Nucleon sigma terms and electromagnetic corrections to ligh hadrons from the lattice

Alberto Ramos <alberto.ramos@cpt.univ-mrs.fr>

NIC, DESY

November 28, 2011

Dürr, Fodor, Frison, Hemmert, Hoelbling, Hoffman, Katz, Krieg, Kurth, Lellouch, Lippert, Portelli, Ramos, Schaefer, Szabo
[Budapest-Marseille-Wuppertal collaboration]

Fundamental interactions

Standard model and general relativity

$SU_L(2) \times U_Y(1)$

- Massless photons mediate electromagnetic interactions between charged particles.
- Massive vector bosons mediate short range weak interactions.

$SU_c(3)$

- Massless gluons mediate strong interactions between quarks.

General Relativity

- Gravity as a geometric property of space-time.

Strong interactions

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{i=\text{u,d,s,c,b,t}} \overline{\psi}_i\gamma^\mu(\partial_\mu - \imath g A_\mu - m_i)\psi_i$$

Fundamental degrees of freedom

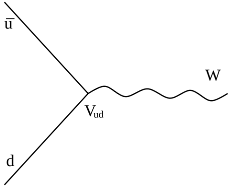
- Massless gluons that mediate strong interactions.
- Quarks that feel the strong interaction.

We observe

- Physical particle states are massive.
- Physical particle states are "white".
- Spontaneous chiral symmetry breaking.

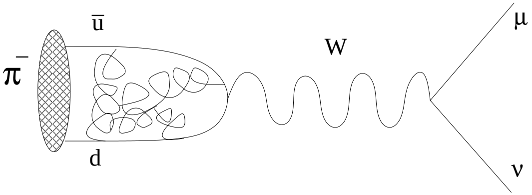
"...there does not exist a convincing, whether or not mathematically complete, theoretical computation demonstrating any of the three properties in QCD..."
– Arthur Jaffe and Edward Witten –

Flavour physics



$$\propto K(m_u, m_d, m_W) \times V_{ud}$$

Flavour physics

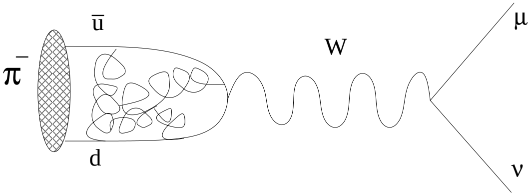


$$\Gamma(\pi \to \mu\nu_\mu) = \frac{G^2 M_\pi m_\mu^2}{8\pi}\left(1 - \frac{m_\mu^2}{M_\pi^2}\right)^2 \times |V_{ud}|^2 \times |F_\pi|^2$$

Common structure for weak decays: Product of

- Kinematic factor
- CKM Matrix element
- Non perturbative QCD factor

Flavour physics



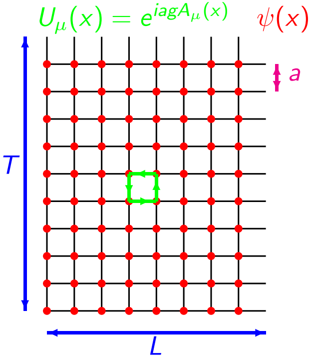
$$\Gamma(\pi \to \mu\nu_\mu) = \frac{G^2 M_\pi m_\mu^2}{8\pi}\left(1 - \frac{m_\mu^2}{M_\pi^2}\right)^2 \times |V_{ud}|^2 \times |F_\pi|^2$$

Common structure for weak decays: Product of

- Kinematic factor
- CKM Matrix element
- Non perturbative QCD factor

Lattice QCD in one slide

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



$$\langle O \rangle = \int \mathcal{D}[U]\mathcal{D}\overline{\psi}\mathcal{D}\psi\, O(U,\overline{\psi},\psi)e^{-S_G[U]-S_F[U,}$$
$$= \int \mathcal{D}[U]\, O(U)_{\rm Wick}\, e^{-S_G[U]}\det(D)$$

- Compute the integral numerically $\rightarrow$ Monte Carlo sampling of $e^{-S_G[U]}\det(D) \geq 0$.
- Observable computed averaging over samples

$$\langle O \rangle = \frac{1}{N_{\rm conf}} \sum_{i=1}^{N_{\rm conf}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\rm conf}})$$

NOT A MODEL: Lattice QCD IS real world QCD ($a \rightarrow 0$, $L \rightarrow \infty$, ...)

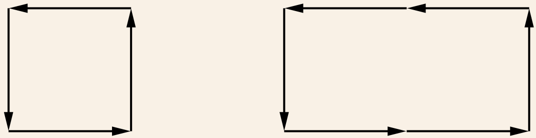
Action details

Gauge action

Tree level improved Lüscher-Weisz action [Lüscher et al (1985)]

$$S_g = \beta \left\{ \frac{c_0}{3} \sum_{\Box} \mathrm{ReTr}(1 - U_{\Box}) + \frac{c_1}{3} \sum_{\mathrm{Rec.}} \mathrm{ReTr}(1 - U_{\mathrm{Rec}}) \right\}$$

with $c_0 = 5/3$ and $c_1 = -1/12$



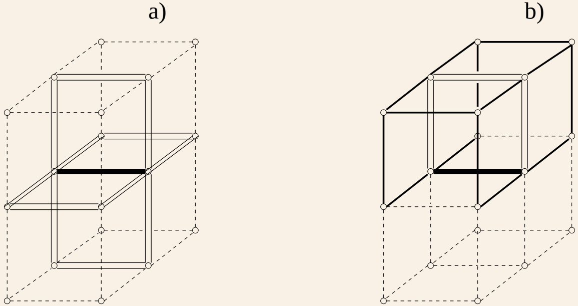
Action details

Fermion action

Tree level $\mathcal{O}(a)$ improved Wilson fermions [Sheikholeslami et al (1985)]

$$S_f = S_W[U^{(2)}] - \frac{c_{SW}}{2} \sum_x \sum_{\mu<\nu} (\overline{\psi}\sigma_{\mu\nu} F_{\mu\nu}[U^{(2)}]\psi)(x)$$

Coupled to smeared links $U^{(2)}(x)$



Action details

Fermion action

Combination of HYP setup and EXP recipe [Capitani, Dürr, Hoelbling (2006)].

$$\Gamma^{(1)}_{\mu;\nu\rho} = \sum_{\pm\sigma\neq(\mu,\nu\rho)} U_\sigma(x) U_\mu(x+\sigma) U^+_\sigma(x+\mu)$$

$$V^{(1)}_{\mu;\nu\rho} = \exp\left\{ \frac{\alpha_3}{2} \mathcal{P}\left(\Gamma^{(1)}_{\mu;\nu\rho} U^+_\mu(x) \right) \right\} U_\mu(x)$$

$$\Gamma^{(2)}_{\mu;\nu} = \sum_{\pm\sigma\neq(\mu,\nu)} V^{(1)}_{\sigma;\mu\nu}(x) V^{(1)}_{\mu;\nu\sigma}(x+\sigma) V^{(1)+}_{\sigma;\mu\nu}(x+\mu)$$

$$V^{(2)}_{\mu;\nu} = \exp\left\{ \frac{\alpha_2}{4} \mathcal{P}\left(\Gamma^{(2)}_{\mu;\nu} U^+_\mu(x) \right) \right\} U_\mu(x)$$

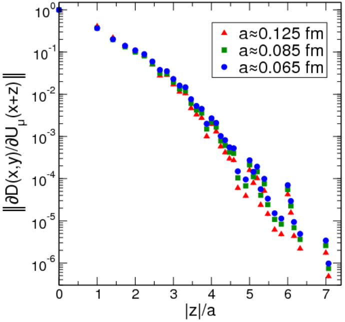
$$\Gamma^{(3)}_{\mu} = \sum_{\pm\sigma\neq(\mu)} V^{(2)}_{\sigma;\mu}(x) V^{(2)}_{\mu;\nu}(x+\sigma) V^{(2)+}_{\sigma;\mu}(x+\mu)$$

$$U^{(1)}_{\mu} = \exp\left\{ \frac{\alpha_3}{6} \mathcal{P}\left(\Gamma^{(3)}_{\mu} U^+_\mu(x) \right) \right\} U_\mu(x)$$

Smearing and locality [Dürr et al Science (2008)]

Our Dirac operator is ultralocal

- $\overline{\psi}(x)D(x,y)\psi(y)$, $D(x,y) \equiv 0$ for $|x-y| > a$.
- $D(x,y)$ depends on $U_\mu(x+z)$ for $|z| > a$, but



$$\left\| \frac{\partial D(x,y)}{\partial U_\mu(x+z)} \right\| \equiv 0 \quad \text{for} \quad |z| > 7.1a$$

and

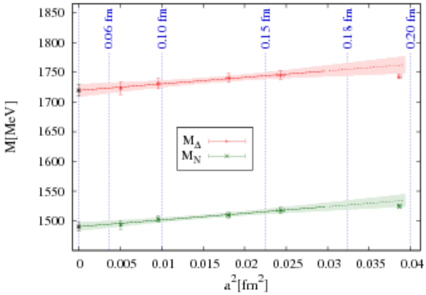
$$\left\| \frac{\partial D(x,y)}{\partial U_\mu(x+z)} \right\| \propto e^{-2.2|z|/a}$$

Not a problem

$2.2a^{-1}$ is much larger than any scale of interests (masses, ...)

Smearing and the continuum limit

Detailed scaling study: $N_f = 3$ with our preferred action(s). 5 lattice spacings, $M_\pi L \gtrsim 4$ and $m_q \sim m_s^{\text{phys}}$. [Dürr et al. Phys.Rev. D79 (2009)].



M_N and M_Δ linear in a^2 for $a \in [0.065, 0.16]$ fm.

Very good scaling properties

Looks non-perturbatively $\mathcal{O}(a)$ improved.

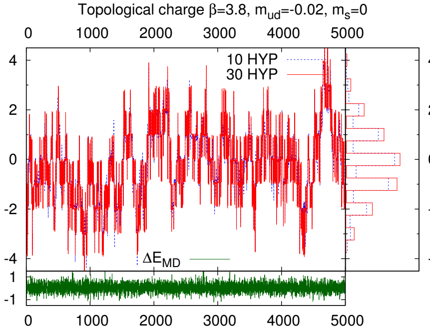
Topological charge sampling on finest lattice

Long autocorrelations in top. charge as $a \to 0$ have been observed [(Schaefer et al (2010))]

5000 trajectory autocorrelation-check run

Absolute worst case: $a \simeq 0.054\, fm$ and $M_\pi \simeq 260\, MeV$ on $48^3 \times 64$ lattice

$$Q_{\text{naive}} = \frac{a^4}{(4\pi)^2} \sum_x Tr[F^{\text{HYP}}_{\mu\nu} \tilde{F}^{\text{HYP}}_{\mu\nu}](x)$$



- Q fluctuates and evolves: $\tau_{\text{int}} \sim 30$
- Q falls into integer centered bins

- Q distribution is reasonably symmetric

$\Rightarrow$ no obvious ergodicity problem

Properties of the action

- The action differs from the usual one by terms that are both ultralocal and irrelevant.
- Action in the same universality class as QCD (true for any number of smearing steps).
- Chiral symmetry breaking is reduced.
- One can reach smaller quark masses before one runs into the problem of "exceptional" configurations.
- Action with tree level C_{SW} is close to be non perturbatively $\mathcal{O}(a)$ improved.

Nice properties for phenomenological studies.

Reaching the physical point

Figure: “Our” particle accelerators.

Reaching the physical point

β	am_{ud}	am_s	$L^3 \times T$	traj.	aM_π	aM_K
3.3	-0.0960	-0.057	$16^3 \times 32$	10000	0.4115(6)	0.4749(6)
	-0.1100	-0.057	$16^3 \times 32$	1450	0.322(1)	0.422(1)
	-0.1200	-0.057	$16^3 \times 64$	4500	0.2448(9)	0.3826(6)
	-0.1233	-0.057	$24^3 \times 64$	2000	0.2105(8)	0.3668(6)
	-0.1233	-0.057	$32^3 \times 64$	1300	0.211(1)	0.3663(8)
	-0.1265	-0.057	$24^3 \times 64$	2100	0.169(1)	0.3500(7)
3.57	-0.0318	0,-0.010	$24^3 \times 64$	1650,1650	0.2214(7),0.2178(5)	0.2883(7),0.2657(5)
	-0.0380	0,-0.010	$24^3 \times 64$	1350,1550	0.1837(7),0.1778(7)	0.2720(6),0.2469(6)
	-0.0440	0,-0.007	$32^3 \times 64$	1000,1000	0.1348(7),0.1320(7)	0.2531(6),0.2362(7)
	-0.0483	0,-0.007	$48^3 \times 64$	500,1000	0.0865(8),0.0811(5)	0.2401(8),0.2210(5)
3.7	-0.007	0.0	$32^3 \times 96$	1100	0.2130(4)	0.2275(4)
	-0.013	0.0	$32^3 \times 96$	1450	0.1830(4)	0.2123(3)
	-0.020	0.0	$32^3 \times 96$	2050	0.1399(3)	0.1920(3)
	-0.022	0.0	$32^3 \times 96$	1350	0.1273(5)	0.1882(4)
	-0.025	0.0	$40^3 \times 96$	1450	0.1021(4)	0.1788(4)

Table: 20 ensembles. $a = 0.125, 0.085, 0.065$ fm. $M_\pi L \gtrsim 4$.

Reaching the physical point

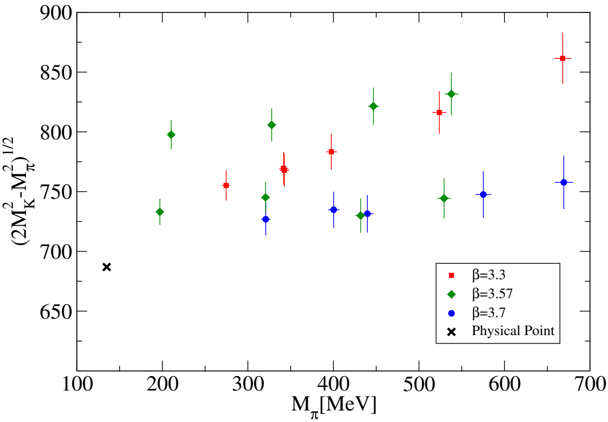


Figure: 20 ensembles. $a = 0.125, 0.085, 0.065$ fm. $M_\pi L \gtrsim 4$.

Reaching the physical point

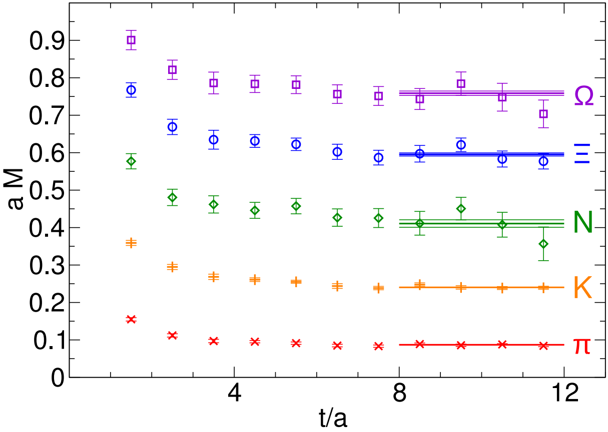


Figure: Effective masses for $\beta = 3.57$ and $M_\pi = 190$ MeV

Dark matter

- Discrepancy between measurements of the mass of structures larger than galaxies made trough dynamical (GR) means and measurements based on the "luminous" matter these objects contains.

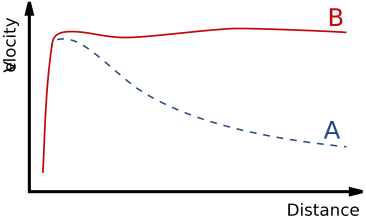


Figure: Rotation curve of a typical spiral galaxy: predicted (A) and observed (B). Dark matter can explain the velocity curve having a 'flat' appearance out to a large radius.

Dark matter

- Discrepancy between measurements of the mass of structures larger than galaxies made trough dynamical (GR) means and measurements based on the "luminous" matter these objects contains.
- Under standard interpretation (BB, FRW), "dark" matter accounts for 23% of the mass of the visible universe.
- Dark matter candidates not in the list of known particles, but arise naturally in many extensions of the SM.
- Direct evidence of its existence and a concrete understanding of its nature have remained elusive.

Dark matter

- Discrepancy between measurements of the mass of structures larger than galaxies made trough dynamical (GR) means and measurements based on the "luminous" matter these objects contains.
- Under standard interpretation (BB, FRW), "dark" matter accounts for 23% of the mass of the visible universe.
- Dark matter candidates not in the list of known particles, but arise naturally in many extensions of the SM.
- Direct evidence of its existence and a concrete understanding of its nature have remained elusive.

Dark matter

- Discrepancy between measurements of the mass of structures larger than galaxies made trough dynamical (GR) means and measurements based on the "luminous" matter these objects contains.
- Under standard interpretation (BB, FRW), "dark" matter accounts for 23% of the mass of the visible universe.
- Dark matter candidates not in the list of known particles, but arise naturally in many extensions of the SM.
- Direct evidence of its existence and a concrete understanding of its nature have remained elusive.

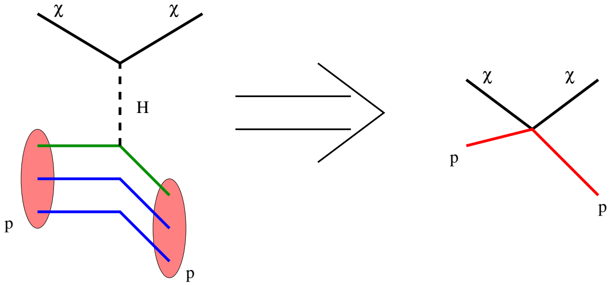
Dark matter

- Discrepancy between measurements of the mass of structures larger than galaxies made trough dynamical (GR) means and measurements based on the "luminous" matter these objects contains.
- Under standard interpretation (BB, FRW), "dark" matter accounts for 23% of the mass of the visible universe.
- Dark matter candidates not in the list of known particles, but arise naturally in many extensions of the SM.
- Direct evidence of its existence and a concrete understanding of its nature have remained elusive.

Dark matter detection

Dark mater detection is one of the challenges of the decade.

Direct DM detection



$$\mathcal{L}_{int} = \lambda_q \overline{n} n \overline{\chi} \chi \qquad \mathcal{L}_{int} = \lambda_N \overline{q} q \overline{\chi} \chi$$

Nucleon form factors relate these couplings

$$\lambda_N \longrightarrow \sum_{q=1}^{6} f_q^N \lambda_q$$

Direct DM detection

Hadronic Uncertainties in the Elastic Scattering of Supersymmetric Dark Matter

John Ellis,[1,*] Keith A. Olive,[2,†] and Christopher Savage[2,‡]

[1] *TH Division, Physics Department, CERN, 1211 Geneva 23, Switzerland*

[2] *William I. Fine Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA*

(Dated: February 7, 2008)

Abstract

We review the uncertainties in the spin-independent and -dependent elastic scattering cross sections of supersymmetric dark matter particles on protons and neutrons. We propagate the uncertainties in quark masses and hadronic matrix elements that are related to the π-nucleon σ term and the spin content of the nucleon. By far the largest single uncertainty is that in spin-independent scattering induced by our ignorance of the $\langle N|\bar{q}q|N\rangle$ matrix elements linked to the π-nucleon σ term, which affects the ratio of cross sections on proton and neutron targets as well as their absolute values. This uncertainty is already impacting the interpretations of experimental searches for cold dark matter. *We plead for an experimental campaign to determine better the π-nucleon σ term.* Uncertainties in the spin content of the proton affect significantly, but less strongly, the calculation of rates used in indirect searches.

Nucleon sigma terms

Definitions

$$\sigma_{\pi N} = \hat{m}\langle N(p) \mid (\bar{u}u + \bar{d}d)(0) \mid N(p)\rangle$$
$$\sigma_{\bar{s}sN} = m_s\langle N(p) \mid (\bar{s}s)(0) \mid N(p)\rangle$$
$$y = \frac{2\langle N \mid \bar{s}s \mid N\rangle}{\langle N \mid \bar{u}u + \bar{d}d \mid N\rangle}$$

Important for

- Hadron spectrum
- The quark mas ratio $m_s/\hat{m}$
- $\pi - N$ and $K - N$ scattering amplitudes
- Counting rates in searches of the Higgs boson

Nucleon sigma terms

Definitions

$$
\begin{aligned}
\sigma_{\pi N} &= \hat{m}\langle N(p) \mid (\overline{u}u + \overline{d}d)(0) \mid N(p)\rangle \\
\sigma_{\bar{s}sN} &= m_s\langle N(p) \mid (\bar{s}s)(0) \mid N(p)\rangle \\
y &= \frac{2\langle N \mid \bar{s}s \mid N\rangle}{\langle N \mid \overline{u}u + \overline{d}d \mid N\rangle}
\end{aligned}
$$

Important for

- Hadron spectrum
- The quark mas ratio $m_s/\hat{m}$
- $\pi - N$ and $K - N$ scattering amplitudes
- Counting rates in searches of the Higgs boson

Nucleon sigma terms

Definitions

$$\sigma_{\pi N} = \hat{m}\langle N(p) \mid (\overline{u}u + \overline{d}d)(0) \mid N(p)\rangle$$

$$\sigma_{\bar{s}sN} = m_s\langle N(p) \mid (\bar{s}s)(0) \mid N(p)\rangle$$

$$y = \frac{2\langle N \mid \bar{s}s \mid N\rangle}{\langle N \mid \overline{u}u + \overline{d}d \mid N\rangle}$$

Important for

- Hadron spectrum
- The quark mas ratio $m_s/\hat{m}$
- $\pi - N$ and $K - N$ scattering amplitudes
- Counting rates in searches of the Higgs boson

Nucleon sigma terms

Definitions

$$\begin{aligned}
\sigma_{\pi N} &= \hat{m}\langle N(p) \mid (\bar{u}u + \bar{d}d)(0) \mid N(p)\rangle \\
\sigma_{\bar{s}sN} &= m_s\langle N(p) \mid (\bar{s}s)(0) \mid N(p)\rangle \\
y &= \frac{2\langle N \mid \bar{s}s \mid N\rangle}{\langle N \mid \bar{u}u + \bar{d}d \mid N\rangle}
\end{aligned}$$

Important for

- Hadron spectrum
- The quark mas ratio $m_s/\hat{m}$
- $\pi - N$ and $K - N$ scattering amplitudes
- Counting rates in searches of the Higgs boson

Nucleon sigma terms

Definitions

$$\sigma_{\pi N} = \hat{m}\langle N(p) \mid (\overline{u}u + \overline{d}d)(0) \mid N(p)\rangle$$

$$\sigma_{\bar{s}sN} = m_s\langle N(p) \mid (\bar{s}s)(0) \mid N(p)\rangle$$

$$y = \frac{2\langle N \mid \bar{s}s \mid N\rangle}{\langle N \mid \overline{u}u + \overline{d}d \mid N\rangle}$$

Important for

- Hadron spectrum
- The quark mas ratio $m_s/\hat{m}$
- $\pi - N$ and $K - N$ scattering amplitudes
- Counting rates in searches of the Higgs boson

Nucleon sigma terms

The nucleon mass is given by

$$M_N = \langle N \mid T_{\mu\mu} \mid N \rangle = \sum_q m_q \langle N \mid \overline{q} q \mid N \rangle + \text{Gluonic contribution}$$

then

$$\begin{aligned} m_{ud} \frac{\partial M_N}{\partial m_{ud}} = m_{ud} \langle N \mid \overline{u} u + \overline{d} d \mid N \rangle &= \sigma_{\pi N} \\ m_s \frac{\partial M_N}{\partial m_s} = m_s \langle N \mid \overline{s} s \mid N \rangle &= \sigma_{\overline{s} s N} \end{aligned}$$

- The sigma terms measures how much the nucleon mass changes when you change quark masses
- Is this useful ??

Nucleon sigma terms

The nucleon mass is given by

$$M_N = \langle N \mid T_{\mu\mu} \mid N\rangle = \sum_q m_q \langle N \mid \overline{q}q \mid N\rangle + \text{Gluonic contribution}$$

then

$$\begin{aligned} m_{ud}\frac{\partial M_N}{\partial m_{ud}} = m_{ud}\langle N \mid \overline{u}u + \overline{d}d \mid N\rangle &= \sigma_{\pi N} \\ m_s\frac{\partial M_N}{\partial m_s} = m_s\langle N \mid \overline{s}s \mid N\rangle &= \sigma_{\overline{s}sN} \end{aligned}$$

- The sigma terms measures how much the nucleon mass changes when you change quark masses
- Is this useful ??

Thanks to the lattice

Quark masses are constant in the real world but can (and are) varied in the lattice.

Regular extrapolations

Any physical quantity is analytic in the quark masses if you do not expand around $m_q = 0$.

Expansion variables

$$\begin{aligned} (M_\pi^{\exp})^2 &= \frac{1}{2}[(M_\pi^{\Phi})^2 + (M_\pi^{\max})^2] \\ M_{\bar{s}s}^2 &= 2M_K^2 - M_\pi^2 \end{aligned}$$

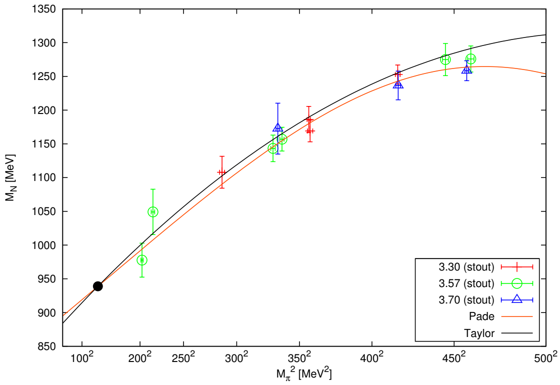
Nucleon mass dependence

$$M_N = M_0 + \sum_{i=1}^{N_\pi} \alpha_i' \left[M_\pi^2 - (M_\pi^{\exp})^2\right]^i + \sum_{i=1}^{N_{\bar{s}s}} \beta_i' \left[M_{\bar{s}s}^2 - \left(M_{\bar{s}s}^{\Phi}\right)^2\right]^j$$

$$M_N = \frac{M_0}{1 + \sum_{i=1}^{N_\pi} \alpha_i' \left[M_\pi^2 - (M_\pi^{\exp})^2\right]^i + \sum_{i=1}^{N_{\bar{s}s}} \beta_i' \left[M_{\bar{s}s}^2 - \left(M_{\bar{s}s}^{\Phi}\right)^2\right]^j}$$

One example fit

Figure: $M_\pi < 420$ MeV; $\chi^2/dof \approx 4.9/7$.



$$\sigma_{\pi N} = 53(10)_{\text{stat}}\mathrm{MeV} \qquad \text{Taylor}$$

$$\sigma_{\pi N} = 44(6)_{\text{stat}}\mathrm{MeV} \qquad \text{Pade}$$

χPT

Nucleon mass as a function of quark masses

$$M_N = M_0 + \alpha M_\pi^2 + \text{Higher order terms}$$

Higer order terms

- HBχPT: $\propto g_A M_\pi^3$
- CBχPT: $\propto g_A h(M_\pi)$

$$h(M_\pi) = -\frac{M_\pi^3}{4\pi^2}\left\{\sqrt{1-\left(\frac{M_\pi}{2M_0}\right)^2}\arccos\frac{M_\pi}{2M_0} + \frac{M_\pi}{2M_0}\log\frac{M_\pi}{M_0}\right\}$$

Three flavour CBχPT [S. Dürr @ LAT10]

$$M_X = M_0 - 4c_X^\pi M_\pi^2 - 4c_X^s M_{\bar{s}s}^2 + \sum_{\alpha=\pi,K,\eta} \frac{g_X^\alpha}{F_\alpha^2} M_0^3 h\left(\frac{M_\alpha}{M_0}\right) + d^\pi M_\pi^4 + d^s M_{\bar{s}s}^4$$

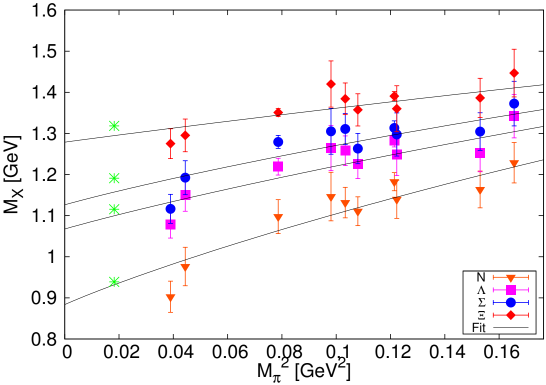
The good thing: All the constants $c_X^{\pi,s}, g_X^\alpha$ depend only on 5 independent parameters: b_0, b_D, b_F, g_A, ξ

How to fit lattice data

- First we set the scale (using Ω).
- Fit data in physical units using the formula above.

CBχPT fits

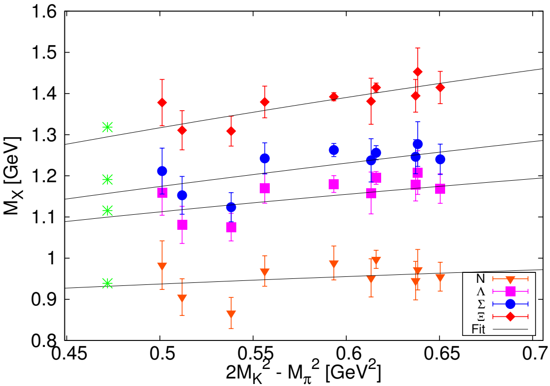
Figure: $M_\pi < 420$ MeV; χ^2/points $\approx 9/10$.



$$\sigma_{\pi N} = 47(9)_{\text{stat}} \qquad \text{CB}\chi\text{PT}$$

CBχPT fits

Figure: $M_\pi < 420$ MeV; χ^2/points $\approx 9/10$.



Complete analysis of the sigma term

Analysis

Simultaneously analyse all four octect members: N, Λ, Σ, Ξ.

Complete analysis of the sigma term

- Use a total of 8 different functional forms for the chiral extrapolation.
  - 4 Based of χPT. Fitting or not g_a, ξ, fitting or not h.o.t. $d_\pi, d_{\bar{s}s}$.
  - 2 General regular expansions. 1 Taylor, 1 Padé.
  - 2 $SU(3)$ constrained regular expansions. 1 Taylor, 1 Padé.
- Impose two pion mass cuts $M_\pi < 410\ \mathrm{MeV}, 550\ \mathrm{MeV}$
- Do a correlated fit for all four channels.
- Formally cutoff effects are $\mathcal{O}(\alpha_s a)$, but they are small in our data,and compatible with them being absent or $\mathcal{O}(a^2)$. $M_X \to M_X(1 + \eta a^p)$, with $p = 0, 1, 2$.
- Finite volume effects below the 1% in our data.
- Use 18 different fitting intervals for the correlators.

Idea of the analysis

Use all the 864 different method to obtain the physical quantity:

- Weight them by the fit quality and build a distribution.
- The median is our final result (typical result of our analysis).
- The 68% confidence interval gives the systematic error.

Complete analysis of the sigma term

- Use a total of 8 different functional forms for the chiral extrapolation.
  - 4 Based of χPT. Fitting or not g_a, ξ, fitting or not h.o.t. $d_\pi, d_{\bar{s}s}$.
  - 2 General regular expansions. 1 Taylor, 1 Padé.
  - 2 $SU(3)$ constrained regular expansions. 1 Taylor, 1 Padé.
- Impose two pion mass cuts $M_\pi < 410\ \mathrm{MeV}, 550\ \mathrm{MeV}$
- Do a correlated fit for all four channels.
- Formally cutoff effects are $\mathcal{O}(\alpha_s a)$, but they are small in our data,and compatible with them being absent or $\mathcal{O}(a^2)$. $M_X \to M_X(1+\eta a^p)$, with $p = 0, 1, 2$.
- Finite volume effects below the 1% in our data.
- Use 18 different fitting intervals for the correlators.

Idea of the analysis

Use all the 864 different method to obtain the physical quantity:

- Weight them by the fit quality and build a distribution.
- The median is our final result (typical result of our analysis).
- The 68% confidence interval gives the systematic error.

Complete analysis of the sigma term

- Use a total of 8 different functional forms for the chiral extrapolation.
  - 4 Based of χPT. Fitting or not g_a, ξ, fitting or not h.o.t. $d_\pi, d_{\bar{s}s}$.
  - 2 General regular expansions. 1 Taylor, 1 Padé.
  - 2 $SU(3)$ constrained regular expansions. 1 Taylor, 1 Padé.
- Impose two pion mass cuts $M_\pi < 410\ \mathrm{MeV}, 550\ \mathrm{MeV}$
- Do a correlated fit for all four channels.
- Formally cutoff effects are $\mathcal{O}(\alpha_s a)$, but they are small in our data,and compatible with them being absent or $\mathcal{O}(a^2)$. $M_X \to M_X(1 + \eta a^p)$, with $p = 0, 1, 2$.
- Finite volume effects below the 1% in our data.
- Use 18 different fitting intervals for the correlators.

Idea of the analysis

Use all the 864 different method to obtain the physical quantity:

- Weight them by the fit quality and build a distribution.
- The median is our final result (typical result of our analysis).
- The 68% confidence interval gives the systematic error.

Complete analysis of the sigma term

- Use a total of 8 different functional forms for the chiral extrapolation.
  - 4 Based of χPT. Fitting or not g_a, ξ, fitting or not h.o.t. $d_\pi, d_{\bar{s}s}$.
  - 2 General regular expansions. 1 Taylor, 1 Padé.
  - 2 $SU(3)$ constrained regular expansions. 1 Taylor, 1 Padé.
- Impose two pion mass cuts $M_\pi < 410 \text{ MeV}, 550 \text{ MeV}$
- Do a correlated fit for all four channels.
- Formally cutoff effects are $\mathcal{O}(\alpha_s a)$, but they are small in our data,and compatible with them being absent or $\mathcal{O}(a^2)$. $M_X \to M_X(1 + \eta a^p)$, with $p = 0, 1, 2$.
- Finite volume effects below the 1% in our data.
- Use 18 different fitting intervals for the correlators.

Idea of the analysis

Use all the 864 different method to obtain the physical quantity:

- Weight them by the fit quality and build a distribution.
- The median is our final result (typical result of our analysis).
- The 68% confidence interval gives the systematic error.

Complete analysis of the sigma term

- Use a total of 8 different functional forms for the chiral extrapolation.
  - 4 Based of χPT. Fitting or not g_a, ξ, fitting or not h.o.t. $d_\pi, d_{\bar{s}s}$.
  - 2 General regular expansions. 1 Taylor, 1 Padé.
  - 2 $SU(3)$ constrained regular expansions. 1 Taylor, 1 Padé.
- Impose two pion mass cuts $M_\pi < 410 \text{ MeV}, 550 \text{ MeV}$
- Do a correlated fit for all four channels.
- Formally cutoff effects are $\mathcal{O}(\alpha_s a)$, but they are small in our data,and compatible with them being absent or $\mathcal{O}(a^2)$. $M_X \to M_X(1 + \eta a^p)$, with $p = 0, 1, 2$.
- Finite volume effects below the 1% in our data.
- Use 18 different fitting intervals for the correlators.

Idea of the analysis

Use all the 864 different method to obtain the physical quantity:

- Weight them by the fit quality and build a distribution.
- The median is our final result (typical result of our analysis).
- The 68% confidence interval gives the systematic error.

Complete analysis of the sigma term

- Use a total of 8 different functional forms for the chiral extrapolation.
  - 4 Based of χPT. Fitting or not g_a, ξ, fitting or not h.o.t. $d_\pi, d_{\bar{s}s}$.
  - 2 General regular expansions. 1 Taylor, 1 Padé.
  - 2 $SU(3)$ constrained regular expansions. 1 Taylor, 1 Padé.
- Impose two pion mass cuts $M_\pi < 410 \text{ MeV}, 550 \text{ MeV}$
- Do a correlated fit for all four channels.
- Formally cutoff effects are $\mathcal{O}(\alpha_s a)$, but they are small in our data,and compatible with them being absent or $\mathcal{O}(a^2)$. $M_X \to M_X(1 + \eta a^p)$, with $p = 0, 1, 2$.
- Finite volume effects below the 1% in our data.
- Use 18 different fitting intervals for the correlators.

Idea of the analysis

Use all the 864 different method to obtain the physical quantity:

- Weight them by the fit quality and build a distribution.
- The median is our final result (typical result of our analysis).
- The 68% confidence interval gives the systematic error.

Complete analysis of the sigma term

- Use a total of 8 different functional forms for the chiral extrapolation.
  - 4 Based of χPT. Fitting or not g_a, ξ, fitting or not h.o.t. $d_\pi, d_{\bar{s}s}$.
  - 2 General regular expansions. 1 Taylor, 1 Padé.
  - 2 $SU(3)$ constrained regular expansions. 1 Taylor, 1 Padé.
- Impose two pion mass cuts $M_\pi < 410 \text{ MeV}, 550 \text{ MeV}$
- Do a correlated fit for all four channels.
- Formally cutoff effects are $\mathcal{O}(\alpha_s a)$, but they are small in our data,and compatible with them being absent or $\mathcal{O}(a^2)$. $M_X \to M_X(1 + \eta a^p)$, with $p = 0, 1, 2$.
- Finite volume effects below the 1% in our data.
- Use 18 different fitting intervals for the correlators.

Idea of the analysis

Use all the 864 different method to obtain the physical quantity:

- Weight them by the fit quality and build a distribution.
- The median is our final result (typical result of our analysis).
- The 68% confidence interval gives the systematic error.

Complete analysis of the sigma term

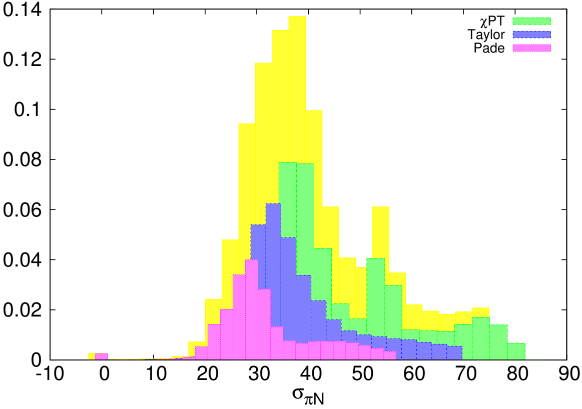


Figure: Final result $\sigma_{\pi N} = 39(4)_{\rm stat}(^{+18}_{-7})_{\rm sys}$

Complete analysis of the sigma term

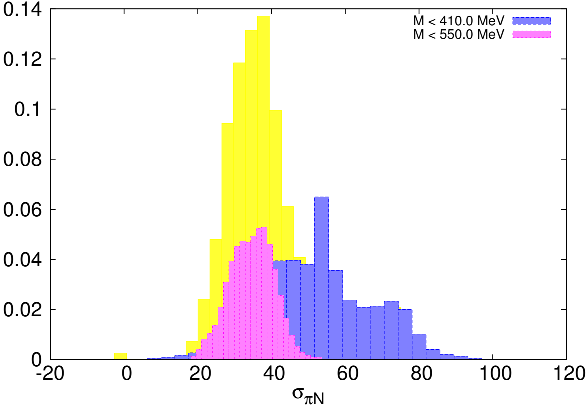


Figure: Final result $\sigma_{\pi N} = 39(4)_{\rm stat}(^{+18}_{-7})_{\rm sys}$

Complete analysis of the sigma term

Source of systematic error	error on $\sigma_{\pi N}$ [MeV]
Chiral Extrapolation:	
- Pion mass range	9.0
- Functional form	5.5
Continuum extrapolation	1.9

Table: Different sources of systematic error.

Complete analysis of the sigma term

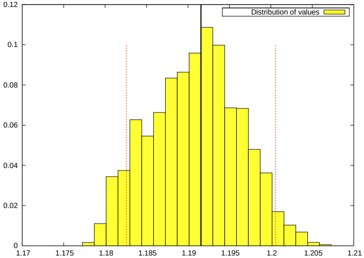
	$\sigma_{\pi X}$ [MeV]	$\sigma_{\bar{s}sX}$ [MeV]	y_X	f_{udX}	$f_{\bar{s}sX}$
N	$39(4)(^{+18}_{-7})$	$67(27)(^{+55}_{-47})$	$0.20(7)(^{+13}_{-17})$	$0.042(5)(^{+21}_{-4})$	$0.036(14)(^{+30}_{-25})$
Λ	$29(3)(^{+11}_{-5})$	$180(26)(^{+48}_{-77})$	$0.51(15)(^{+48}_{-27})$	$0.027(3)(^{+5}_{-10})$	$0.083(12)(^{+23}_{-31})$
Σ	$23(3)(^{+19}_{-3})$	$245(29)(^{+50}_{-72})$	$0.82(21)(^{+87}_{-39})$	$0.019(3)(^{+17}_{-3})$	$0.104(12)(^{+23}_{-31})$
Ξ	$16(2)(^{+8}_{-3})$	$312(32)(^{+72}_{-77})$	$1.7(5)(^{+1.9}_{-0.7})$	$0.0116(18)(^{+59}_{-22})$	$0.120(13)(^{+30}_{-30})$

Table: Final results. All quantities, all octet members.

2010 BMWc dataset

Main source of uncertainty

Chiral extrapolation being the main source of uncertainty is a general characteristic of lattice QCD computations.



Source of systematic error	error on F_K/F_π
Chiral Extrapolation:	
- Functional form	3.3×10^{-3}
- Pion mass range	3.0×10^{-3}
Continuum extrapolation	3.3×10^{-3}
Excited states	1.9×10^{-3}
Scale setting	1.0×10^{-3}
Finite volume	6.2×10^{-4}

Result for F_K/F_π [Dürr et al. Phys. Rev. D81 (2010)]

Result for the ratio $F_K/F_\pi = 1.192(7)_{\rm st}(6)_{\rm sy}$

2010 BMWc dataset

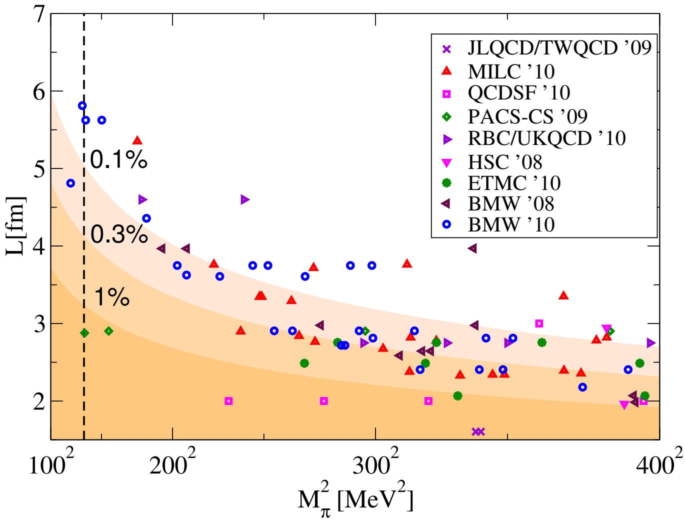
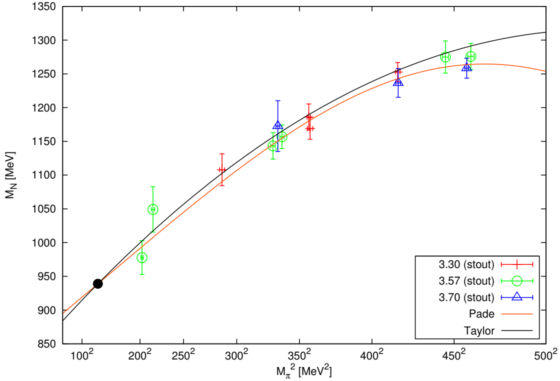


Figure: [arXiv:1011.2403, arXiv:1011.2711]

2010 BMWc dataset

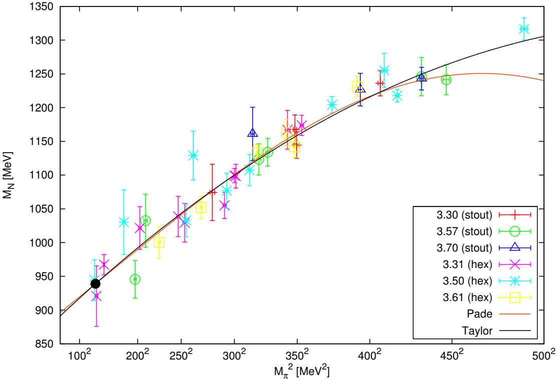
Figure: $M_\pi < 420$ MeV; $\chi^2/dof \approx 4.9/7$.



$$\sigma_{\pi N} = 53(10)_{\text{stat}}\mathrm{MeV} \qquad \text{Taylor}$$
$$\sigma_{\pi N} = 44(6)_{\text{stat}}\mathrm{MeV} \qquad \text{Pade}$$

2010 BMWc dataset

Figure: $M_\pi < 420$ MeV; $\chi^2/dof \approx 4.9/7$.



$$\sigma_{\pi N} = 42(5)_{\text{stat}}\mathrm{MeV} \qquad \text{Taylor}$$
$$\sigma_{\pi N} = 46(3)_{\text{stat}}\mathrm{MeV} \qquad \text{Pade}$$

2010 BMWc dataset

Impact DM searches

A precise determination (uncertainty $\lesssim 8$ MeV) can have a significant impact in DM searches.

We are convinced this is possible

- 5 values of the lattice spacing: $a \approx 0.115$ fm, $a \approx 0.093$ fm, $a \approx 0.077$ fm, $a \approx 0.065$ fm, $a \approx 0.054$ fm.
- Reaching the physical point, and even below ($M_\pi = 120$ MeV).
- Big volumes (up to $L = 6$ fm). All ensembles $M_\pi L > 4$ fm.
- Good statistics. More than 47 ensembles. 35 ensembles with $M_\pi < 400$ MeV. 18 ensembles with $M_\pi < 300$ Mev. 6 ensembles with $M_\pi < 200$ MeV.

Proof of concept

Proof of concept

Use all available information

- Use M_N^{phys} to set scale.
- 5 values of β. 33 ensembles. Subset of BMW 2010 dataset.
- Only Taylor and Pade to interpolate both in m_{ud} and m_s.
- $M_\pi < 410$ MeV and $M_\pi < 350$ MeV
- Cutoff for $\sigma_{\pi N}$: Absent, $\mathcal{O}(a)$, $\mathcal{O}(a^2)$.
- 32 time intervals to estimate excited state contributions.
- Total 384 analysis.

$$
\begin{aligned}
(aM_N) &= (aM_N^\Phi)\left\{1 + \sum_{i=1}^{N_\pi} \alpha_i \left[\left(\frac{aM_\pi}{aM_N^\Phi}\right)^2 - \left(\frac{M_\pi^\Phi}{M_N^\Phi}\right)^2\right]^i + \right. \\
&\left. + \sum_{j=1}^{N_K} \beta_j \left[\left(\frac{aM_{ks}}{aM_N^\Phi}\right)^2 - \left(\frac{M_{ks}^\Phi}{M_N^\Phi}\right)^2\right]^j \right\}
\end{aligned}
$$

Proof of concept

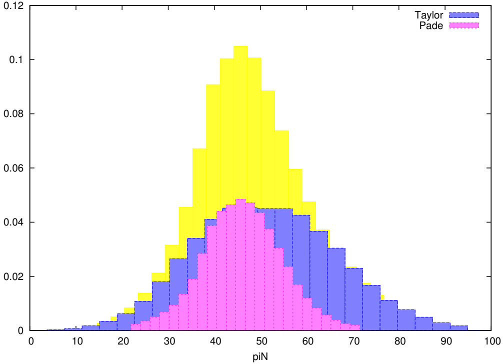


Figure: ONLY PROOF OF CONCEPT: $\sigma_{\pi N} = 50.1(6.1)_{\rm stat}(9.9)_{\rm sys}$

Not enough for strange content

$$y_N = 0.54(2.2)_{\rm stat}(0.71)_{\rm sys}$$

Proof of concept

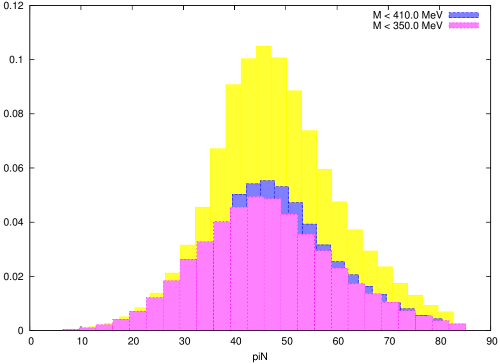


Figure: ONLY PROOF OF CONCEPT: $\sigma_{\pi N} = 50.1(6.1)_{\rm stat}(9.9)_{\rm sys}$

Not enough for strange content

$$y_N = 0.54(2.2)_{\rm stat}(0.71)_{\rm sys}$$

Isospin breaking

In nature there are two sources of isospin breaking

- $m_u \neq m_d$
- $q_u \neq q_d$

QED

Electromagnetic effects are key to understand isospin breaking effects in nature.

Important for

- Describe the universe we see $M_n > M_p$.
- EM corrections can play a role in quantities that the lattice determine very precisely (i.e. F_K/F_π at 0.25%).
- Isospin breaking effects allow a first principle determination of m_u and m_d (strong CP problem).
- SM at low energies is only QCD + QED + Some operators.
- ...

Isospin breaking

In nature there are two sources of isospin breaking

- $m_u \neq m_d$
- $q_u \neq q_d$

QED

Electromagnetic effects are key to understand isospin breaking effects in nature.

Important for

- Describe the universe we see $M_n > M_p$.
- EM corrections can play a role in quantities that the lattice determine very precisely (i.e. F_K/F_π at 0.25%).
- Isospin breaking effects allow a first principle determination of m_u and m_d (strong CP problem).
- SM at low energies is only QCD + QED + Some operators.
- ...

Isospin breaking

In nature there are two sources of isospin breaking

- $m_u \neq m_d$
- $q_u \neq q_d$

QED

Electromagnetic effects are key to understand isospin breaking effects in nature.

Important for

- Describe the universe we see $M_n > M_p$.
- EM corrections can play a role in quantities that the lattice determine very precisely (i.e. F_K/F_π at 0.25%).
- Isospin breaking effects allow a first principle determination of m_u and m_d (strong CP problem).
- SM at low energies is only QCD + QED + Some operators.
- ...

Isospin breaking

In nature there are two sources of isospin breaking

- $m_u \neq m_d$
- $q_u \neq q_d$

QED

Electromagnetic effects are key to understand isospin breaking effects in nature.

Important for

- Describe the universe we see $M_n > M_p$.
- EM corrections can play a role in quantities that the lattice determine very precisely (i.e. F_K/F_π at 0.25%).
- Isospin breaking effects allow a first principle determination of m_u and m_d (strong CP problem).
- SM at low energies is only QCD + QED + Some operators.
- ...

Isospin breaking

In nature there are two sources of isospin breaking

- $m_u \neq m_d$
- $q_u \neq q_d$

QED

Electromagnetic effects are key to understand isospin breaking effects in nature.

Important for

- Describe the universe we see $M_n > M_p$.
- EM corrections can play a role in quantities that the lattice determine very precisely (i.e. F_K/F_π at 0.25%).
- Isospin breaking effects allow a first principle determination of m_u and m_d (strong CP problem).
- SM at low energies is only QCD + QED + Some operators.
- ...

EM with periodic boundary conditions

On $\mathbb{T}^4$ Maxwell equations

$$\partial_\mu F^{\mu\nu} = j^\nu$$

Impose a zero total charge condition

$$Q = \int_{\mathbb{T}^4} j^0 = \int_{\mathbb{T}^4} \partial_\mu F^{\mu 0} = 0$$

Hodge decomposition theorem

$$A_\mu = \partial_\mu g + \epsilon_{\mu\nu\rho\sigma}\partial_\nu h_{\rho\sigma} + \eta_\mu$$

Fix the harmonic part $\eta_\mu = 0$.

EM with periodic boundary conditions

- We formulate the theory in the non compact form (avoids self photon interactions). The Boltzmann factor is normal (i.e. a free theory).
- Easy (numerically cheap) to sample.
- When computing quark propagators we "add" the EM interaction

$$U_\mu \longrightarrow e^{\imath q e A_\mu} U_\mu$$

- Quenched simulation for EM interaction. Unquenching posses deep problems related with the triviality of the theory.
- FV from EM can be large!

Dashen's theorem

$$\Delta M_K^2|_{m_u=m_d} = \Delta M_\pi^2|_{m_u=m_d} + \mathcal{O}(\alpha^2, \alpha m_q)$$

As a first test of our program we will compute corrections to the Dashen's theorem.

Not clear in the literature

The relative correction of Dashen's theorem

$$\Delta_R = \frac{\Delta M_K^2|_{m_u=m_d}}{\Delta M_\pi^2|_{m_u=m_d}} - 1$$

predictions range from $\approx 100\%$ [Bijnens'1993] to $\approx 30\%$ [RBC'2007], with almost all intermediate possibilities.

Dashen's theorem

$$\Delta M_K^2|_{m_u=m_d} = \Delta M_\pi^2|_{m_u=m_d} + \mathcal{O}(\alpha^2, \alpha m_q)$$

As a first test of our program we will compute corrections to the Dashen's theorem.

- Taylor expansion in $M_{\pi^+}^2$ and $M_{\bar{s}s}^2 = \frac{1}{2}(M_{K^+}^2 + M_{K^0}^2 - M_{\pi^+}^2)$ (isospin symmetric quantity up to order $\mathcal{O}((m_u - m_d)^2)$).

$$X(M_{\pi^+}^2, M_{\bar{s}s}^2) = X^\Phi \left\{ 1 + \sum_{i=1}^{N_\pi} \alpha_i' \left[M_{\pi^+}^2 - \left(M_{\pi^+}^\Phi\right)^2 \right]^i + \sum_{i=1}^{N_{\bar{s}s}} \beta_i' \left[M_{\bar{s}s}^2 - \left(M_{\bar{s}s}^\Phi\right)^2 \right]^j \right\}$$

Dashen's theorem

$$\Delta M_K^2|_{m_u=m_d} = \Delta M_\pi^2|_{m_u=m_d} + \mathcal{O}(\alpha^2, \alpha m_q)$$

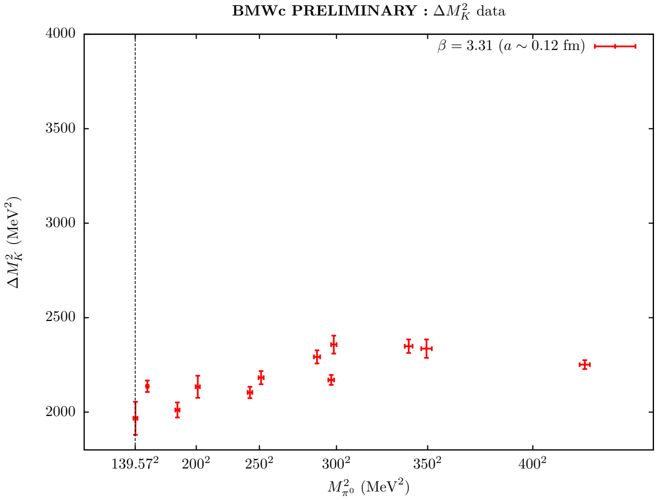
As a first test of our program we will compute corrections to the Dashen's theorem.

Dashen's theorem

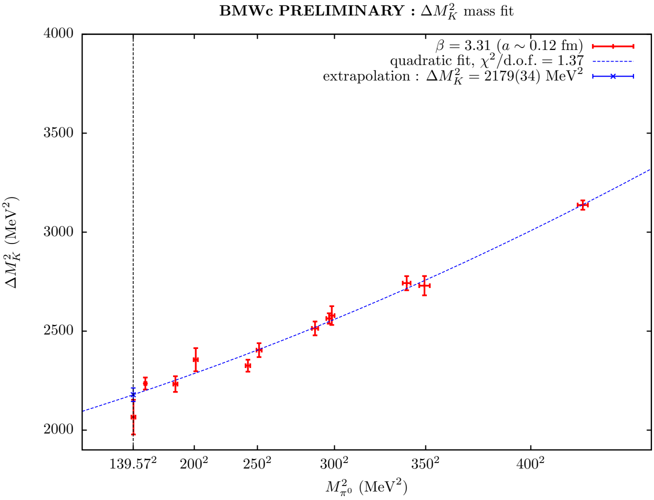
$$\Delta M_K^2|_{m_u=m_d} = \Delta M_\pi^2|_{m_u=m_d} + \mathcal{O}(\alpha^2, \alpha m_q)$$

As a first test of our program we will compute corrections to the Dashen's theorem.

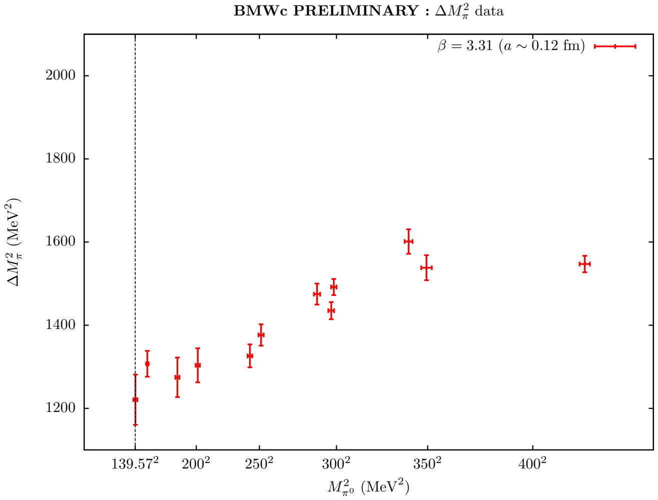
Kaon EM mass splitting



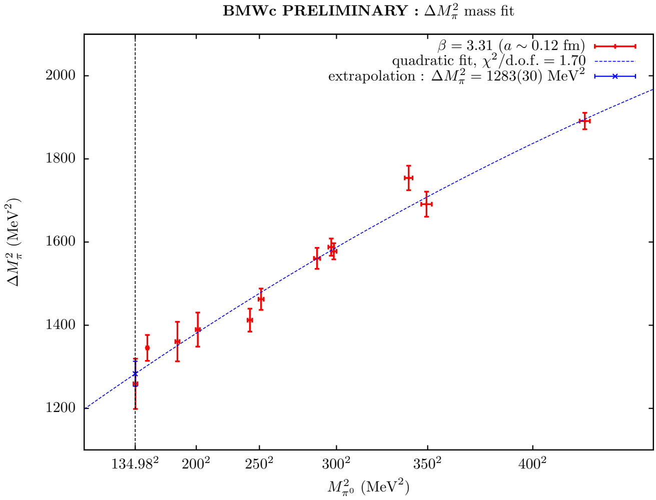
Kaon EM mass splitting



Pion EM mass splitting



Pion EM mass splitting



Preliminary results

Preliminary results

$$
\begin{aligned}
M_{\pi^+} &= 139.57 \\
M_{\pi^0} &= 135.4(1)_{\mathrm{st}}(?)_{\mathrm{sy}}\ \mathrm{MeV} \\
M_{K^0} &= 494.70(3)_{\mathrm{st}}(?)_{\mathrm{sy}}\ \mathrm{MeV} \\
M_{K^+} &= 496.61(3)_{\mathrm{st}}(?)_{\mathrm{sy}}\ \mathrm{MeV} \\
\Delta_R &= 0.66(4)_{\mathrm{st}}(?)_{\mathrm{sy}}\ \mathrm{MeV}
\end{aligned}
$$

Still lot of work to do

- $m_u \neq m_d$
- Unquench QED (??)
- Other (more interesting) applications: Flavour physics, $M_n - M_p$.

Conclusions

- Much progress done, but after 40 years we have poor analytical understanding of QCD.
- Lattice QCD able to make accurate predictions with all the sources of error under control (at least for some quantities).
- Results for a full QCD computation of the octet sigma terms.
  - $\sigma_{\pi N} = 39(^{+18}_{-7})$ MeV, $\sigma_{\bar{s}s} = 67(27)(^{+55}_{-47})$ MeV.
  - $f_{udN} = 0.042(5)(^{+21}_{-4})$, $f_{\bar{s}sN} = 0.036(14)(^{+30}_{-25})$
  - We need to do better to impact significantly DM searches...
  - ... But we can (removing our main source of uncertainty).
  - Preliminary fits with data at the physical point.
- Very preliminary results for EM corrections to meson masses
  - $\Delta_R = 0.65(3)_{\rm st}(?)_{\rm sy}$ MeV