Finite Volume Scaling of the Wilson Dirac Spectrum

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What Spectrum of the QCD Dirac operator

New Lattice QCD effects included ($a \neq 0$)

Why Extract continium physics from the lattice

How Wilson Chiral Perturbation Theory

Warm up: Zero a

CPT in the ϵ-regime $m\Sigma V \sim 1$ $\qquad a = 0$

The partition function in a sector of topological charge ν

$$Z^{\nu}_{N_f}(m; a = 0) \;=\; \int_{U(N_f)} dU \ \det{}^{\nu}(U) e^{\frac{m}{2}\Sigma V \mathrm{Tr}(U+U^{\dagger})}$$

A group integral (*not a path integral*)

Σ is the chiral condensate

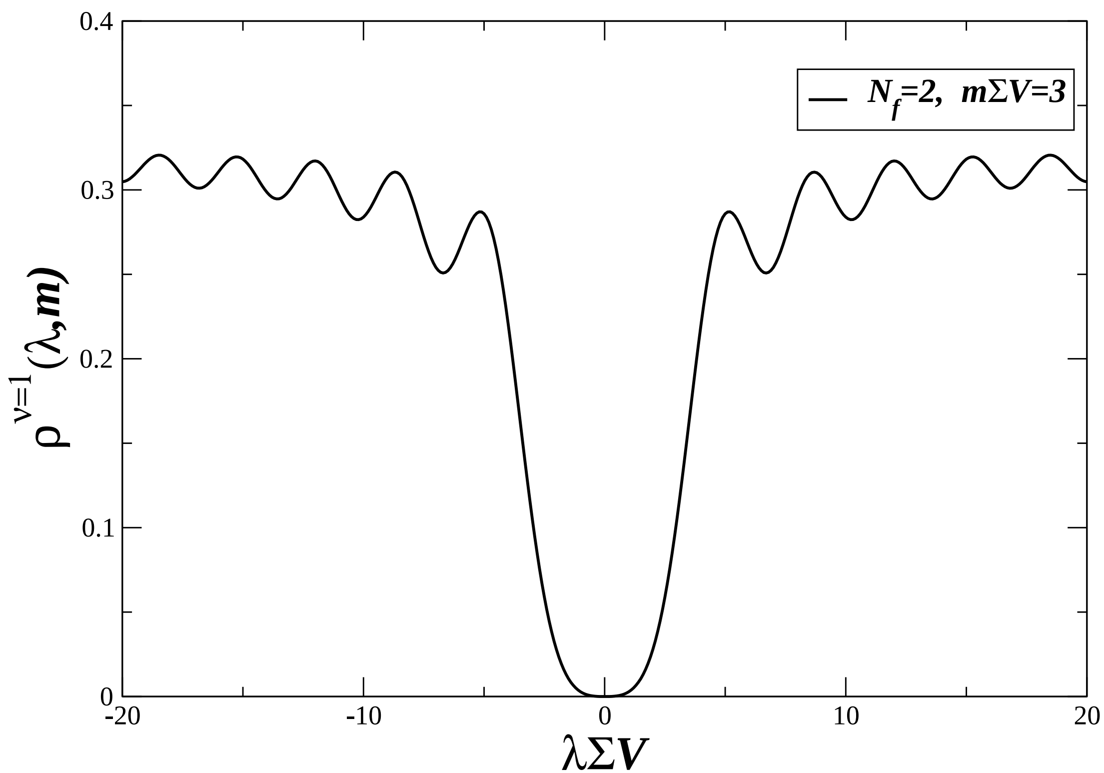
Gasser, Leutwyler, PLB **188**(1987) 477; NPB **307** (1988) 763

Leutwyler, Smilga, PRD 46 (1992) 5607

Eigenvalue density at $a = 0$:

$$\gamma_5 D = -D\gamma_5 \qquad \text{eigenvalues in pairs } (i\lambda, -i\lambda)$$

ν zero ev's



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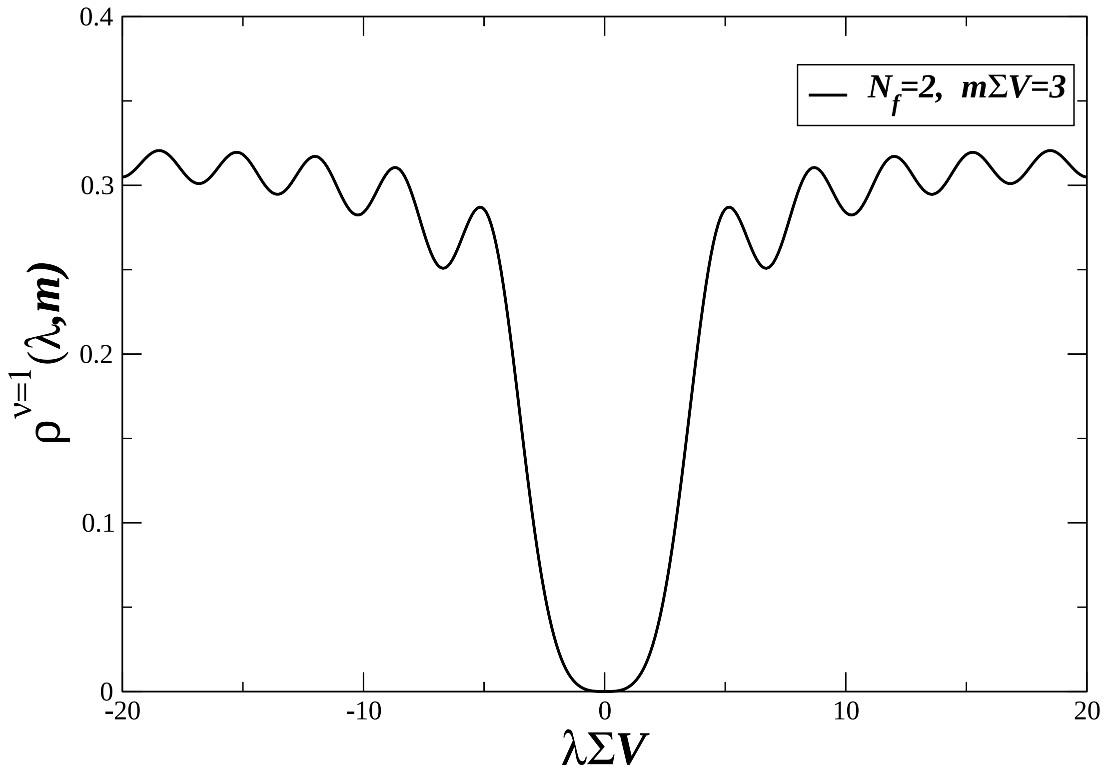

One fit parameter Σ

Verbaarschot Wettig Ann.Rev.Nucl.Part.Sci. 50 (2000) 343, hep-ph/0003017

New: non zero lattice spacing a

Goal: *analytic predictions for the Dirac spectrum with* $a \neq 0$

Discretization effects depend on the discretization

Here: Wilson fermions

$$\gamma_5 D_W \neq -D_W \gamma_5$$
$$D_W^\dagger \neq -D_W$$

γ_5-hermiticity

$$D_W^\dagger = \gamma_5 D_W \gamma_5$$

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γ_5-hermiticity

$$D_W^\dagger = \gamma_5 D_W \gamma_5$$

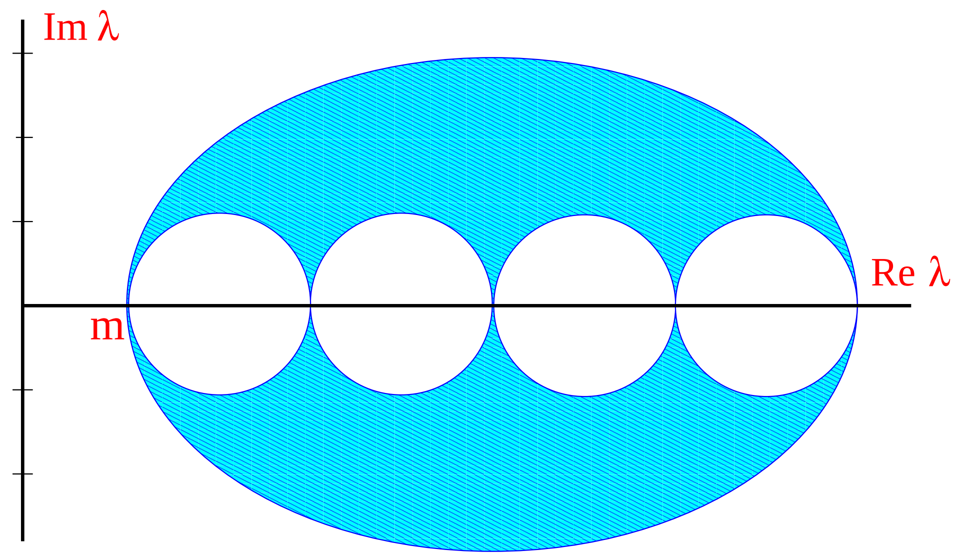
Eigenvalues, z, of D_W

- complex conjugate pairs (z, z^*)
- exact real eigenvalues

Itho, Iwasaki, Yoshie, PRD 36 (1987) 527

Free Wilson fermions.

Spectrum of $D_W + m$



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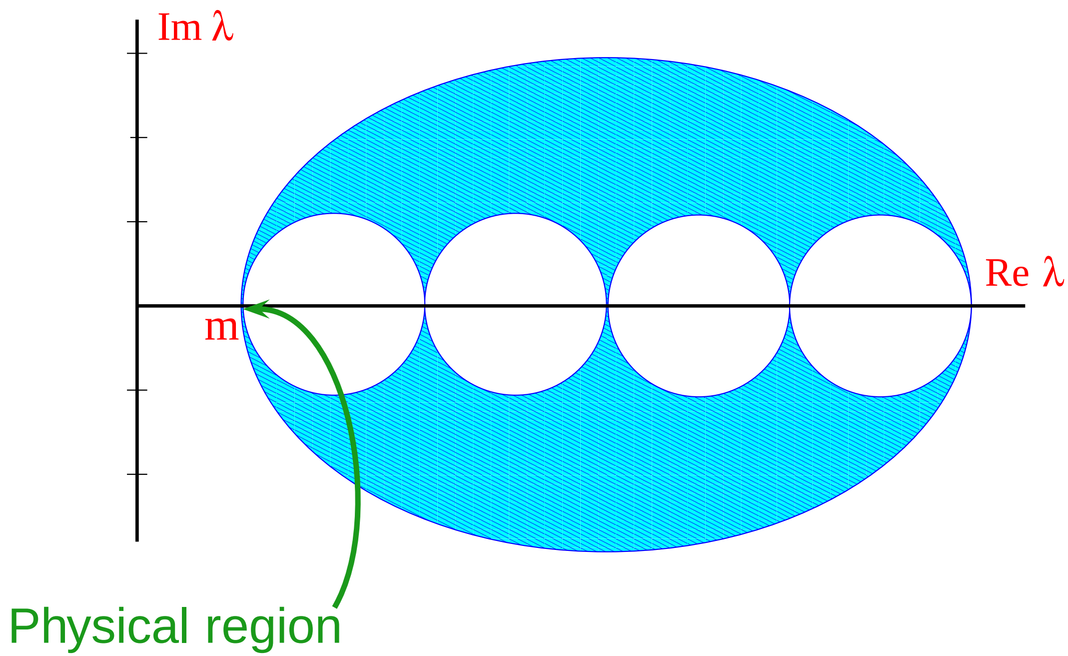
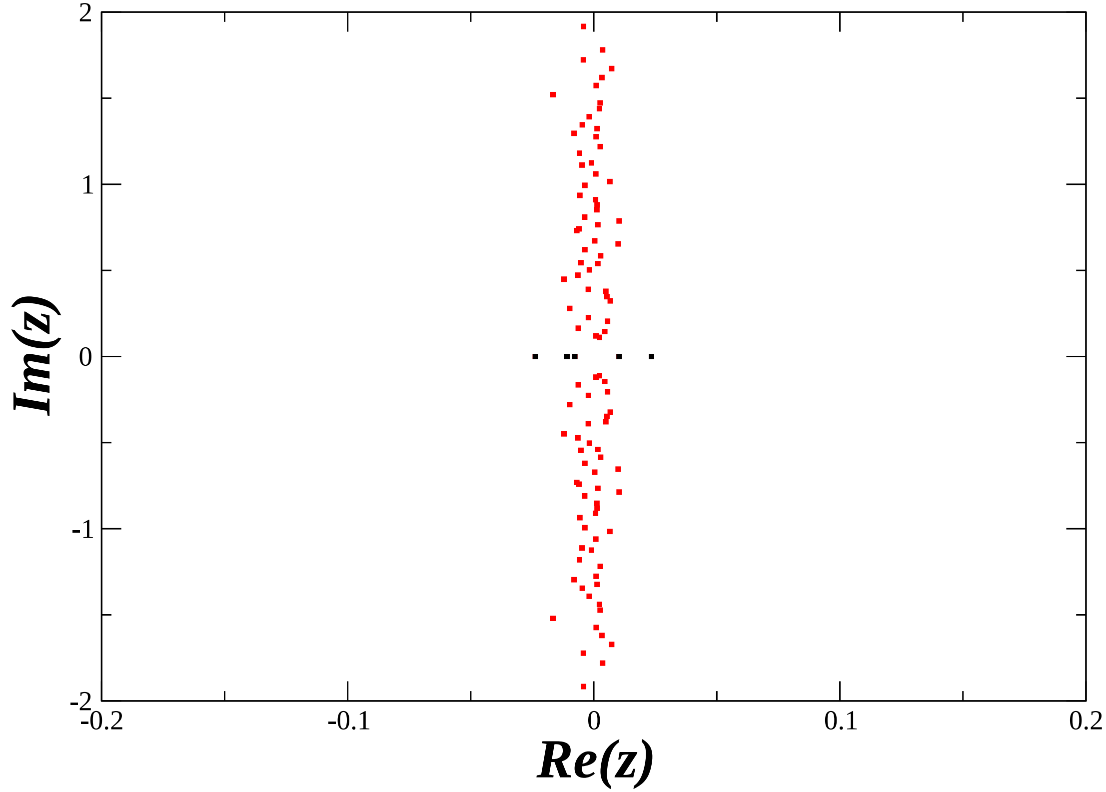


Creutz Annals Phys.322:1518-1540,2007

Eigenvalues, z, of D_W (*illustration*)



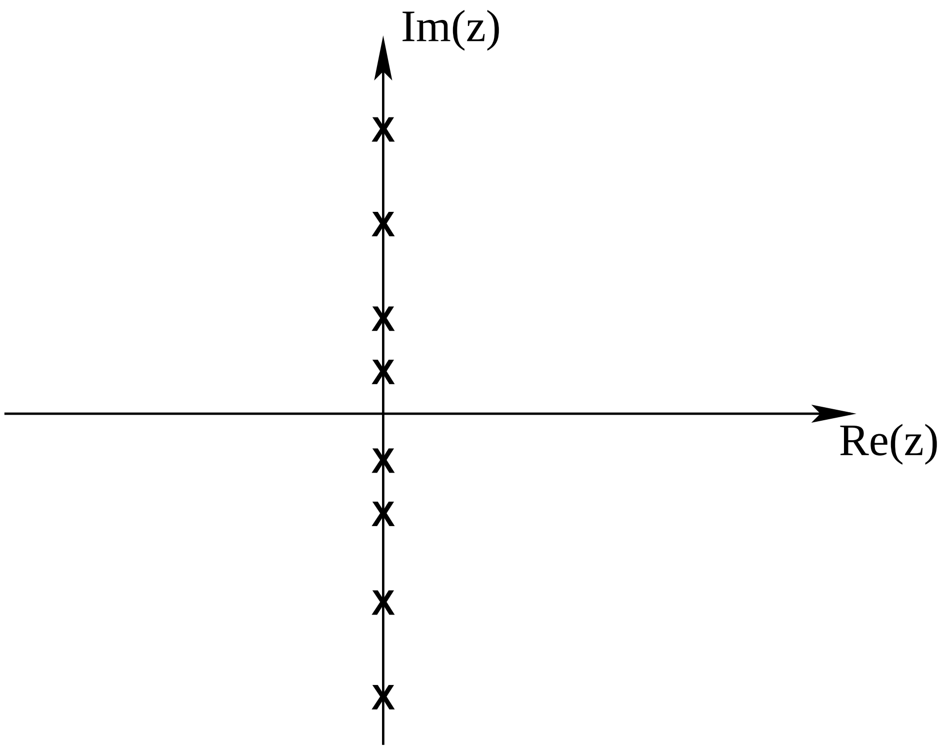
Goal: *analytic predictions for the Wilson Dirac spectrum with* $a \neq 0$

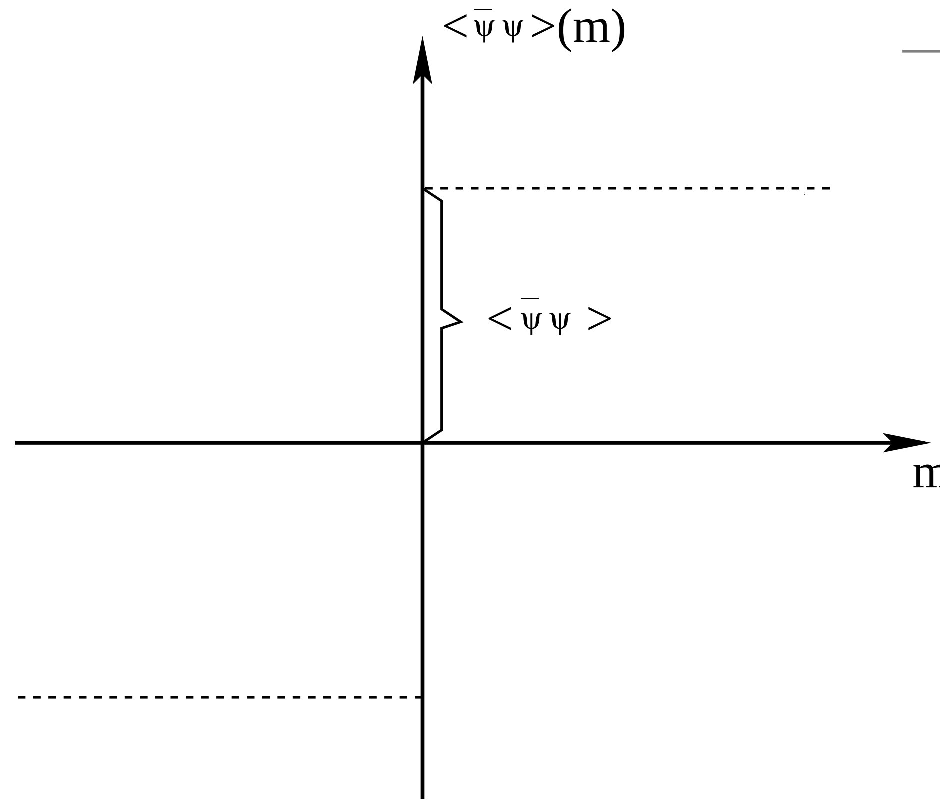
Lüscher JHEP0707:081,2007

Del Debbio Giusti Lüscher Petronzio Tantalo JHEP0702:082,2007

$a = 0$

Banks Casher

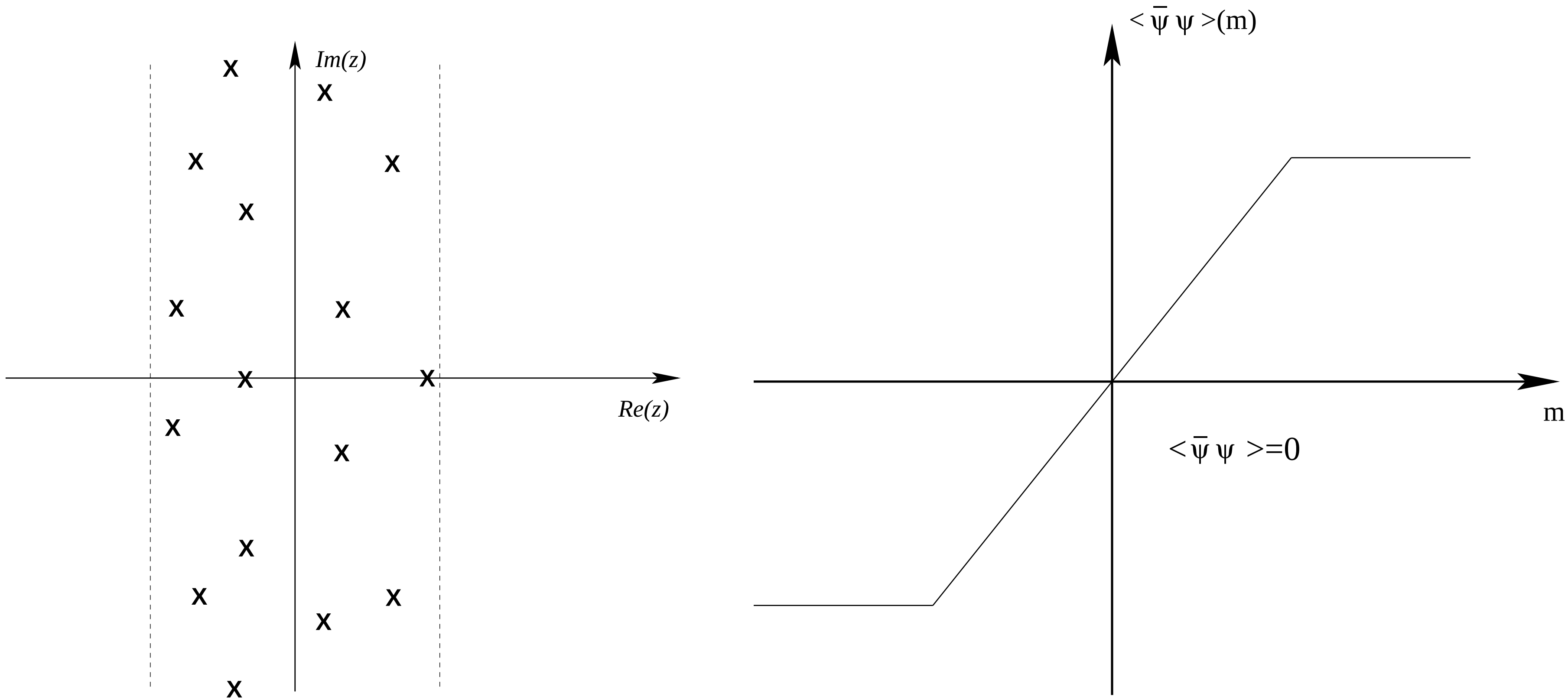




$$\langle \bar{\psi}\psi \rangle = \frac{\pi}{V}\rho(0)$$

Banks Casher NPB 169 (1980) 103

$a \neq 0$ Aoki phase (parity broken phase)



Electrostatic analogy:

Eigenvalues = charges, quark mass = test charge

Aoki PRD **30** 2653 (1984)

Barbour et al. NPB 275 (1986) 296 (nonzero μ)

Method: *Wilson Chiral Perturbation Theory*

Sharpe PRD 74 (2006) 014512: p-regime

Wilson CPT

The chiral Lagrangian for Wilson fermions has new terms

$$\begin{aligned}\mathcal{L} &= \frac{F_\pi^2}{4}\mathrm{Tr}\left(d_\mu U d_\mu U^\dagger\right) + \frac{m}{2}\Sigma\mathrm{Tr}(U+U^\dagger) \\ &-a^2W_6[\mathrm{Tr}\left(U+U^\dagger\right)]^2 - a^2W_7[\mathrm{Tr}\left(U-U^\dagger\right)]^2 \\ &-a^2W_8\mathrm{Tr}(U^2+{U^\dagger}^2)\end{aligned}$$

Sharpe Singleton PRD **58**, 074501 (1998)
Rupak Shoresh PRD **66**, 054503 (2002)
Aoki PRD 68:054508,2003
Bar Rupak Shoresh PRD **70**, 034508 (2004)
Sharpe Wu PRD **70**, 094029 (2004)
Aoki Baer PRD 70 (2004) 116011
Golterman Sharpe Singleton PRD **71**, 094503 (2005)
Del Debbio Frandsen Panagopoulos Sannino JHEP0806:007 (2008)
Shindler PLB 672, 82 (2009)
Bar Necco Schaefer JHEP 0903, 006 (2009)
Bar Necco Shindler JHEP 1004:053,2010

with new constants W_6, W_7 and W_8

Wilson CPT in the ϵ-regime $(m\Sigma V \sim a^2 V W_i \sim 1)$

The partition function in a sector ν

$$Z_{N_f}^{\nu} = \int_{U(N_f)} dU \, \det{}^{\nu}(U) \, e^{S}$$

with

$$\begin{aligned} S &= +\frac{m}{2}\Sigma V \mathrm{Tr}(U + U^{\dagger}) \\ &\quad -a^2 V W_6 [\mathrm{Tr}\left(U + U^{\dagger}\right)]^2 - a^2 V W_7 [\mathrm{Tr}\left(U - U^{\dagger}\right)]^2 \\ &\quad -a^2 V W_8 \mathrm{Tr}(U^2 + {U^{\dagger}}^2) \end{aligned}$$

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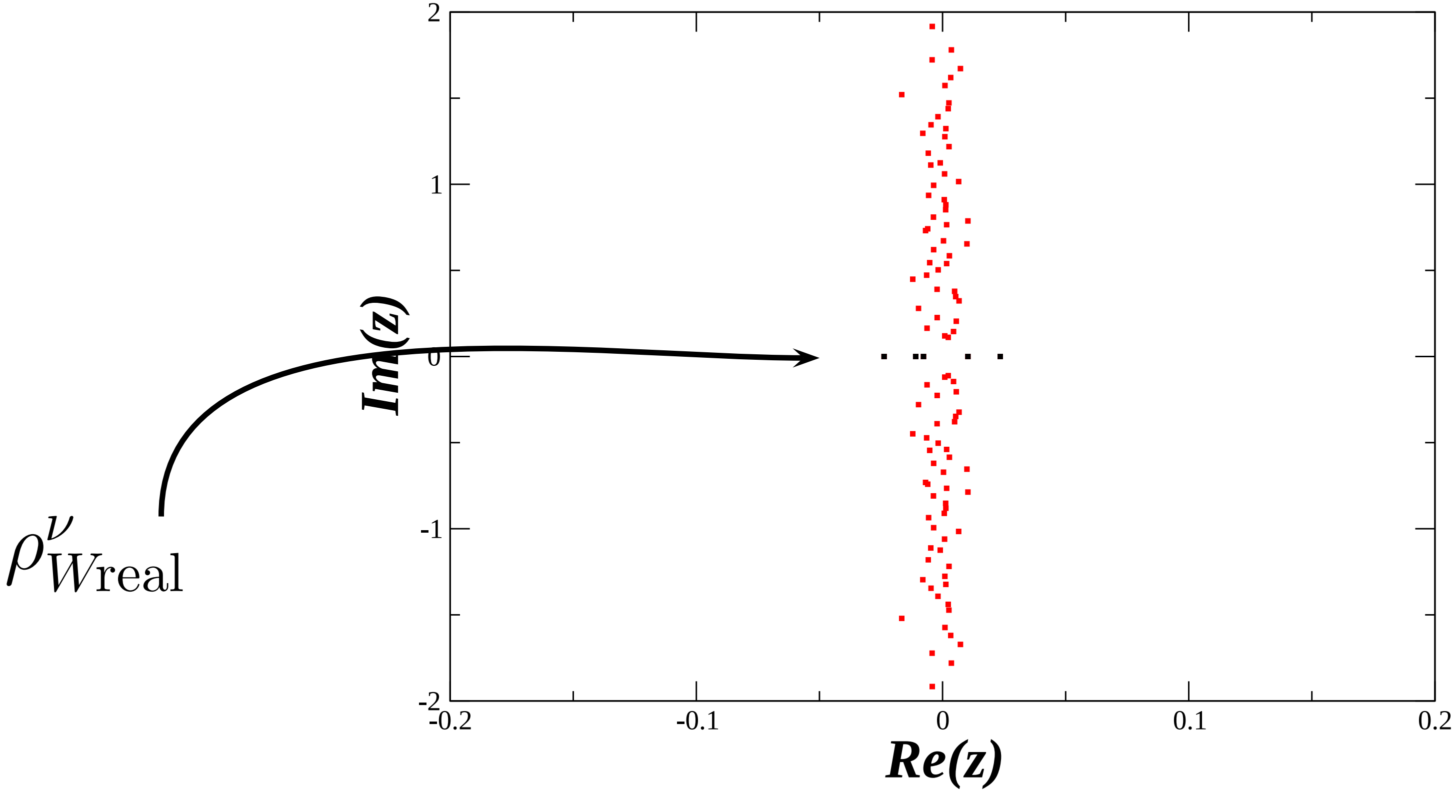
Non trivial fact: In sector ν the Wilson Dirac operator D_W has index ν

index $= \sum_k \mathrm{sign}(\langle k|\gamma_5|k\rangle)$

Damgaard Splittorff Verbaarschot PRL 105:162002, 2010

Akemann, Damgaard, Splittorff, Verbaarschot, PRD 83:085014, 2011

Eigenvalues, z, of D_W (*illustration*)



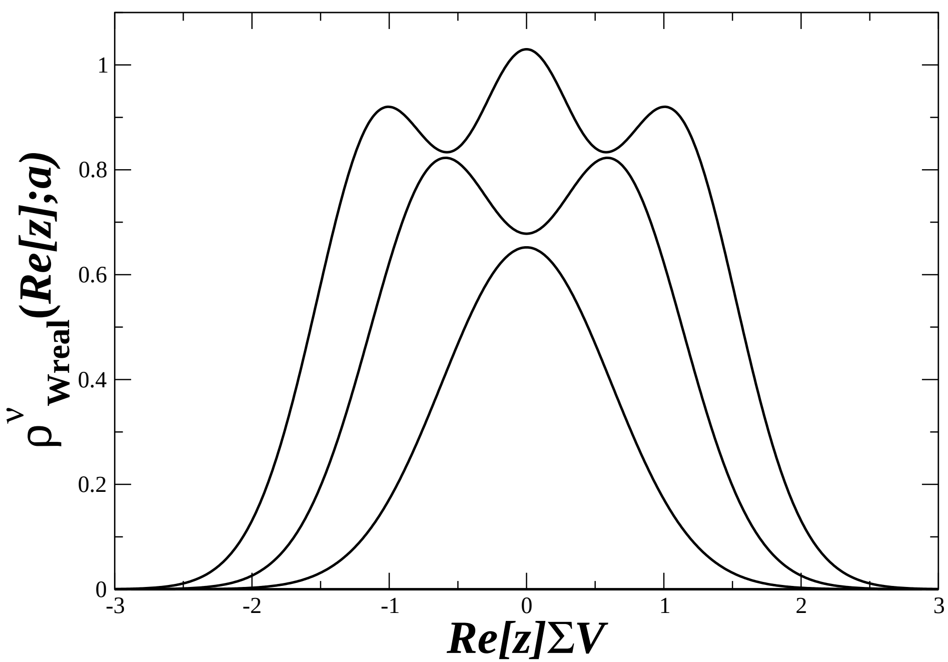
Microscopic density of D_W

The real eigenvalues of D_W in sector $\nu = 0, 1, 2, 3$

$N_f = 0$

$a\sqrt{W_8 V} = 0.2$

$W_6 = W_7 = 0$



Gattringer Hip Lang NPB 508 (1997) 329
Hernandez NPB 536 (1998) 345
Damgaard Splittorff Verbaarschot PRL 105:162002,2010

Kieburg, Verbaarschot, Zafeiropoulos arXiv:1109.0656

The Hermitian Wilson Dirac operator D_5

Introduce

$$D_5 \equiv \gamma_5 (D_W + m)$$

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γ_5-Hermiticity of D_W Hermiticity of D_5

$$D_W^\dagger = \gamma_5 D_W \gamma_5 \qquad \Rightarrow \qquad D_5^\dagger = D_5$$

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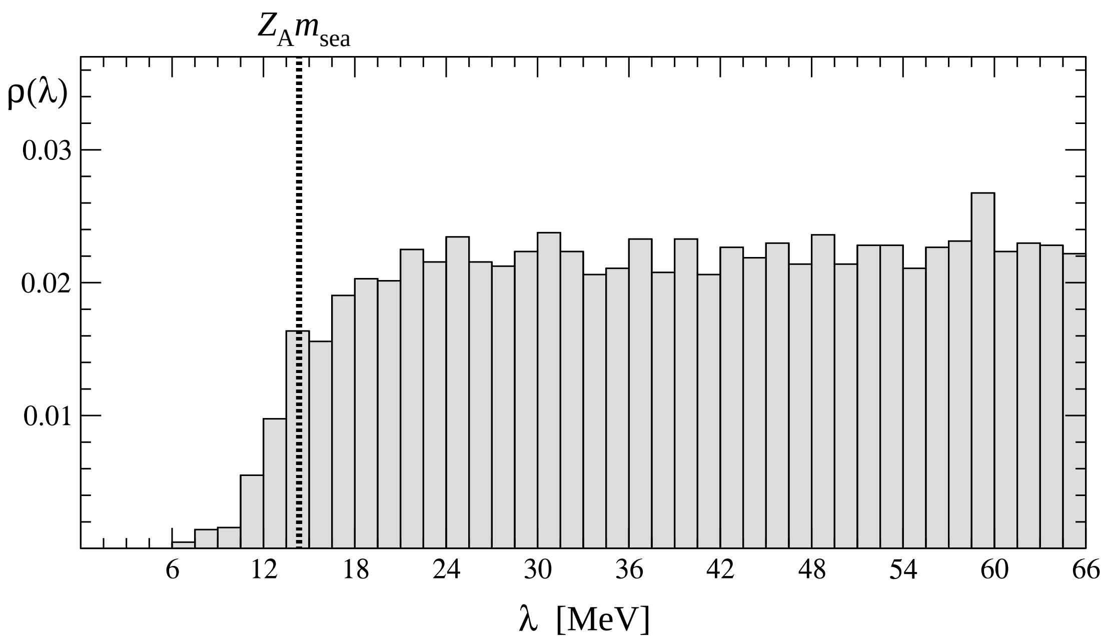
D_5 is hermitian but spectrum *not* symmetric: *not* $(\lambda^5, -\lambda^5)$

Lattice

Spectrum of D_5 for $N_f = 2$



- Aoki phase when gap closes

Lüscher JHEP0707:081,2007
Del Debbio Giusti Lüscher Petronzio Tantalo JHEP0702:082,2007
Aoki PRD 30 (1984) 2653

Bitar Heller Narayanan PLB **418** 167 (1998)

From Wilson CPT to the spectrum of D_5

The SUSY method

The SUSY way of writing the eigenvalue density

$$\rho_5^{N_f}(\lambda^5, m; a) = \frac{1}{\pi}\mathrm{Im}[\lim_{z'\to z}\frac{d}{dz}Z^{\nu}_{N_f+1|1}(m, m, z, z'; a)]$$

SUSY/graded *generating function* for the eigenvalue density

$$Z^{\nu}_{N_f+1|1}(m, m, z, z'; a) = \int \mathrm{d}A\,\det(D_W + m)^{N_f}\,\frac{\det(D_W + m + z\gamma_5)}{\det(D_W + m + z'\gamma_5)}\; e^{-S_{\mathrm{YM}}(A)}$$

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integrate over the gauge fields

Efetov *Supersymmetry in disorder and chaos* Cambridge Uni Press 1997

The SUSY method in Wilson CPT

The SUSY way of writing the eigenvalue density

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SUSY/graded *generating function* for the eigenvalue density

$$\mathcal{Z}_{N_f+1|1}(m, m, z, z'; a) =$$

$$\int dU \ \mathrm{Sdet}(U)^{\nu}$$

$$\times e^{i\frac{1}{2}\mathrm{Str}(\mathcal{M}[U-U^{-1}])+i\frac{1}{2}\mathrm{Str}(\mathcal{Z}[U+U^{-1}])+a^2W_8V\mathrm{Str}(U^2+U^{-2})}$$

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integrate over graded Goldstone manifold $Gl(N_f + 1|1)$

Damgaard Osborn Toublan Verbaarschot NPB **547** 305 (1999): $a = 0$

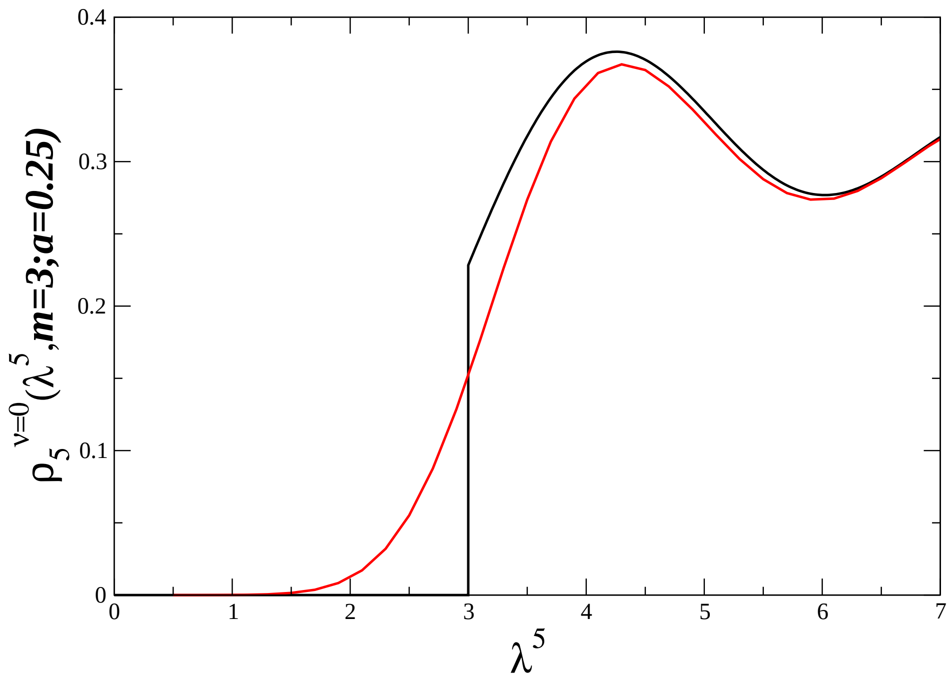
Splittorff, Verbaarschot, NPB 683 (2004) 467: $\mu \neq 0$

Spectrum of D_5 $\qquad N_f = 2$

$m\Sigma V = 3$

$\nu = 0$

$a\sqrt{VW_8} = 0.25$



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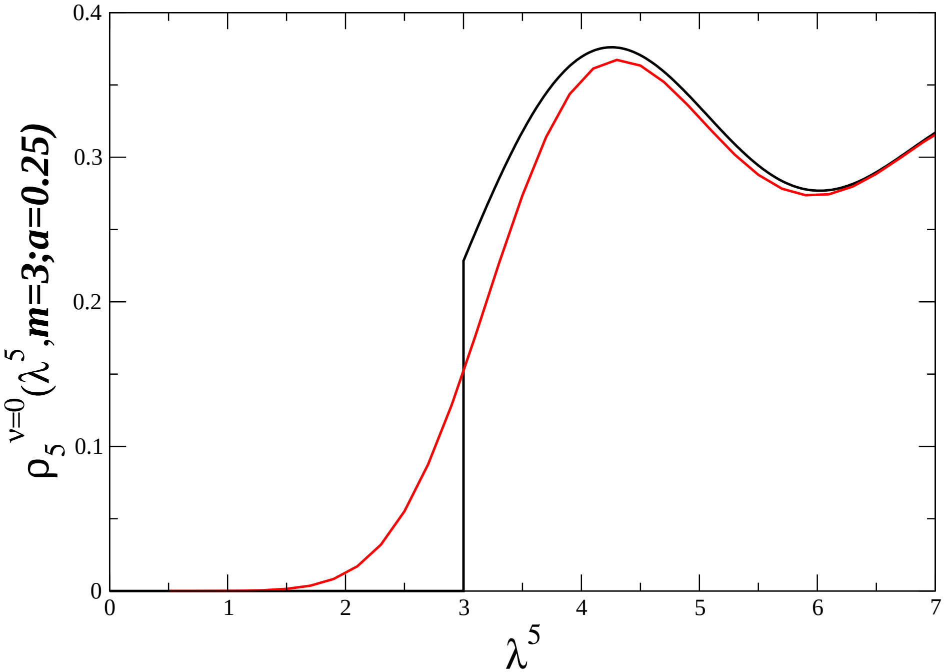

Explains $1/\sqrt{V}$ scaling of width of smallest eigenvalues

Del Debbio Giusti Lüscher Petronzio Tantalo JHEP0702:082,2007
Damgaard Splittorff Verbaarschot PRL 105:162002,2010
Akemann Damgaard Splittorff Verbaarschot PoS LATTICE2010 (2010) 079

Splittorff Verbaarschot PRD 84 (2011) 065031

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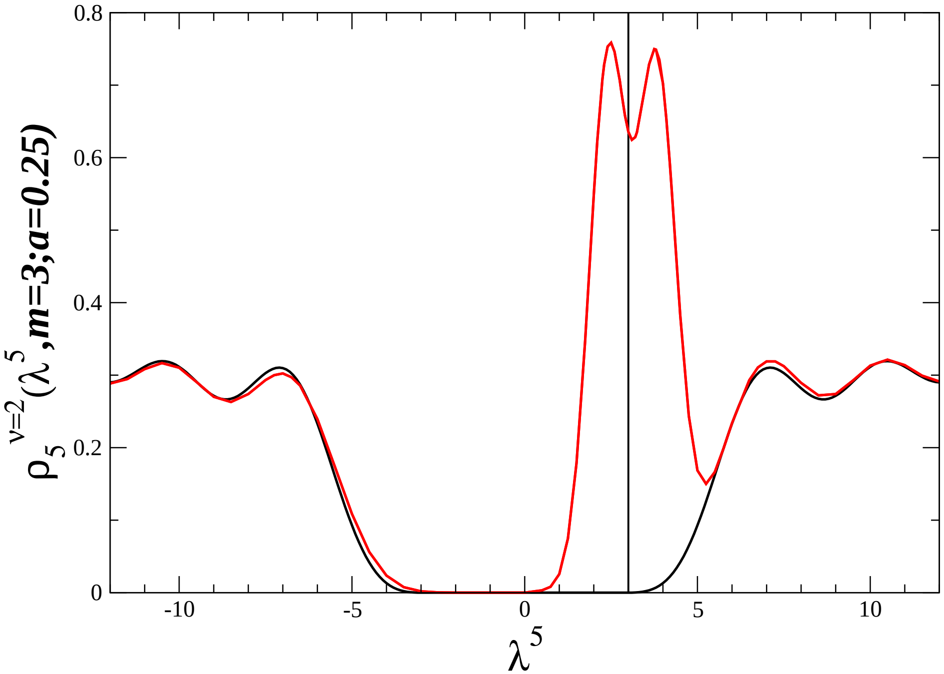

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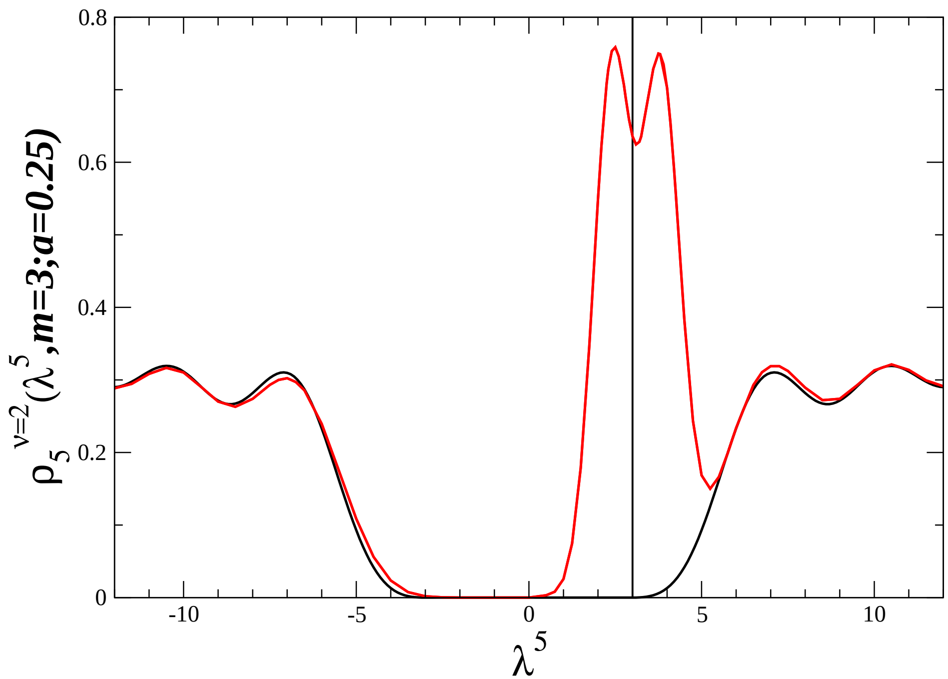

Unquenched: $\rho_5(\lambda^5 = 0, m; a) = 0$ since

$$\det{}^2(D_W + m) = \det{}^2 D_5(m) = \prod_j \lambda_j^5(m)^2$$

Damgaard Splittorff Verbaarschot PRL 105:162002,2010

Akemann Damgaard Splittorff Verbaarschot PoS LATTICE2010 (2010) 079

Splittorff Verbaarschot PRD 84 (2011) 065031

Twisted mass

$$\det(D_W + m + iz_t\gamma_5\tau_3) = \det(D_5(m) + iz_t\tau_3)$$
$$= \prod_j (\lambda_j^5(m) + iz_t)(\lambda_j^5(m) - iz_t) = \prod_j (\lambda_j^5(m)^2 + z_t^2)$$

Twisted mass

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Maximal twist ($m = 0$):

spectrum of $D_5(m = 0)$

$$\frac{d}{dz_t}\log Z(m = 0, z_t; a) = \int d\lambda^5 \frac{2z_t}{{\lambda^5}^2 + z_t^2}\ \rho_5(\lambda^5, m = 0, z_t; a)$$

Banks-Casher relation

$$\Sigma = \frac{\pi\rho_5(\lambda^5 = 0; a)}{V}$$

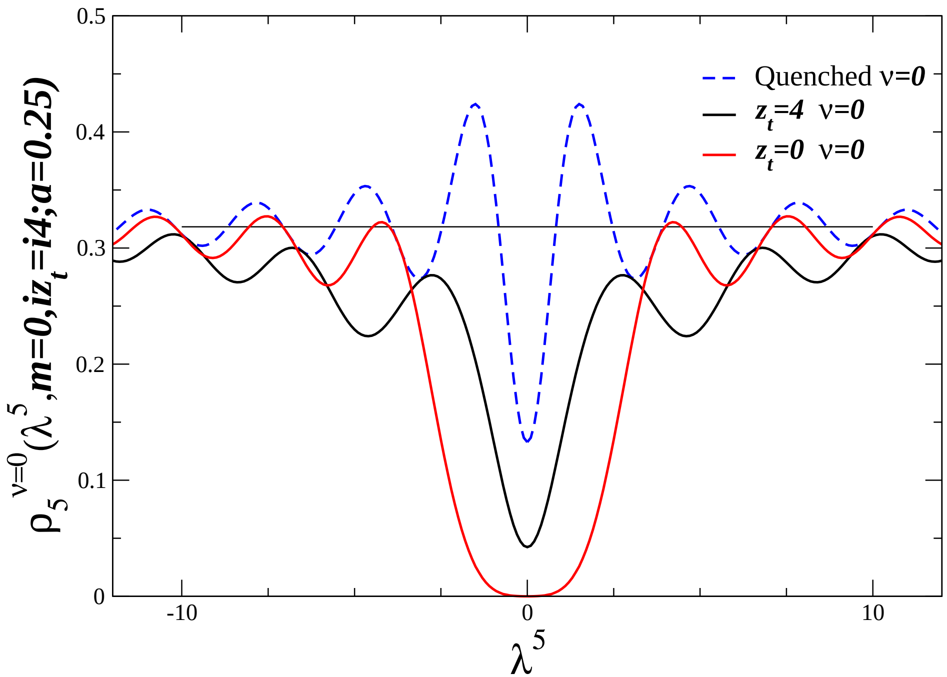
Frezzotti Grassi Sint Weisz JHEP **0108**, 058 (2001)

Banks Casher NPB 169, 103 (1980)

Spectrum of D_5 at maximally twisted mass

$\nu = 0$

$a\sqrt{VW_8} = 0.25$



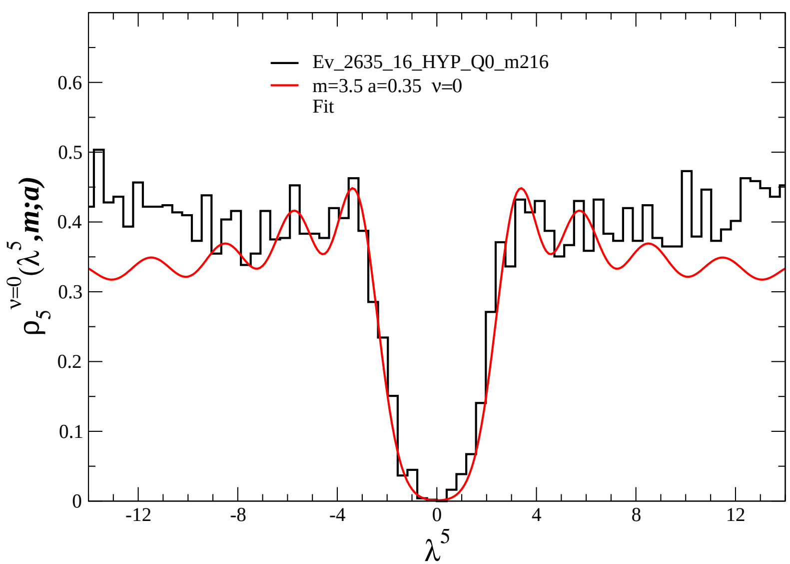
$$\prod_j (\lambda_j^5(m=0)^2 + z_t^2)$$

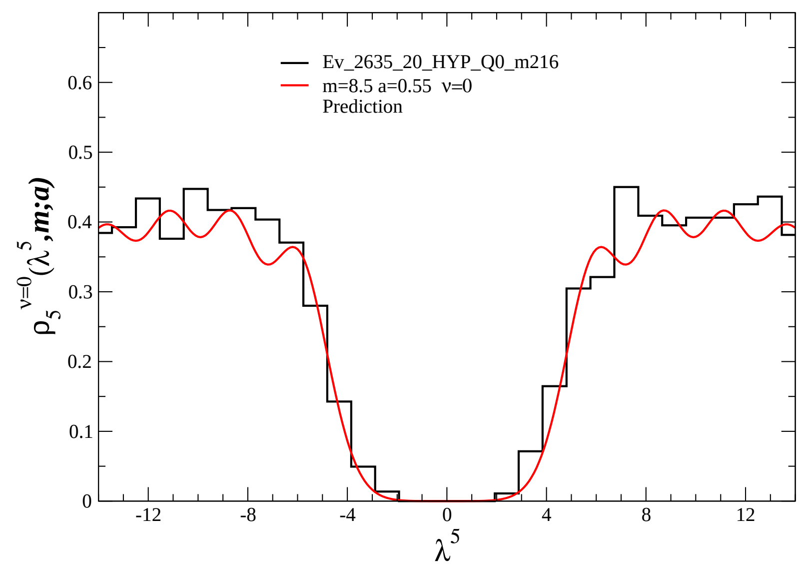
Splittorff Verbaarschot *to appear*

Does it work ?

Spectrum of D_5 for $N_f = 0$ on 16^4 and 20^4 lattice

Histograms: lattice   Curves: WCPT



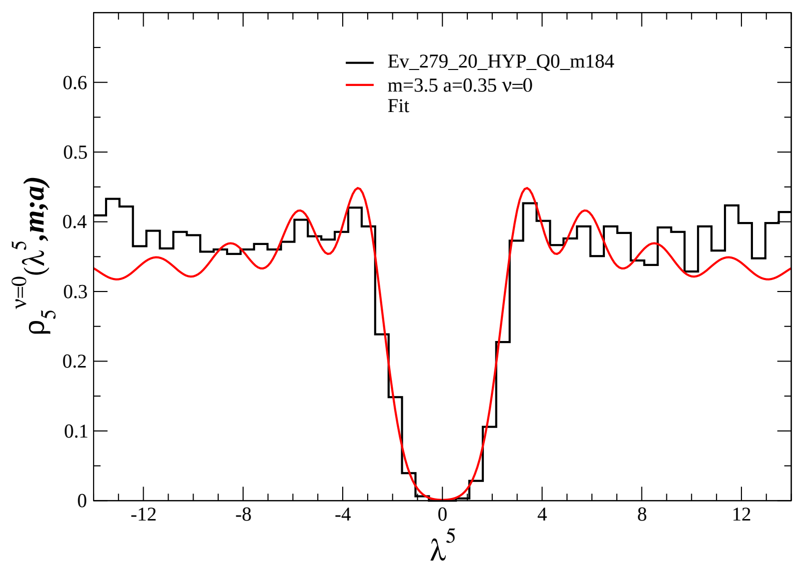


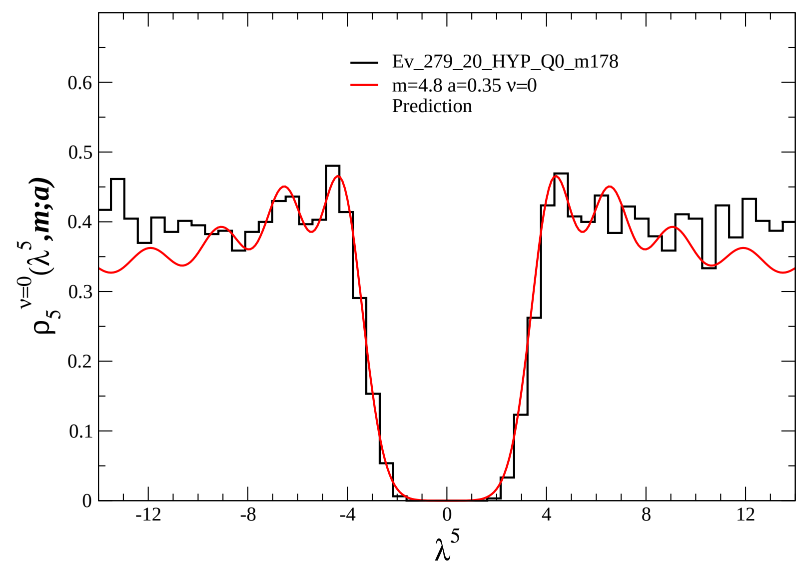
LHS fit (ΣV, $m\Sigma V$ and a_8) RHS prediction: Volume scaling

Damgaard Heller Splittorff arXiv:1110.2851

Spectrum of D_5 for $N_f = 0$ on 20^4 lattice smaller coupling

Histograms: lattice Curves: WCPT





LHS fit (ΣV, $m\Sigma V$ and a_8) RHS prediction: mass scaling

Damgaard Heller Splittorff arXiv:1110.2851

Deuzeman Wenger Wuilloud arXiv:1110.4002

Wilson CPT

The chiral Lagrangian for Wilson fermions

$$\begin{aligned}\mathcal{L} &= \frac{F_\pi^2}{4}\mathrm{Tr}\left(d_\mu U d_\mu U^\dagger\right) + \frac{m}{2}\Sigma\mathrm{Tr}(U+U^\dagger) \\ &-a^2W_6[\mathrm{Tr}\left(U+U^\dagger\right)]^2 - a^2W_7[\mathrm{Tr}\left(U-U^\dagger\right)]^2 \\ &-a^2W_8\mathrm{Tr}(U^2+{U^\dagger}^2)\end{aligned}$$

We used $W_6 = W_7 = 0$ and $W_8 > 0$

Sharpe Singleton PRD **58**, 074501 (1998)
Rupak Shoresh PRD **66**, 054503 (2002)
Aoki PRD 68:054508,2003
Bar Rupak Shoresh PRD **70**, 034508 (2004)
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The sign of W_8

γ_5-Hermiticity $\Rightarrow \det^2(D_W + m) \geq 0$

$$\text{QCD inequality} \qquad\qquad Z^{\nu}_{N_f=2}(m;a) \geq 0$$

The sign of W_8

γ_5-Hermiticity $\Rightarrow \det^2(D_W + m) \geq 0$

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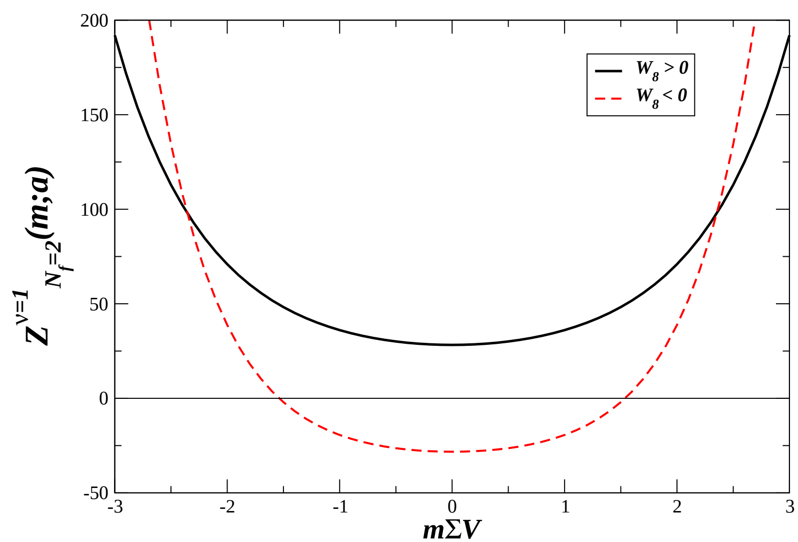
Only satisfied if $W_8 > 0$ (for $W_6 = W_7 = 0$)



$a^2 V W_8 = 1$ (full) $\qquad a^2 V W_8 = -1$ (dashed)

Akemann Damgaard Splittorff Verbaarschot PRD 83 (2011) 085014

Hansen Sharpe arXiv:1111.2404

The sign of W_8

$$W_8 > 0$$

$W_8 < 0$: Wilson CPT with $W_8 < 0$ corresponds to an *anti-hermitian* D_W

The sign of W_8

$W_8 > 0$

$W_8 < 0$: Wilson CPT with $W_8 < 0$ corresponds to an *anti-hermitian* D_W

γ_5-Hermitian

$$D_W = \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2}\nabla_\mu\nabla^*_\mu \qquad \rightleftharpoons \qquad W_8 > 0$$

Anti-Hermitian (not γ_5-Hermitian)

$$D_{OS} = \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - i\frac{ar}{2}\nabla_\mu\nabla^*_\mu \qquad \rightleftharpoons \qquad W_8 < 0$$

sign problem

Akemann Damgaard Splittorff Verbaarschot PRD 83 (2011) 085014

The sign of W_6 and W_7

$$W_6 < 0$$

$W_6 > 0$: lattice theory where the spectrum of D_W can fluctuate vertically

Not allowed by γ_5 hermiticity !

The sign of W_6 and W_7

$$W_6 < 0$$

$W_6 > 0$: lattice theory where the spectrum of D_W can fluctuate vertically

Not allowed by γ_5 hermiticity !

$$W_7 < 0$$

$W_7 > 0$: lattice theory where the spectrum of D_5 can fluctuate vertically

Not allowed by γ_5 hermiticity !

Splittorff Verbaarschot *to appear*

The sign of W_6, W_7 and W_8

Constraints from γ_5-Hermiticity

$$W_6 < 0, \quad W_7 < 0, \quad W_8 > 0$$

The sign of W_6, W_7 and W_8

Constraints from γ_5-Hermiticity

$$W_6 < 0, \quad W_7 < 0, \quad W_8 > 0$$

Aoki (2nd order PT)

$$W_8 + 2W_6 > 0$$

Sharpe Singleton (1st order PT)

$$W_8 + 2W_6 < 0$$

The sign of W_6, W_7 and W_8

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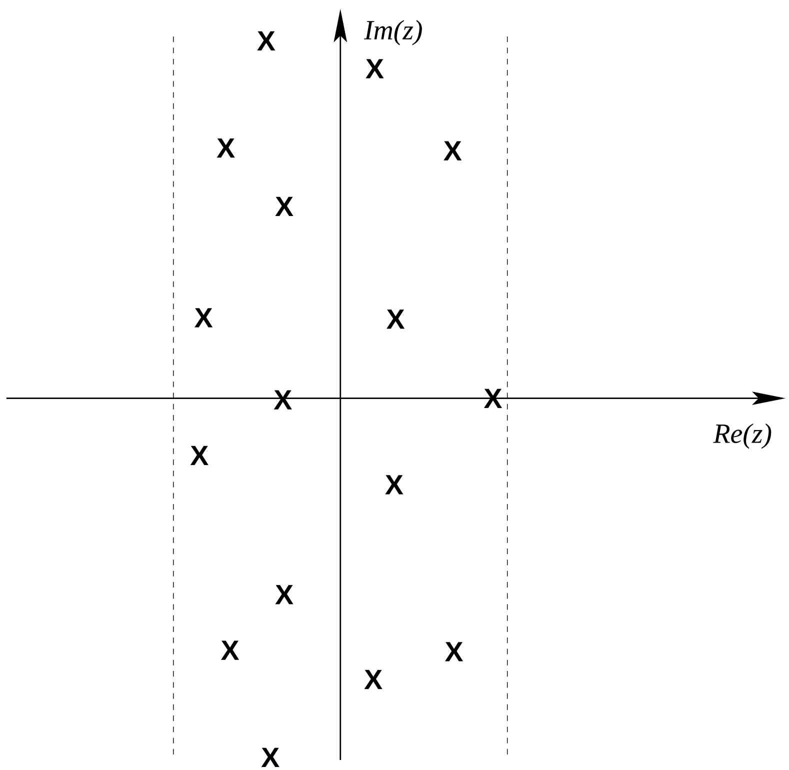
$$W_8 + 2W_6 < 0$$

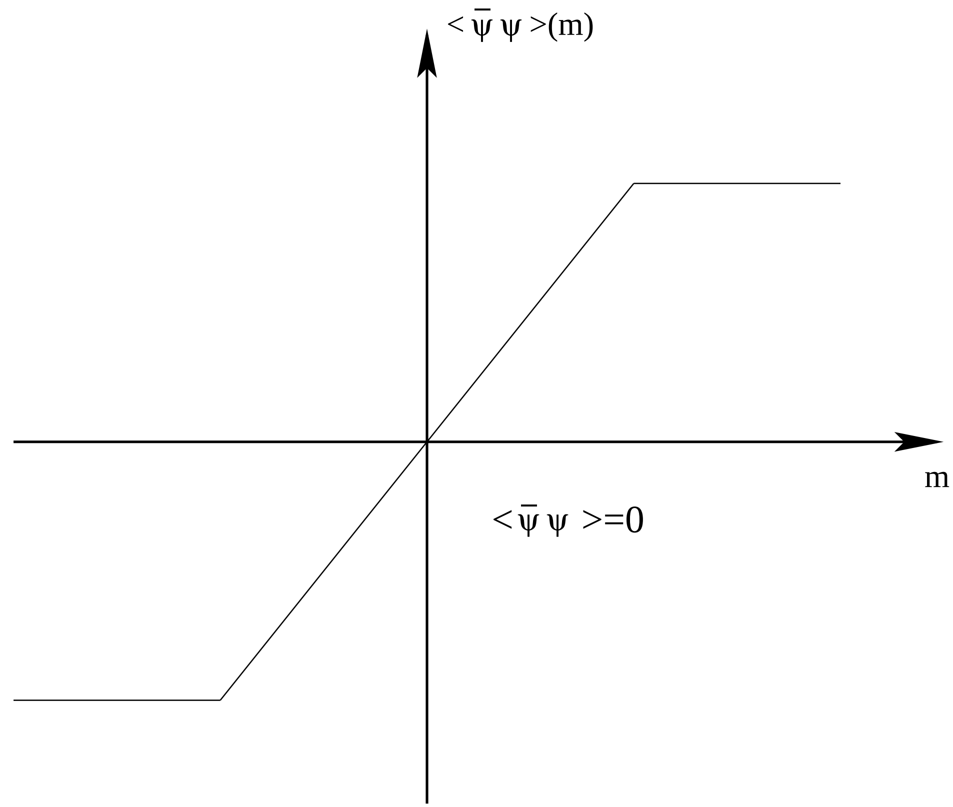
Both allowed by γ_5 hermiticity !

Farchioni et al. Eur.Phys.J.C47:453-472,2006
Baron et al. (ETM collab) JHEP08(2010)097

Bernardoni Bulava Sommer arXiv:1111.4351

Aoki phase $\quad W_8 + 2W_6 > 0$





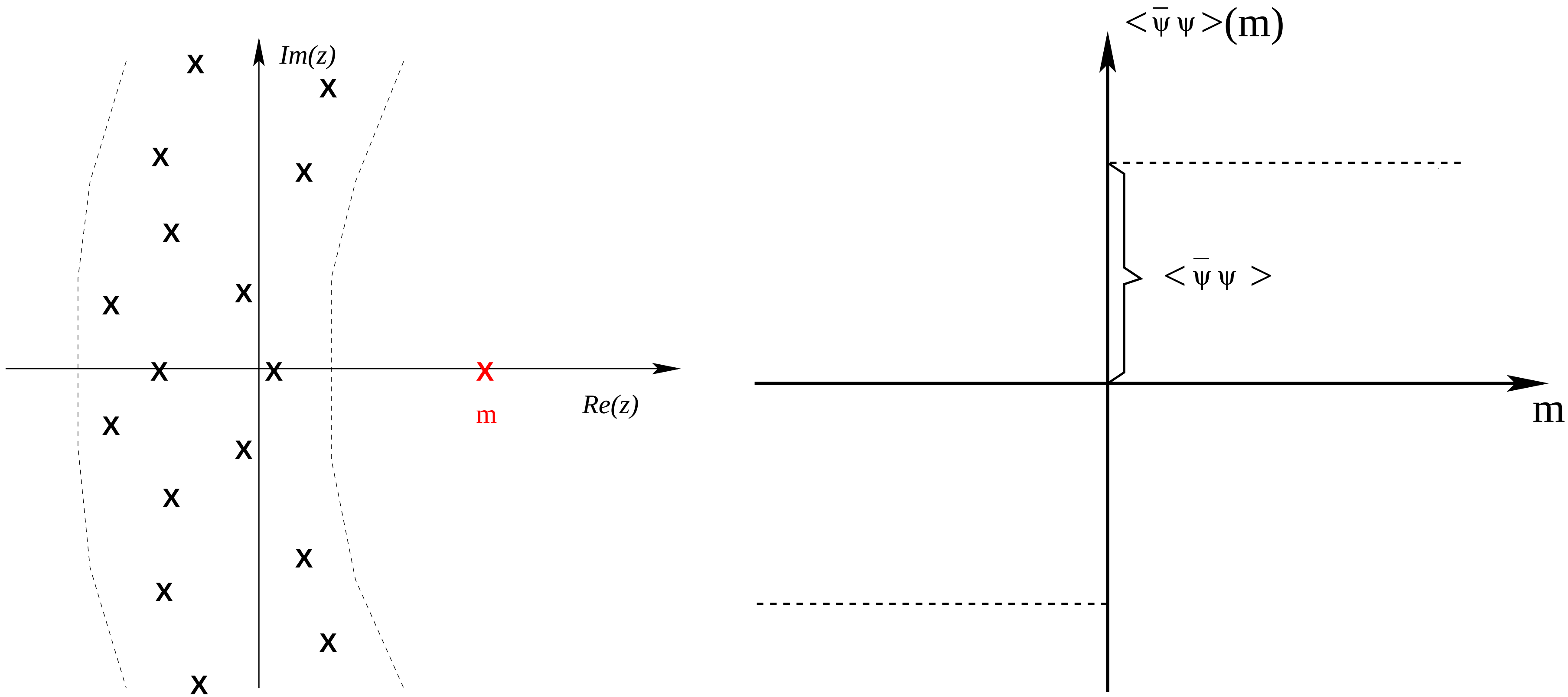
Electrostatic analogy:

Eigenvalues = charges, quark mass = test charge

Aoki PRD **30** 2653 (1984)

Barbour et al. NPB 275 (1986) 296 (nonzero μ)

Sharpe Singleton $W_8 + 2W_6 < 0$



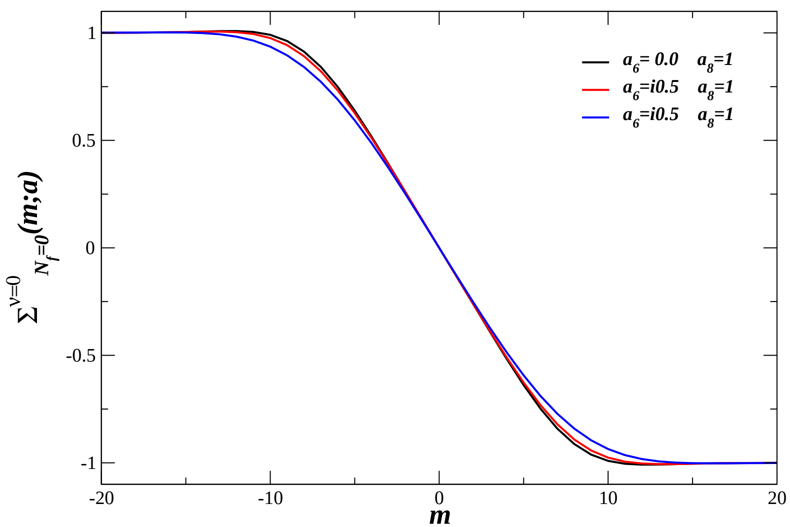
$W_6 < 0$ prefers the same signs of all $Re[z]$

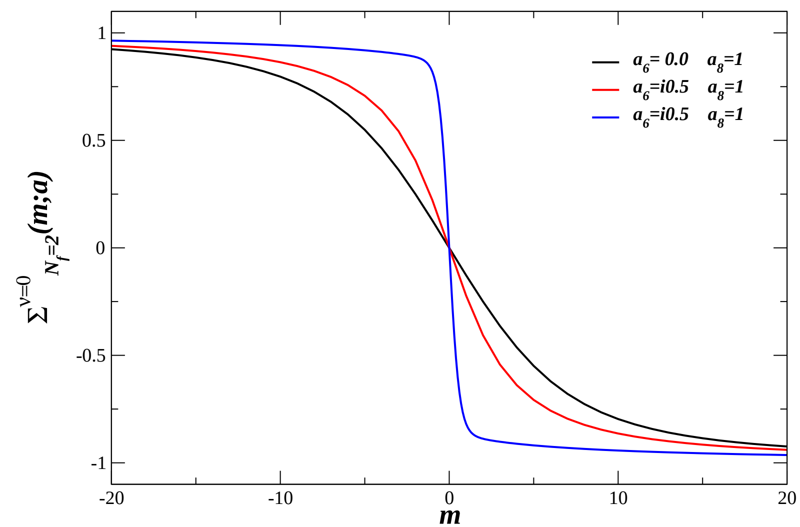
Splittorff Verbaarschot *to appear*

Quenched and unquenched condensate

$W_8 + 2W_6 > 0$ $\quad$ $W_8 + 2W_6 = 0$ $\quad$ $W_8 + 2W_6 < 0$

Quenched $\qquad$ $N_f = 2$





Sharpe Singleton $\qquad$ only for $N_f > 0$

Splittorff Verbaarschot *to appear*

Conclusions

Derived the microscopic eigenvalue density from WCPT

- for the real eigenvalues of D_W

- for $D_5 = \gamma_5(D_W + m)$ (also for twisted mass)

in sectors with fixed index of D_W

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Constraints on the parameters of WCPT from γ_5-Hermiticity

More: Unquenched lattice, individual eigenvalues, complex eigenvalues of D_W, dist of chirality ... *suggestions are welcome!*

Akemann, Nagao arXiv:1108.3035

Kieburg, Verbaarschot, Zafeiropoulos arXiv:1109.0656

Additional slides

Quenched microscopic density of $D_5 = \gamma_5(D_W + m)$

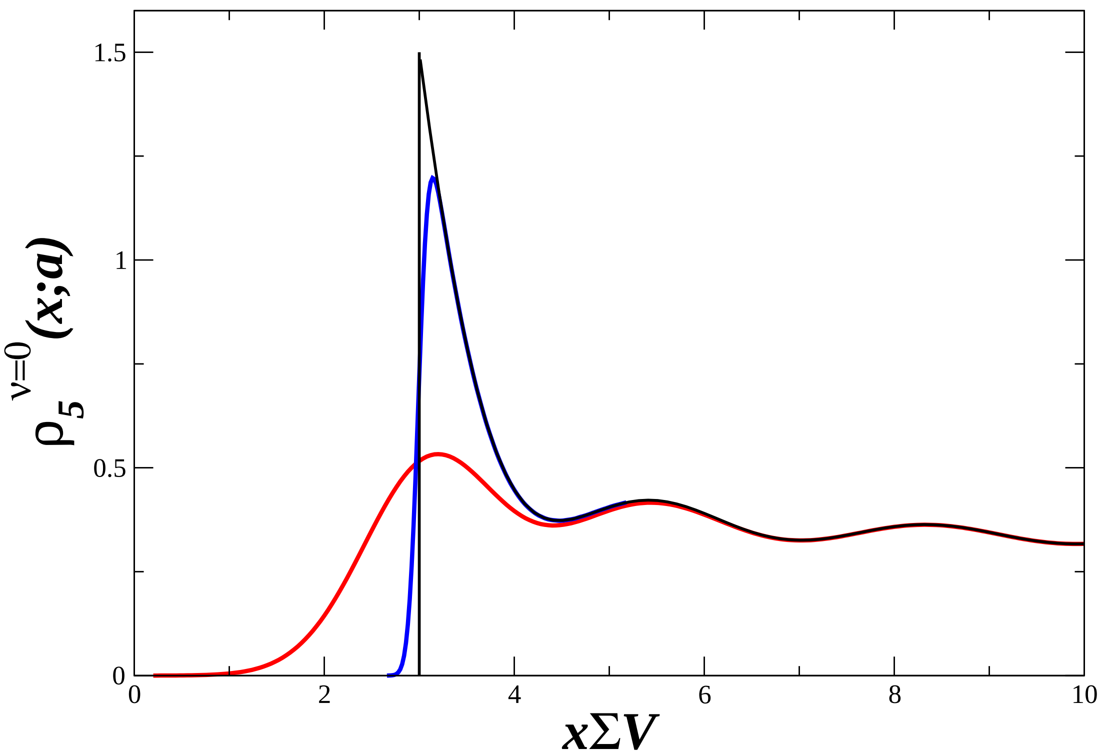
Sector $\nu = 0$

$m\Sigma V = 3$

$a\sqrt{W_8 V} = 0$

$a\sqrt{W_8 V} = 0.03$

$a\sqrt{W_8 V} = 0.250$



For $\nu = 0$ the density is symmetric: $\rho_5^{\nu=0}(x;a) = \rho_5^{\nu=0}(-x;a)$

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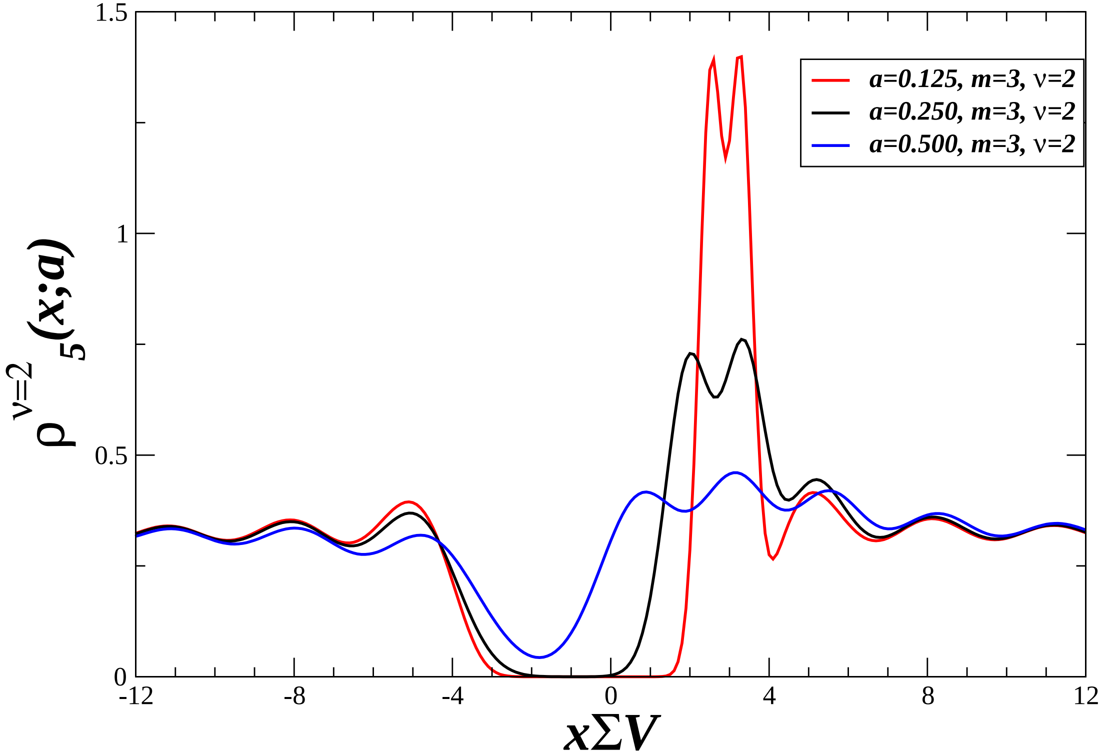
Quenched microscopic density of $D_5 = \gamma_5(D_W + m)$

Sector $\nu = 2$ increasing $a\sqrt{W_8 V}$

$m\Sigma V = 3$

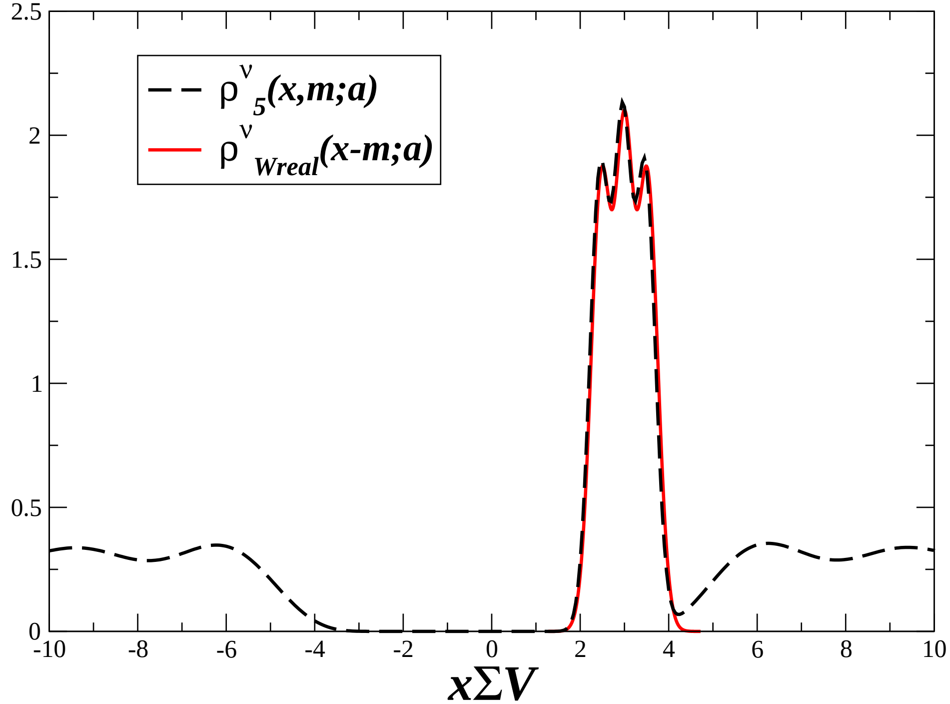


Quenched microscopic density of D_5 and D_W

Sector $\nu = 3$:

$a\sqrt{W_8 V} = 0.1$

$m\Sigma V = 3$



The real modes, ϕ, of D_W are almost chiral: $\phi^\dagger \gamma_5 \phi \simeq 1$

Itho Iwasaki Yoshie PRD 36 (1987) 527
Gattringer Hip Lang NPB 508 (1997) 329
Gattringer Hip NPB 536 (1998) 363
Hernandez NPB 536 (1998) 345

Akemann Damgaard Splittorff Verbaarschot PRD 83 (2011) 085014

W_6 and W_7

The double-trace terms re-expressed as gaussian integrals

$$Z_{N_f}^{\nu}(m, x; a_6, a_8) = \frac{1}{4\sqrt{\pi} a_6} \int_{-\infty}^{\infty} dy \, e^{-\frac{y^2}{16|a_6^2|}} \, Z_{N_f}^{\nu}(m + y, x; a_6 = 0, a_8)$$

where $a_6 = a\sqrt{W_6 V}$ and $a_8 = a\sqrt{W_8 V}$

W_6 **and** W_7

The double-trace terms re-expressed as gaussian integrals

$$Z^{\nu}_{N_f}(m,x;a_6,a_8) = \frac{1}{4\sqrt{\pi}a_6}\int_{-\infty}^{\infty} dy\; e^{-\frac{y^2}{16|a_6^2|}}\; Z^{\nu}_{N_f}(m+y,x;a_6=0,a_8)$$

where $a_6 = a\sqrt{W_6 V}$ and $a_8 = a\sqrt{W_8 V}$

Also works for the density

$$\rho^{\nu}_5(x,m;a_6,a_8) = \frac{1}{4\sqrt{\pi}a_6}\int_{-\infty}^{\infty} dy\; e^{-\frac{y^2}{16|a_6^2|}}\; \rho^{\nu}_5(x,m+y;a_6=0,a_8)$$

W_6 and W_7

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W_7 averaged x instead of m

RMT for Wilson Lattice QCD

Properties of the Wilson Dirac operator

γ_5-Hermiticity

$$D^\dagger = \gamma_5 D \gamma_5$$

Properties of the Wilson Dirac operator

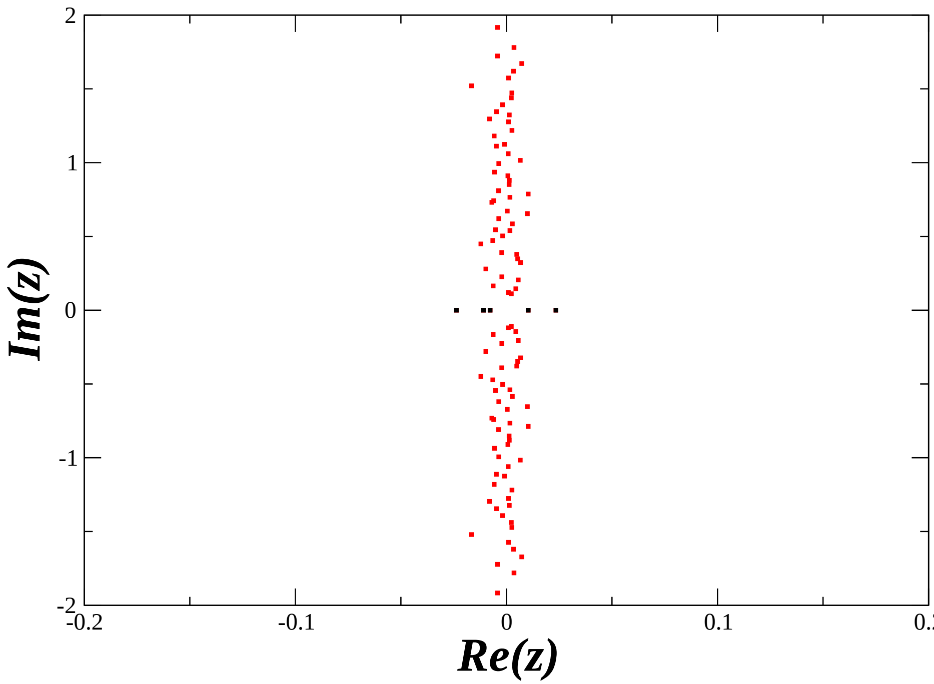
γ_5-Hermiticity

$$D^\dagger = \gamma_5 D \gamma_5$$

$$D = \begin{pmatrix} aA & iW \\ iW^\dagger & aB \end{pmatrix}$$

A ($N \times N$) and B ($(N+\nu) \times (N+\nu)$) are hermitian
W is a general complex matrix

The spectrum of one Random Matrix



Damgaard Splittorff Verbaarschot PRL 105:162002,2010

The Wilson RMT partition function

$$\mathcal{Z}^{\nu}_{N_f} \equiv \int dW\,dA dB\,\det(D+m)^{N_f}\,e^{-\frac{N}{2}\mathrm{Tr}(A^2+B^2)-N\mathrm{Tr}W^{\dagger}W}$$

where

$$D+m=\begin{pmatrix} aA+m & iW \\ iW^{\dagger} & aB+m \end{pmatrix}$$

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where

$$D+m = \begin{pmatrix} aA+m & iW \\ iW^\dagger & aB+m \end{pmatrix}$$

Same low energy theory in the ϵ-regime

Shuryak, Verbaarschot, NPA **560**, 306 (1993), Verbaarschot, PRL **72**, 2531 (1994)

Damgaard Splittorff Verbaarschot PRL 105:162002,2010

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where

$$D+m = \begin{pmatrix} aA+m & iW \\ iW^\dagger & aB+m \end{pmatrix}$$

same flavor symmetries as QCD and same breaking by m and a

$$\mathcal{Z}_{N_f}^{\nu} = \int_{U(N_f)} dU \, \det{}^{\nu}(U) \, e^{Nm\mathrm{Tr}(U+U^\dagger)-\frac{Na^2}{2}\mathrm{Tr}(U^2+U^{\dagger 2})}$$

for $N \to \infty$ with mN and $a^2 N$ fixed

Shuryak Verbaarschot NPA **560**, 306 (1993), Verbaarschot PRL **72**, 2531 (1994)

Damgaard Splittorff Verbaarschot PRL 105:162002,2010

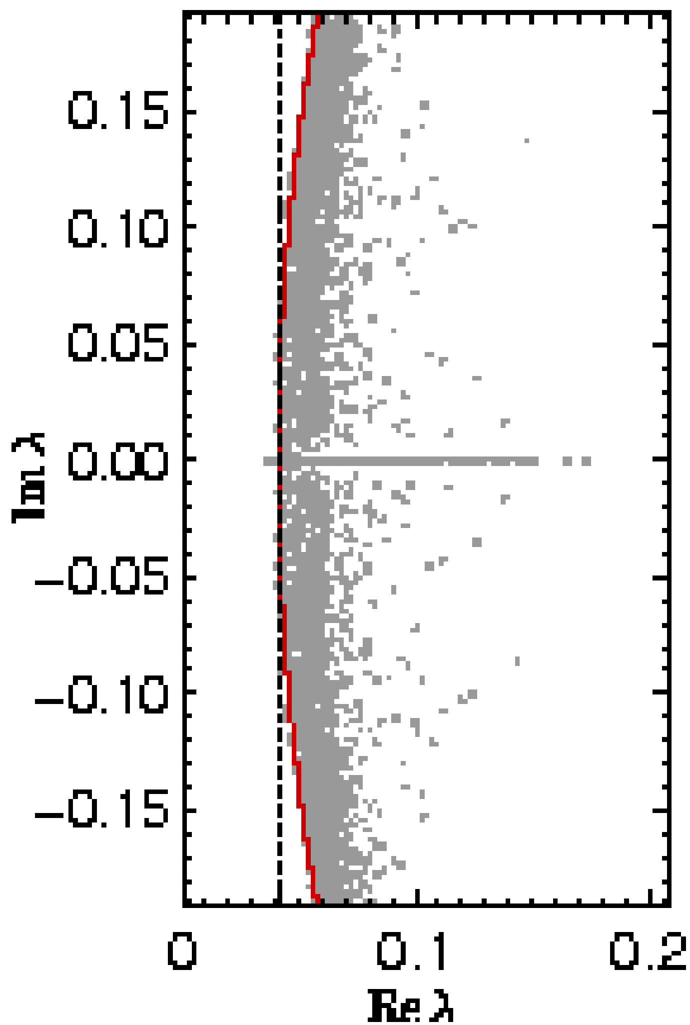
Why Wilson RMT

Usually: *easier to compute spectral correlation functions with RMT than the SUSY method*

- any N_f
- higher order correlation functions
- individual eigenvalue distributions

Splittorff Verbaarschot arXiv:to.appear

Lattice I



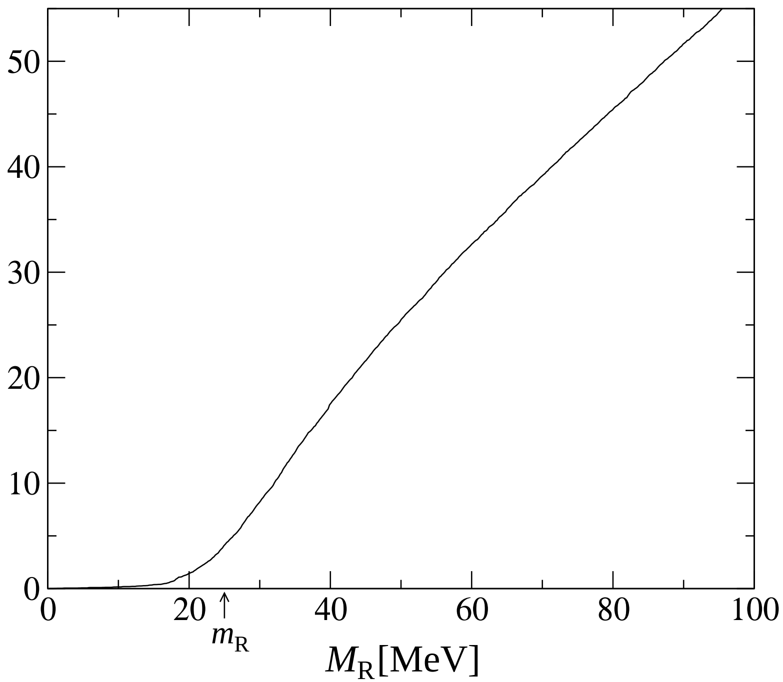
Hasenfratz Hoffmann Schaefer JHEP0705:029,2007

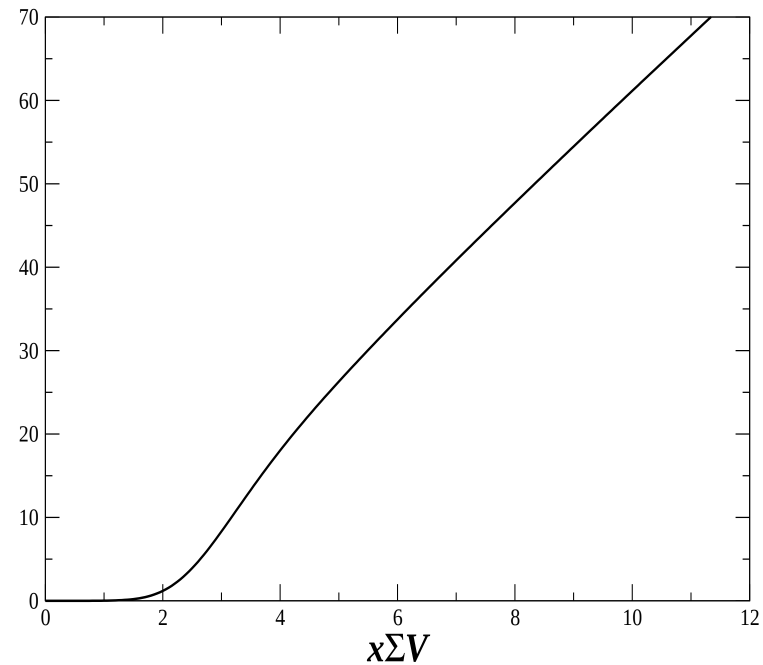
Spectrum of D_5 for $N_f = 0$

- integrated up from zero $\&$ summed over the index

Lattice 64×32^3 $a \simeq 0.07\,fm$

WCPT ($m\Sigma V = 3$, $a_8 = 0.2$)





Lüscher Palombi JHEP09(2010)110 Akemann Damgaard Splittorff Verbaarschot PRD 83 (2011) 085014

Necco Shindler arXiv:1101.1778