|q^2| \to \infty$

$\Rightarrow$ can write it as a Taylor expansion in $z(q^2, t_0)$

Use additional constraint $\mathrm{Im} f_+(q^2) \sim (q^2 - t_+)^{3/2}$ to reduce number of parameters and propose parametrization

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{\mathcal{K}-1} b_k \left[z^k - (-1)^{k-\mathcal{K}} \frac{k}{\mathcal{K}} z^{\mathcal{K}} \right]$$

which can be shown to have desired analytic and scaling properties

Take $t_0 = t_{opt} \equiv (m_B + m_\pi)(\sqrt{m_B} - \sqrt{m_\pi})^2 = 20.1\,\mathrm{GeV}^2$ by default

$\rightarrow$ semileptonic domain mapped onto $|z| \leq 0.279$
$\rightarrow$ maximum truncation error minimized in semileptonic range
$\rightarrow$ gives expansion very close to optimal expansion of Cutkosky et al '68

Can take other t_0 to optimize in specific energy range, using $z(t_0, t_0) = 0$

Unitarity constraint and systematic error

Comparing our expansion w/ old one and using $\sum_{n=0}^{\infty} a_n(t_0)^2 \leq 1$, construct

$$\sum_{j,k=0}^{\mathcal{K}} B_{jk}(t_0) b_j(t_0) b_k(t_0) \leq 1$$

with $B_{j(j+k)} = B_{0k}$ and $B_{jk} = B_{kj}$ (see paper for numerical values, which require OPE evaluation of polarization fn)

Becher et al '06 argue, $1 \to O(1/m_b)^3 \sim 0.001$ and, more generally, 1 is clearly an overestimate

$\to$ however, in absence of precise quantitative information, stick to 1

Systematic error is truncation error. Estimate w/

$$\delta f_+(q^2)_{syst} = \frac{b_{\mathcal{K}+1}^{max} \, |z(q^2, t_0)^{\mathcal{K}+1}|}{1 - q^2/m_{B^*}^2}$$

where $b_{\mathcal{K}+1}^{max}$ the maximum value of $|b_{\mathcal{K}+1}|$ allowed by unitarity bound above w/ $\mathcal{K} \to \mathcal{K}+1$, for fixed, fitted values of b_k, $k \leq \mathcal{K}-1$

Experimental and theoretical input

Experiment: branching fractions in various q^2 bins

- tagged: 4 bins from CLEO '07, 3 from Belle '07 and 3 from BaBar '06
- untagged: 12 bins from BaBar '07 (full covariance matrix is known)
- $\delta\mathcal{B}/\mathcal{B} \sim 15 \div 30\%$ (stat. dominated)

Theory: values of $f_+(q^2)$ at different q^2's

- LCSR: $f_+(0) = 0.26(3)$ (Ball '07)
- LQCD: 3 points from FNAL-MILC '05-'06 and 5 from HPQCD-MILC '06
  - $\delta f_+(q^2)/f_+(q^2) \sim 10 \div 15\%$ (syst. dominated)
  - Assume: 50% correlation in statistical and 100% in systematic errors amongst different q^2 in same calculation; 25% in statistical errors across the 2 calculations
- Include FNAL-MILC '09 in updates

Fitting

Define

$$\chi^2(b_k, |V_{ub}|) = \chi^2_{th} + \chi^2_{exp}$$

where

$$\chi^2_{th} = \sum_{j,k=1}^{8} [f_j^{in} - f_+(q_j^2)] C_{jk}^{-1} [f_k^{in} - f_+(q_k^2)] + (f_+(0) - f_{\rm LCSR})^2/(\delta f_{\rm LCSR})^2$$

$$\chi^2_{exp} = \sum_{j,k=1}^{22} [\mathcal{B}_j^{in} - \mathcal{B}_j(f_+)] C_{\mathcal{B}\,jk}^{-1} [\mathcal{B}_k^{in} - \mathcal{B}_k(f_+)]$$

and $\mathcal{B}_j(f_+)$ are the branching fractions obtained by integrating differential width over bin $[q_j^2, q_{j+1}^2]$, using our parametrization for $f_+(q^2)$

Number of parameters is $N = \mathcal{K} + 1$: b_k w/ $k \leq \mathcal{K} - 1$ and $|V_{ub}|$

Optimize

$$\mathcal{L}(b_j, |V_{ub}|) = \chi^2(b_j, |V_{ub}|) + \lambda \left(\sum_{j,k=0}^{\mathcal{K}} B_{jk} b_j b_k - 1 \right)$$

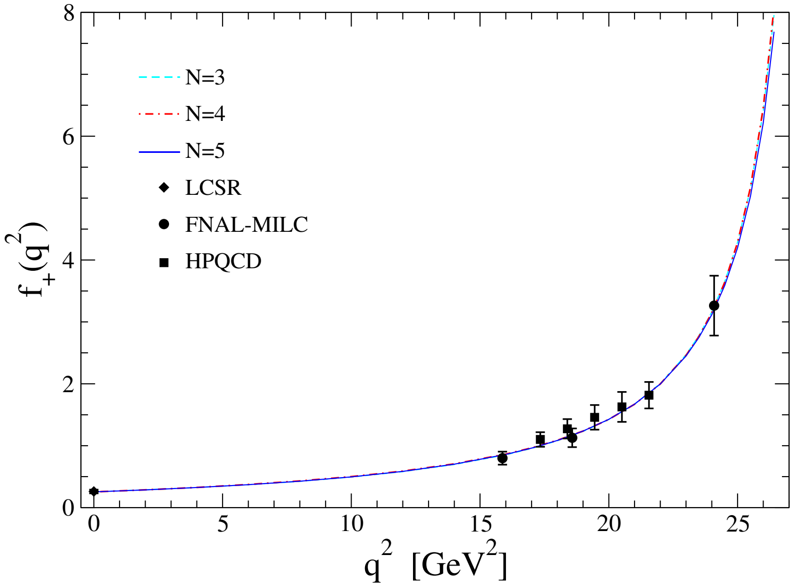
Fitting strategy

Perform full $\Delta\chi^2$ error analysis

Increase number of terms until $\delta f_+(q^2)_{syst} \sim |z(q^2,t_0)^{\mathcal{K}+1}|/(1-q^2/m_{B^*}^2)$ becomes negligible compared to statistical error for whole semileptonic range

$\longrightarrow$ get full envelope of ff's satisfying constraints with statistical error
$\longrightarrow$ occurs for $N = \mathcal{K}+1 = 5$ parameter fits

N	3	4	5
$f_+(0)$	$0.26 \pm 0.02 \pm 0.15$	$0.25 \pm 0.02 \pm 0.04$	$0.25 \pm 0.02 \pm 0.01$
$f_+(t_-)$	$8.0 \pm 0.7 \pm 2.2$	$7.7 \pm 1.0 \pm 0.6$	$8.1 \pm 2.5 \pm 0.2$



- All fits have $\chi^2/dof \simeq 0.8$
- Fits stable as N is increased (more constrained than old parametrization)
- For same N, errors much smaller than old parametrization, e.g. $N=5$:

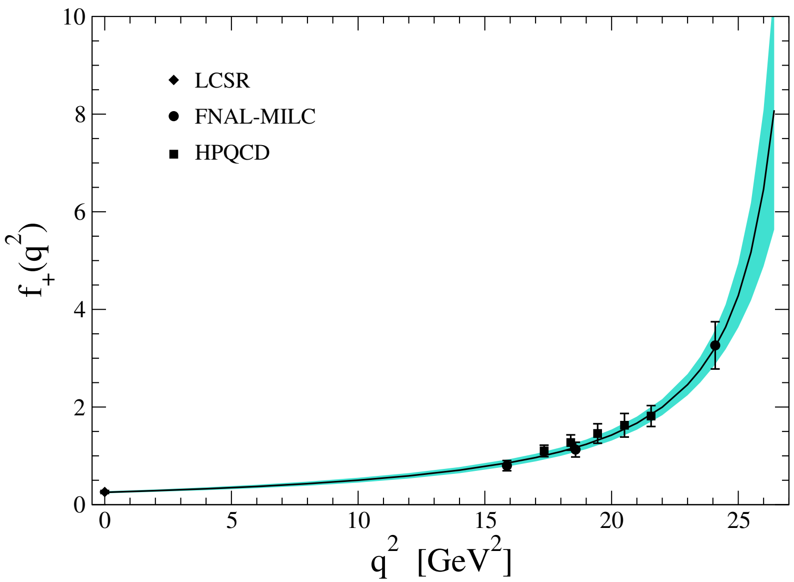
$$f_+(0) = 0.25 \pm 0.02 \pm 0.11$$
$$f_+(t_-) = 8.6 \pm 3.6 \pm 1.8$$

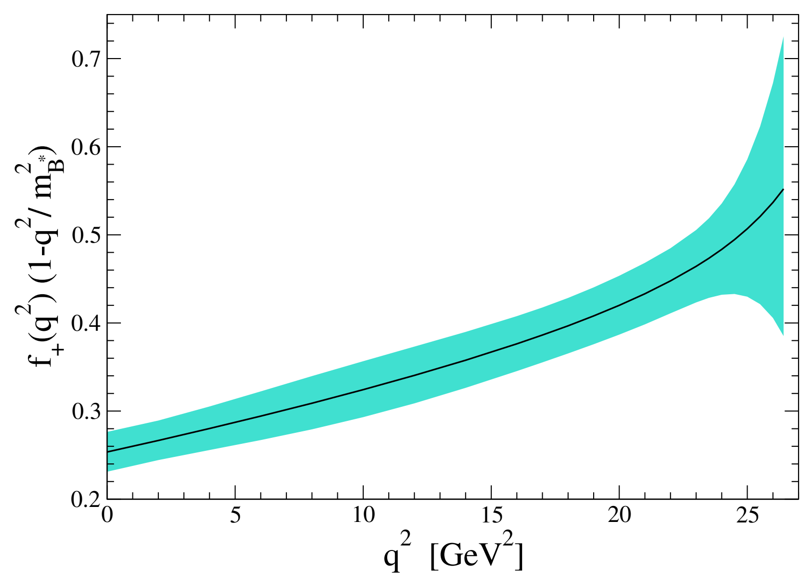
Optimal parametrization in semileptonic domain

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{3} b_k \left[z^k - (-1)^k \frac{k}{4} z^4 \right]$$

j	0	1	2	3
b_j	0.42 ± 0.03	-0.47 ± 0.13	-0.2 ± 1.3	-0.8 ± 4.1

	total	LCSR	LQCD	Belle	CLEO	BaBar-t	BaBar-u
n_{data}	31	1	3+5	3	3	3	12
χ^2	21.0	0.0	5.1	0.0	2.8	4.3	8.7





Some numerical results

$$|V_{ub}| = \begin{cases} (3.54 \pm 0.30) \times 10^{-3} \text{ (w/ LCSR)} \\ (3.60 \pm 0.35) \times 10^{-3} \text{ (no LCSR)} \end{cases}$$ vs $(3.50^{+0.20}_{-0.09}) \times 10^{-3}$ (global fit, CKMfitter '11)

$$|V_{ub}|f_+(0) = \begin{cases} (9.0 \pm 0.7) \times 10^{-4} \text{ (w/ LCSR)} \\ (8.9 \pm 0.8) \times 10^{-4} \text{ (no LCSR)} \end{cases}$$ vs $(7.6 \pm 1.9) \times 10^{-4}$ (SCET, Bauer et al '04)

$$f_+(0) = \begin{cases} 0.25 \pm 0.02 \text{ (w/ LCSR)} \\ 0.25 \pm 0.03 \text{ (no LCSR)} \end{cases}$$ vs 0.26 ± 0.03 (LCSR, Ball '07)

Also get $g_{B^*B\pi} = 2r_+ m_{B^*}/f_{B^*}$ w/ $r_+ = \lim_{q^2 \to m^2_{B^*}} (1 - q^2/m^2_{B^*})\, f_+(q^2)$

$g_{B^*B\pi} = 37. \pm 33. \pm 12. \pm 6.\,(\delta f_{B^*})$ vs $47 \pm 3\,(\mathrm{stat}) \pm 9\,(\mathrm{syst})$ (LQCD, Abada '03)

Extrapolation $\to$ more work needed here

Conclusion

- Proposed a simple, fully model-independent parametrization for leading $B \to \pi$ form factor $f_+(q^2)$:
  - which is an analytic expansion in a small kinematical variable ...
  - ... close to optimal expansion
  - which has correct analyticity properties ...
  - ... and correct scaling for $|q^2| \to \infty$
  - whose coefficients are constrained by unitarity bounds
- With 4 parameters, obtain a description of $f_+(q^2)$ for which systematic/truncation error is negligible over full semileptonic domain
- Combined fit to experimental results for differential rate and theoretical results for $f_+(q^2) \to$ competitive results for:
  - $f_+(q^2)$ over full semileptonic domain
  - $|V_{ub}|$, remarkably close to expectations from global CKM fits
- Important to find new ways of improving unitarity constraint, especially for extrapolations to regions not constrained by data