Fanning about with QCD

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– for QCDSF-UKQCD Collaboration –

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[DESY, Zeuthen, April 2011]

Papers:

- arXiv:0807.0345 (PRD) [action, P c_{sw}]
- arXiv:0901.3302 (PRD) [action, NP c_{sw}]
- arXiv:1003.1114 (PLB) [tuning]
- arXiv:1012.0215 (PRD) [$\langle x \rangle$]
- arXiv:1102.5300 [group theory, spectrum]

Earlier Lattice conferences:

- H. Perlt, RH: Lattice 2008, arXiv:0811.2355, arXiv:0809.4769 [c_{sw}, P and NP]
- RH: Lattice 2009, arXiv:0910.2963 [tuning]

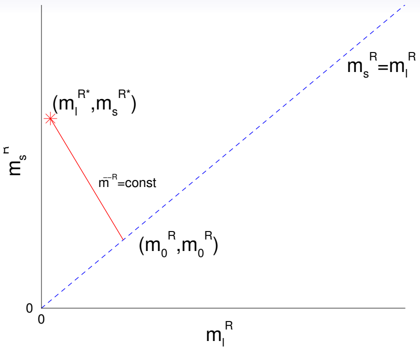
Lattice 2010:

- P. Rakow, RH: arXiv:1012.4371 [theory, spectrum]
- F. Winter, J. Zanotti: arXiv:1102.3407, arXiv:1101.2806 [3-point functions: $g_A, FFs, \ldots$]
- Y. Nakamura, H. Stüben: arXiv:1011.0199 [$N_f = 2 + 1$ BQCD]

Introduction

- The m_l^R–m_s^R plane and our choice of path to physical point
- This path choice with clover fermions
- Tuning results
- Hadron spectrum
    - Constrained flavour expansions
    - The Fan club:
        - Meson octet: $\pi(ll)$, $K(ls)$
        - Vector octet: $\rho(ll)$, $K^*(ls)$
        - Baryon octet: $N(lll)$, $\Lambda(lls)$, $\Sigma(lls)$, $\Xi(lss)$
        - Baryon decuplet: $\Delta(lll)$, $\Sigma^*(lls)$, $\Xi^*(lss)$, $\Omega(sss)$
- Work in progress
    - Partially quenched (pq) constrained flavour expansions
    - f_K/f_π
    - m_s^R/m_l^R
    - (hyperon) sigma terms
    - 3-pt functions: $\langle x \rangle$, g_A
    - ...
- Conclusions

Many paths to approach the physical point



Extrapolate:
from a point on the $SU(3)_F$ flavour symmetry line to the physical point:

$$(m_0^R, m_0^R) \longrightarrow (m_l^{R\,*}, m_s^{R\,*})$$

Choice here: keep the singlet quark mass fixed

$$\overline{m}^R = \frac{1}{3}\,(2m_l^R + m_s^R)$$

Flavour expansion about (flavour) symmetric line with $\overline{m}^R =$ const.

Potential Advantages:

- $\overline{m}^R =$ const. means that as we extrapolate
  $m_l^R \searrow m_l^{R*}$ then $m_s^R \nearrow m_s^{R*}$
  ie $m_\pi \searrow m_\pi^*$, $m_K \nearrow m_K^*$
  so m_K never $> m_K^*$, perhaps better for $SU(3)_F$ χPT
- Flavour singlet quantity (eg r_0) flat at symmetric point, value close to extrapolated value at physical point $\longrightarrow$ scale
- No knowledge of χPT necessary
  (indeed for some quantities, eg r_0 not known)
- Highly constrained hadron mass multiplet fits – alternative to χPT
- Additional natural advantage for clover fermions
  (due to different singlet and non-singlet renormalisations)

Expanding a flavour singlet quantity about a point on the $SU(3)_F$-flavour line:

Let $X_S(m_u^R, m_d^R, m_s^R)$ be a flavour singlet object

[X_S invariant under the quark permutation symmetry between u, d and s]

$$
\begin{aligned}
& X_S(m_0^R + dm_u^R, m_0^R + dm_d^R, m_0^R + dm_s^R) \\
&= \quad X_{S\,sym} + \left.\frac{\partial X_S}{\partial m_u^R}\right|_{sym} dm_u^R + \left.\frac{\partial X_S}{\partial m_d^R}\right|_{sym} dm_d^R + \left.\frac{\partial X_S}{\partial m_s^R}\right|_{sym} dm_s^R + O((dm_q^R)^2)
\end{aligned}
$$

On the symmetric line:

$$
\left.\frac{\partial X_S}{\partial m_u^R}\right|_{sym} = \left.\frac{\partial X_S}{\partial m_d^R}\right|_{sym} = \left.\frac{\partial X_S}{\partial m_s^R}\right|_{sym}
$$

On the chosen trajectory $\overline{m}^R = \frac{1}{3}(m_u^R + m_d^R + m_s^R) = \text{constant}$

$$
dm_u^R + dm_d^R + dm_s^R = 0
$$

Together these imply that

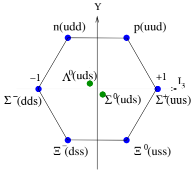
$$
X_S(m_0^R + dm_u^R, m_0^R + dm_d^R, m_0^R + dm_s^R) = X_{S\,sym}^{(0)} + O((dm_q^R)^2)
$$

Examples of singlet quantities

- Octet baryons: (centre of mass)

$$X_N = \frac{1}{3}(m_N + m_\Sigma + m_\Xi) = 1.150\,\text{GeV}$$

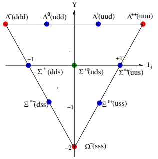
stable under strong ints.



- Decuplet baryons: (centre of mass)

$$X_\Delta = \frac{1}{3}(2m_\Delta + m_\Omega) = 1.379\,\text{GeV}$$

decay under strong ints.



- Gluonic:

$$X_r = 1/r_0$$

$r_0 = 0.5$ fm ?

- Some other possibilities

$$X_S = \begin{cases} \frac{1}{2}(m_\Sigma + m_\Lambda) & \\ m_{\Sigma^*},\ \frac{1}{2}(m_\Delta + m_{\Xi^*}) & \\ \sqrt{\frac{1}{3}(2m_K^2 + m_\pi^2)} & X_\pi \\ \frac{1}{3}(2m_{K^*} + m_\rho) & X_\rho \end{cases}$$

Check using χPT

Choose your favourite χPT result

Expand about a $SU(3)_F$ flavour symmetric point:

$$X_S = X_S^{(0)}|_{sym} + O((dm_q^R)^2)$$

Clover fermions

Above discussion holds for all fermions

- For clover fermions, path $\overline{m}^R = \text{const.}$ solves another problem
  Singlet and non-singlet quarks renormalise differently

$q = l, s$

$$m_q^R = Z_m^{NS}(m_q - \overline{m}) + Z_m^S \overline{m} \equiv Z_m^{NS}(m_q + \alpha_Z \overline{m})$$

$\alpha_Z = (Z_m^S - Z_m^{NS})/Z_m^{NS} \sim O(1)$

$am_q = \frac{1}{2}(1/\kappa_q - 1/\kappa_{0;c})$

[Vanishing of the quark mass along the symmetric line (i.e. for 3 mass degenerate flavours) $\Longrightarrow \kappa_{0;c}$]

$$\overline{m}^R = Z_m^{NS}(1 + \alpha_Z)\overline{m}$$

- Alternatively LO χPT:

$$m_{ps}^2 = B_0^R \left(m_{q_1}^R + m_{q_2}^R\right)$$

giving

$$\begin{aligned} X_\pi^2 &= \tfrac{1}{3}(2m_K^2 + m_\pi^2) \\ &= 2B_0^R \overline{m}^R \\ &= B_0^R Z_m^{NS}(1 + \alpha_Z)\overline{m} \end{aligned}$$

So (equivalently) for our path

$$\underbrace{\overline{m}^R = \text{const.}}_{\text{computer unfriendly}} \iff \underbrace{X_\pi = \text{const.}}_{\text{awkward}} \iff \underbrace{\overline{m} = \text{const.}}_{\text{computer friendly}}$$

So define our trajectory as $\overline{m} = \text{const.} \equiv m_0$ or:

$$\kappa_s = \frac{1}{\frac{3}{\kappa_0} - \frac{2}{\kappa_l}}$$

where κ_0 is a point on the $SU(3)$ flavour symmetric line

[Using (eg) $m_s^R = \text{const.}$ is a more complicated path]

Strategy to determine κ_0:

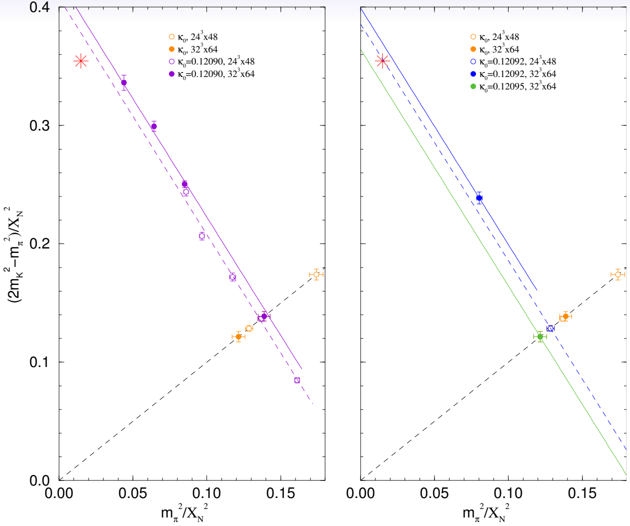
Simulate on flavour symmetric line for various κ_0 until find value where

$$\frac{X_\pi^2}{X_N^2} \equiv \frac{m_\pi^2}{m_N^2} = \underbrace{\left.\frac{X_\pi^2}{X_N^2}\right|^*}_{\text{physical value}} \Longrightarrow \kappa_0$$

On flavour symmetric line $X_\pi = m_\pi$ so as $X_\pi \sim$ constant,

$$m_\pi \sim 410\,\text{MeV}$$

m_l^R–m_s^R plane



Also shown: fit to constant

$$c_N = \frac{X_\pi^2}{X_N^2} = \frac{\frac{1}{3}(2m_K^2 + m_\pi^2)}{X_N^2} \qquad \text{or} \qquad \frac{2m_K^2 - m_\pi^2}{X_N^2} = c_N - 2\,\frac{m_\pi^2}{X_N^2}$$

- Best starting point ?
- Finite size effects ?
- Possible to choose (κ_l, κ_s) values such that $m_l > m_s$. In this 'strange' world, would expect to see an *inversion* of the particle spectrum, with (eg) the nucleon being the heaviest octet particle, so $p \longrightarrow \Sigma$ or $p \longrightarrow \Lambda$

Higher order corrections

- Leading order:
  Similar plots for all

$$\frac{X_\pi^2}{X_S^2} = \text{const.} \qquad S = N, \Delta, \rho$$

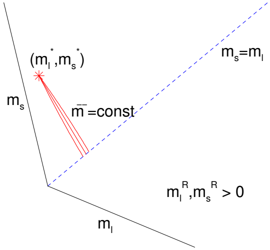
  straight line from one point on the symmetric line to physical point with common gradient -2

- Higher order:

  For *each*

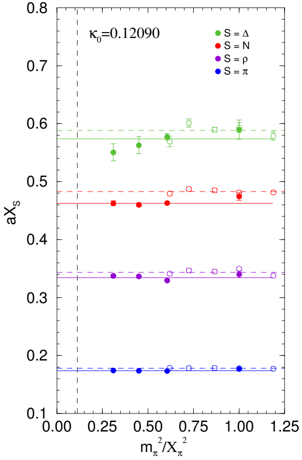
$$\frac{X_\pi^2}{X_S^2} = \text{const.} \qquad S = N, \Delta, \rho$$

  slightly curved line from a point on the symmetric line to physical point with *initial* gradient -2

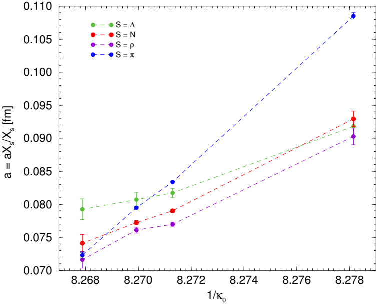


See little sign of this effect at present

Singlet quantities and the Scale



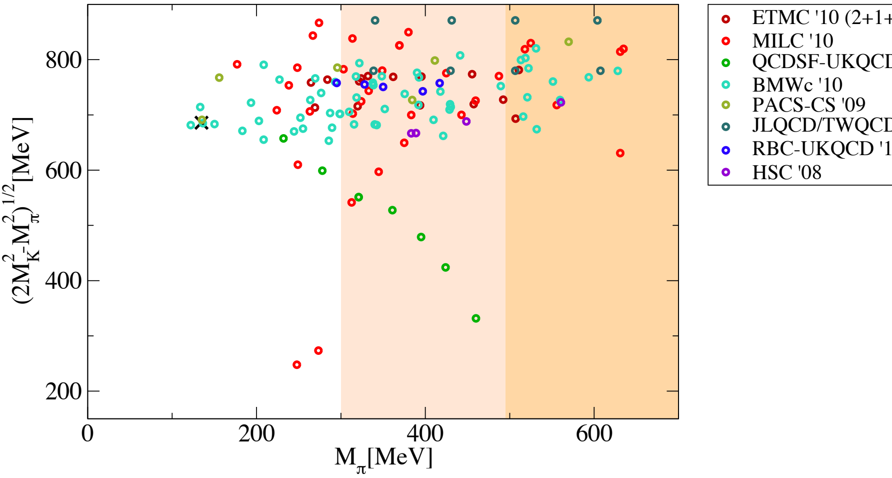
- Flat
- Crude finite size extrapolation indicates that finite size effects on $32^3 x64$ volumes small



- If QCD consistent then expect consistency of different scales to a unique value
- $a \sim 0.078(3)$ fm

Landscape

C. Hoelbling – Plenary talk, Lattice 2010 [arXiv:1102.0410]



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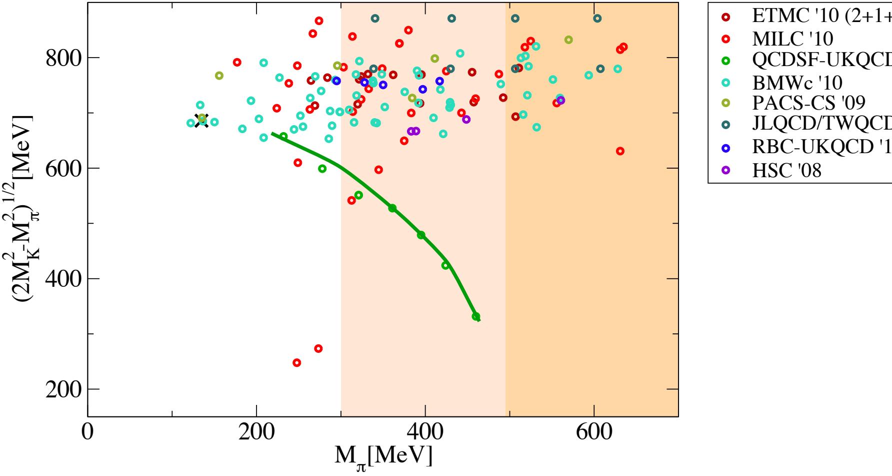
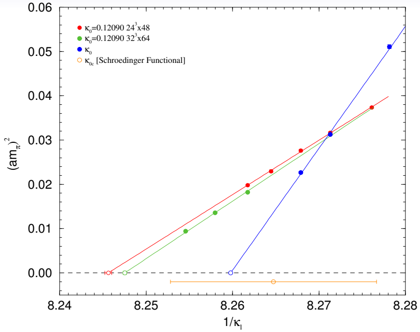
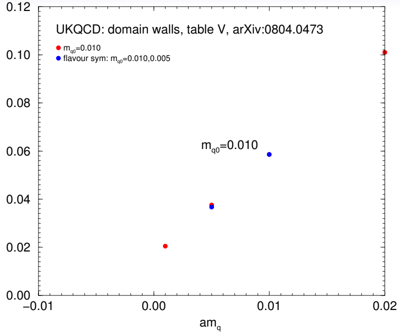


Aside: Clover versus Chiral fermions





- $$m_l = \frac{1}{2}\left(\frac{1}{\kappa_l} - \frac{1}{\kappa_{0c}}\right) \qquad \delta m_l = m_l - \overline{m} = \frac{1}{2}\left(\frac{1}{\kappa_l} - \frac{1}{\kappa_0}\right)$$

  m_l can be negative
  δm_l usually is

- Vanishing of $M_\pi^2 = 2B_0^R m_l^R = 2B_0^R Z^{NS}(m_l + \alpha_Z \overline{m})$ gives an estimate of α_Z using

$$\alpha_Z = -\frac{am_l|_{\kappa=\kappa_c}}{a\overline{m}} = \frac{\left(\frac{1}{\kappa_{0;c}} - \frac{1}{\kappa_c}\right)}{\left(\frac{1}{\kappa_0} - \frac{1}{\kappa_{0;c}}\right)} \sim 1.06$$

Flavour expansion

Step I: S_3, $SU(3)$ classification

Polynomial		S_3	$SU(3)$		
1	✓	A_1	1		
$(\overline{m} - m_0)$		A_1	1		
δm_s	✓	E^+		8	
$(\delta m_u - \delta m_d)$	✓	E^-		8	
$(\overline{m} - m_0)^2$		A_1	1		
$(\overline{m} - m_0)\delta m_s$		E^+		8	
$(\overline{m} - m_0)(\delta m_u - \delta m_d)$		E^-		8	
$\delta m_u^2 + \delta m_d^2 + \delta m_s^2$	✓	A_1	1		27
$3\delta m_s^2 - (\delta m_u - \delta m_d)^2$	✓	E^+		8	27
$\delta m_s(\delta m_d - \delta m_u)$	✓	E^-		8	27

$$\delta m_q = \overline{m} - m_l$$

- All the quark-mass polynomials up to $O(\delta m_q^3)$, classified by symmetry properties [shown here to $O(\delta m_q^2)$]
- A tick (✓) – relevant polynomials on constant $\overline{m}$ surface

Aside: $O(a)$-improvement of quark masses

Need to add terms am_q^2 to quark mass

$$\begin{aligned}
\overline{m}^R &= Z_m^S \left[\overline{m} + a\left\{b_1 \overline{m}^2 + b_2(\delta m_s^2 + \delta m_u^2 + \delta m_d^2)\right\}\right] \\
\delta m_s^R &= Z_m^{NS} \left[\delta m_s + a\left\{b_3 \overline{m}\delta m_s + b_4(3\delta m_s^2 - (\delta m_u - \delta m_d)^2)\right\}\right]
\end{aligned}$$

- Add $SU(3)$-singlet, A_1 (singlet) improvement terms to singlet; add $SU(3)$-octet, E^+ ($u \leftrightarrow d$ symmetric) improvement terms to octet [have expanded about chiral limit $m_0 = 0$]
- δm_u^R, δm_d^R by flavour permuting
- have used only flavour arguments, so expect that expansion results (for masses) remain true whether use bare/improved terms – have checked this
- agrees with Bhattacharya et al, hep-lat/0511014 (upon redefinition of coefficients)

Step II: Mass hierarchy

[determining expansion coeffs.]

Δ^-	Δ^0	Δ^+	Δ^{++}	Σ^{*-}	Σ^{*0}	Σ^{*+}	Ξ^{*-}	Ξ^{*0}	Ω^-	S_3	$SU(3)$
1	1	1	1	1	1	1	1	1	1	A_1	1
−1	−1	−1	−1	0	0	0	1	1	2	E^+	8
−3	−1	1	3	−2	0	2	−1	1	0	E^-	8
3	−1	−1	3	−1	−3	−1	−1	−1	3	A_1	27
−3	7	7	−3	−5	0	−5	−2	−2	6	E^+	27
−3	−1	1	3	3	0	−3	4	−4	0	E^-	27
2	−3	−3	2	−3	12	−3	−3	−3	2	A_1	64
−1	0	0	−1	3	0	3	−3	−3	2	E^+	64
−1	2	−2	1	1	0	−1	−1	1	0	E^-	64
0	−1	1	0	1	0	−1	−1	1	0	A_2	64

- 10×10 dim baryon decuplet diagonal mass matrix classified by diagonal matrices in a particular S_3, $SU(3)$ representation
- eg A_1, 27

$$\begin{aligned} &3M_{\Delta^-} - M_{\Delta^0} - M_{\Delta^+} + 3M_{\Delta^{++}} \\ &-M_{\Sigma^{*-}} - 3M_{\Sigma^{*0}} - M_{\Sigma^{*+}} - M_{\Xi^{*-}} - M_{\Xi^{*0}} + 3M_{\Omega^-} \\ &\qquad = b_{27}\left[\delta m_u^2 + \delta m_d^2 + \delta m_s^2\right] + 9c_{27}\delta m_u \delta m_d \delta m_s + \ldots \end{aligned}$$

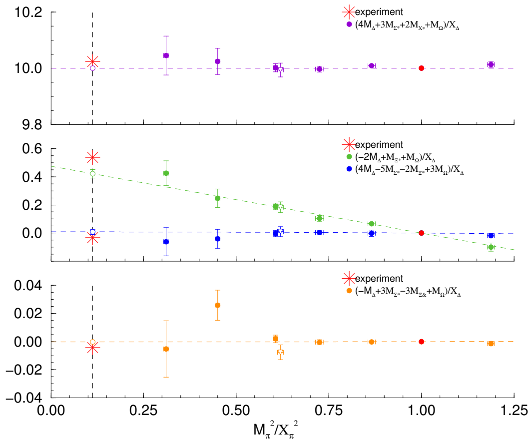
Step II: Mass hierarchy

[determining expansion coeffs.]

$SU(3)$	Mass Combination	Expansion				
1	$4M_\Delta + 3M_{\Sigma^*} + 2M_{\Xi^*} + M_\Omega$	1,		δm_l^2,	δm_l^3, ...	13.8 GeV
8	$-2M_\Delta + M_{\Xi^*} + M_\Omega$		δm_l,	δm_l^2,	δm_l^3, ...	0.742 GeV
27	$4M_\Delta - 5M_{\Sigma^*} - 2M_{\Xi^*} + 3M_\Omega$			δm_l^2,	δm_l^3, ...	−0.044 GeV
64	$-M_\Delta + 3M_{\Sigma^*} - 3M_{\Xi^*} + M_\Omega$				δm_l^3, $\cdots$	−0.006 GeV

- $1 + 1 + 1 \longrightarrow 2 + 1$
- each additional factor δm_l gives $\sim$ order of magnitude reduction – suggests rapidly converging Taylor expansion down to physical point
- invert to give flavour expansions for masses

Mass hierarchy (from lattice)



- order of magnitude drop with each power of δm_l
- $(-2M_\Delta + M_{\Xi^*} + M_\Omega)/X_N$ dominated by linear term

Meson Spectrum

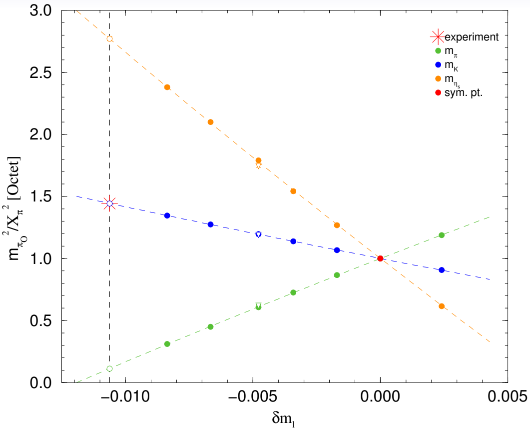
Flavour expansion about the symmetric point (Gell-Mann–Okubo)
$\Longrightarrow$ constrained fits for (pseudoscalar) meson octet:

$$
\begin{aligned}
M_\pi^2 &= M_0^2 + 2\alpha\delta m_l + (\beta_0 + 2\beta_1)\delta m_l^2 \\
M_K^2 &= M_0^2 - \alpha\delta m_l + (\beta_0 + 5\beta_1 + 9\beta_2)\delta m_l^2 \\
M_{\eta_s}^2 &= M_0^2 - 4\alpha\delta m_l + (\beta_0 + 8\beta_1)\delta m_l^2
\end{aligned}
$$

$$
\delta m_q = m_q - \overline{m} = \frac{1}{2a}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_{0;c}}\right) - \frac{1}{2a}\left(\frac{1}{\kappa_0} - \frac{1}{\kappa_{0;c}}\right) = \frac{1}{2a}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_0}\right)
$$

- Linear terms: 1 coefficient; Quadratic terms: 3 coefficients
- m_{η_s} fictitious $s\bar{s}$ particle [η, η' mixing] – but as constrained fit [linear term] then still contains useful information
- Similar expansions for f_π, f_K, f_{η_s}

(Pseudoscalar) Meson Octet 'fan plot'



- $M_\pi^2/X_\pi^2 =$ phys. value defines δm_l^*
- M_{η_s} line, fictitious particle, but still can use for constrained fit
- ratios within the same multiplet tend to give cancellation of finite size effects – adopt this philosophy
- fit is quadratic in δm_l but very little curvature – $\beta_i \sim 0$

Relation of flavour expansion coefficients to χPT

- $SU(3)$ χPT convergent $\lesssim 420\,\mathrm{MeV}$?
- eg for pion:

$$
\begin{aligned}
M_0^2 &= \overline{\chi}\left[1 - \frac{16\overline{\chi}}{f_0^2}\left(3L_4 + L_5 - 6L_6 - 2L_8\right) + \frac{\overline{\chi}}{24\pi^2 f_0^2}\ln\frac{\overline{\chi}}{\Lambda_\chi^2}\right] \\
\alpha &= Q_0\left[1 - \frac{16\overline{\chi}}{f_0^2}\left(3L_4 + 2L_5 - 6L_6 - 4L_8\right) + \frac{\overline{\chi}}{8\pi^2 f_0^2}\ln\frac{\overline{\chi}}{\Lambda_\chi^2}\right] \\
\beta_0 &= -\frac{Q_0^2}{6\pi^2 f_0^2} \\
\beta_1 &= \frac{Q_0^2}{f_0^2}\left[-32\left(L_5 - 2L_8\right) + \frac{1}{24\pi^2}\left(7 + 4\ln\frac{\overline{\chi}}{\Lambda_\chi^2}\right)\right] \\
\beta_2 &= \frac{Q_0^2}{f_0^2}\left[16\left(L_5 - 2L_8\right) - \frac{1}{24\pi^2}\left(3 + 2\ln\frac{\overline{\chi}}{\Lambda_\chi^2}\right)\right]
\end{aligned}
$$

$Q_0 = B_0^R Z_m^{NS}$ $\quad$ $\overline{\chi} = 2Q_0(1+\alpha_z)\overline{m}$

- Will be difficult to determine (combinations of) LECs

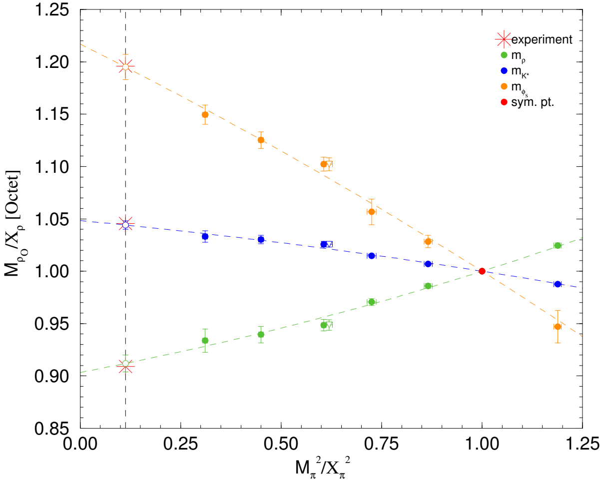
Vector Meson Spectrum

Flavour expansion about the symmetric point (Gell-Mann–Okubo)
$\Longrightarrow$ constrained fits for vector meson octet:

$$\begin{aligned} M_\rho &= M_0 + 2\alpha\delta m_l + (\beta_0 + 2\beta_1)\delta m_l^2 \\ M_{K^*} &= M_0 - \alpha\delta m_l + (\beta_0 + 5\beta_1 + 9\beta_2)\delta m_l^2 \\ M_\phi &= M_0 - 4\alpha\delta m_l + (\beta_0 + 8\beta_1)\delta m_l^2 \end{aligned}$$

- Same structure as for pseudoscalar octet:
  Linear terms: 1 coefficient; Quadratic terms: 3 coefficients
- M_ϕ 'pure' $s\bar{s}$ state [almost perfect η_8, η_1 mixing]

Vector Meson Octet 'fan plot'



- little curvature

Baryon Spectrum

Flavour expansion about the symmetric point (Gell-Mann–Okubo)
$\Longrightarrow$ constrained fits for baryon octet/decuplet:

$$
\begin{aligned}
M_N &= M_0 + 3A_1\delta m_l + (B_0 + 3B_1)\delta m_l^2 \\
M_\Lambda &= M_0 + 3A_2\delta m_l + (B_0 + 6B_1 - 3B_2 + 9B_4)\delta m_l^2 \\
M_\Sigma &= M_0 - 3A_2\delta m_l + (B_0 + 6B_1 + 3B_2 + 9B_3)\delta m_l^2 \\
M_\Xi &= M_0 - 3(A_1 - A_2)\delta m_l + (B_0 + 9B_1 - 3B_2 + 9B_3)\delta m_l^2
\end{aligned}
$$

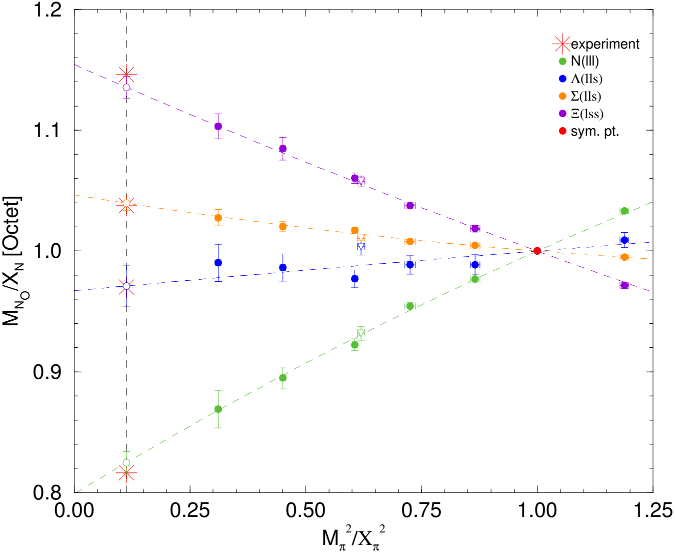
and

$$
\begin{aligned}
M_\Delta &= M_0 + 3A\delta m_l + (B_0 + 3B_1)(\delta m_l)^2 \\
M_{\Sigma^*} &= M_0 + (B_0 + 6B_1 + 9B_2)(\delta m_l)^2 \\
M_{\Xi^*} &= M_0 - 3A\delta m_l + (B_0 + 9B_1 + 9B_2)(\delta m_l)^2 \\
M_\Omega &= M_0 - 6A\delta m_l + (B_0 + 12B_1)(\delta m_l)^2
\end{aligned}
$$

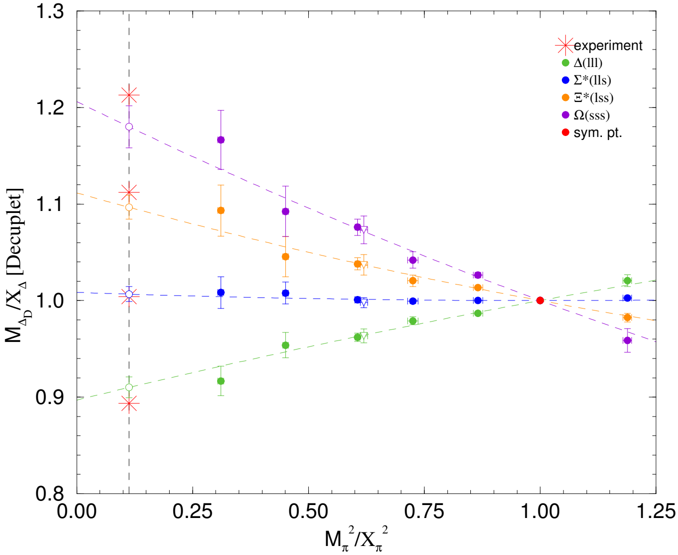
[$M_0 = \alpha + \beta\overline{m}$]

Linear terms: Octet 2 coefficients; Decuplet 1 coefficient

Baryon Octet 'fan plot'



Baryon Decuplet 'fan plot'



Work in progress

Quadratic mass terms and partially quenched Baryon Spectrum

To help determine coefficients generalise to non-unitary valence quarks – partially quenched or *pq*

- m_l, m_s sea quarks, constrained by $\frac{1}{3}(2m_l + m_s) = \overline{m} = \text{const.}$; $\delta m_q = m_q - \overline{m}$ — sea quarks: singlet
- μ_l, μ_s valence quarks, unconstrained $\delta\mu_q = \mu_q - \overline{m}$

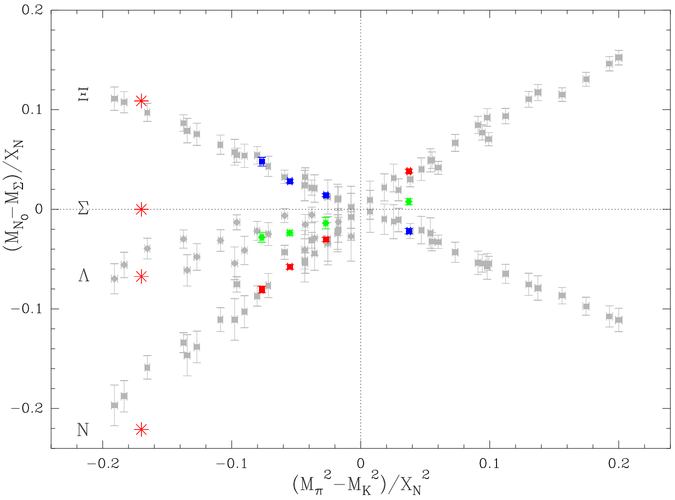
On unitary line, $\mu \longrightarrow m$; together with $2\delta m_l + \delta m_S = 0$ results collapse to previous results

$$\begin{aligned}
M_\pi^2 &= M_0^2 + 2\alpha\delta\mu_l + \beta_0\delta m_l^2 + 2\beta_1\delta\mu_l^2 \\
M_K^2 &= M_0^2 + \alpha(\delta\mu_l + \delta\mu_s) + \beta_0\delta m_l^2 + \beta_1(\delta\mu_l^2 + \delta\mu_s^2) + \beta_2(\delta\mu_s - \delta\mu_l)^2 \\
M_{\eta_s}^2 &= M_0^2 + 2\alpha\delta\mu_s + \beta_0\delta m_l^2 + 2\beta_1\delta\mu_s^2
\end{aligned}$$

$$\begin{aligned}
M_\Delta &= M_0 + 3A\delta\mu_l + B_0\delta m_l^2 + 3B_1\delta\mu_l^2 \\
M_{\Sigma^*} &= M_0 + A(2\delta\mu_l + \delta\mu_s) + B_0\delta m_l^2 + B_1(2\delta\mu_l^2 + \delta\mu_s^2) + B_2(\delta\mu_s - \delta\mu_l)^2 \\
M_{\Xi^*} &= M_0 + A(\delta\mu_l + 2\delta\mu_s) + B_0\delta m_l^2 + B_1(\delta\mu_l^2 + 2\delta\mu_s^2) + B_2(\delta\mu_s - \delta\mu_l)^2 \\
M_\Omega &= M_0 + 3A\delta\mu_s + B_0\delta m_s^2 + 3B_1\delta\mu_s^2
\end{aligned}$$

No additional constants required

Baryon octet ‘fan plot’



- $24^3 \times 48$ results – coloured points
- *pq* results using these (sea) results
- *pq* results have potential to be a good predictor

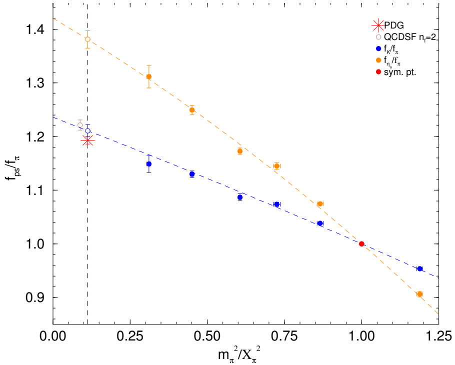
$M_\pi^2 - M_K^2 \sim \mu_l - \mu_s$

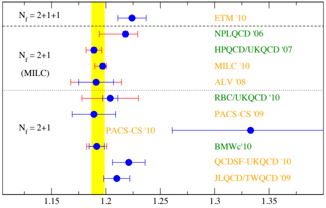
f_K/f_π

$SU(3)$ flavour structure allows the same structures as for the (pseudoscalar) meson octet:

$$
\begin{aligned}
f_\pi &= F + 2G\delta m_l + (H_0 + 2H_1)\delta m_l^2 \\
f_K &= F - G\delta m_l + (H_0 + 5H_1 + 9H_2)\delta m_l^2 \\
f_{\eta_s} &= F - 4G\delta m_l + (H_0 + 8H_1)\delta m_l^2
\end{aligned}
$$

f_K / f_π





C. Hoelbling – Plenary talk, Lattice 2010

m_s^R / m_l^R quark mass ratio I:

$$\frac{1}{r} = \frac{m_s^R}{m_l^R} = \frac{-2\delta m_l + (1+\alpha_Z)\overline{m}}{\delta m_l + (1+\alpha_Z)\overline{m}}$$

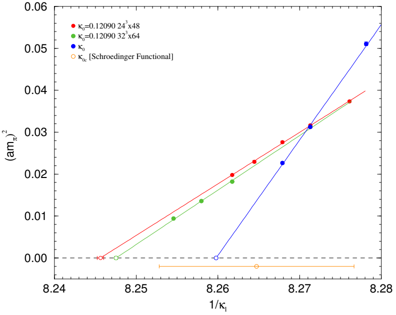
$$m_q^R = Z_m^{NS}(m_q + \alpha_Z \overline{m})$$

Vanishing of

$$\begin{aligned} M_\pi^2 &= 2B_0^R m_l^R \\ &= 2B_0^R Z^{NS}(m_l + \alpha_Z \overline{m}) \end{aligned}$$

gives an estimate of α_Z using

$$\begin{aligned} \alpha_Z &= -\frac{am_l|_{\kappa=\kappa_c}}{a\overline{m}} = \frac{\left(\frac{1}{\kappa_{0;c}} - \frac{1}{\kappa_c}\right)}{\left(\frac{1}{\kappa_0} - \frac{1}{\kappa_{0;c}}\right)} \\ &\sim 1.06 \end{aligned}$$



$$\frac{1}{r^*} = \begin{cases} \dfrac{-2\delta m_l^* + (1+\alpha_Z)\overline{m}}{\delta m_l^* + (1+\alpha_Z)\overline{m}} & \sim 27 \\[2ex] \dfrac{m_s^R}{m_l^R} = \dfrac{2M_K^2 - M_\pi^2}{M_\pi^2}\Bigg|^* & \sim 25 \end{cases}$$

consistent

m_s^R/m_l^R quark mass ratio II:

PCAC quark mass:

$$am_q^{AWI} \overset{t\gg 0}{=} \frac{\langle \partial_4^{LAT} \mathcal{A}_4(t)\mathcal{P}(0)\rangle}{2\langle \mathcal{P}(t)\mathcal{P}(0)\rangle}$$

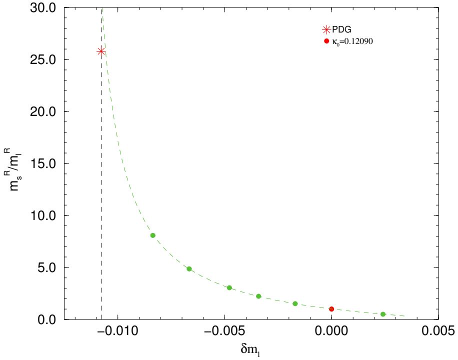
expect

$$m_q^R = \frac{Z_A}{Z_P} m_q^{AWI} \qquad m_q^R = Z_m^{NS}(m_q + \overline{m})$$

gives

$$\frac{1}{r} = \frac{m_s^R}{m_l^R} = \frac{m_s^{AWI}}{m_l^{AWI}} = \frac{-2\delta m_l + (1+\alpha_Z)\overline{m}}{\delta m_l + (1+\alpha_Z)\overline{m}}$$

take $(1+\alpha_Z)\overline{m}$ as fit parameter



$$(1+\alpha_Z)\overline{m} \sim \begin{cases} 0.01186 & \text{[fit]} \\ (1+1.062)\times 0.00576 \sim 0.01187 & \end{cases}$$

consistent

(Hyperon) Sigma terms

$$\sigma_l^{(H)} = m_l^R \langle H | (\overline{u}u + \overline{d}d)^R | H \rangle \qquad \sigma_s^{(H)} = m_s^R \langle H | (\bar{s}s)^R | H \rangle$$

- $\sigma_s^{(N)}$
  - QCD phase transition at large baryon density
  - detection of dark matter (helps constrain cross section)
- $\sigma_{\pi N} \equiv \sigma_l^{(N)}$
  - the π–N scattering amplitude

Bare scalar matrix elements

Feynman–Hellman theorem

$$\frac{\partial M_H}{\partial m_l} = \langle H|(\overline{u}u + \overline{d}d)|H\rangle \qquad \frac{\partial M_H}{\partial m_s} = \langle H|\overline{s}s|H\rangle$$

with

$$M_H = M_0 + M_1(\overline{m} - m_0) + c_H \delta m_l$$

nucleon octet:

$$c_H = \begin{cases} 3A_1 & H = N \\ 3A_2 & H = \Lambda \\ -3A_2 & H = \Sigma \\ -3(A_1 - A_2) & H = \Xi \end{cases}$$

Renormalisation

$$m_q^R = Z^{NS}\left[m_q + \alpha_Z \tfrac{1}{3}(m_u + m_d + m_s)\right]$$

In the action the term $\sum_q m_q \overline{q}q = \sum_q m_q^R (\overline{q}q)^R$ ie is a RGI so gives

$$(\overline{q}q)^R = \frac{1}{Z^{NS}}\left[\overline{q}q - \frac{\alpha_Z}{1+\alpha_Z}\tfrac{1}{3}(\overline{u}u + \overline{d}d + \overline{s}s)\right]$$

so for $\alpha_Z \neq 0$ then always mixing between bare operators

Putting it all together gives:

$$M_H = M_0 + M_1(\overline{m} - m_0) + c_H \delta m_l$$

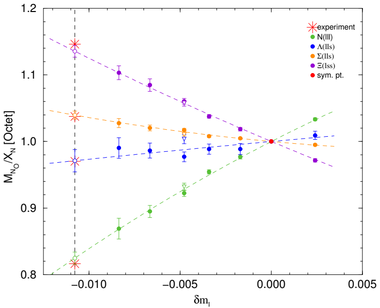
Equation I

$$\sigma_l^{(H)} = -\frac{3r}{1-r} c_H \delta m_l + 2r\sigma_s^{(H)}$$

where ratio of quark masses is

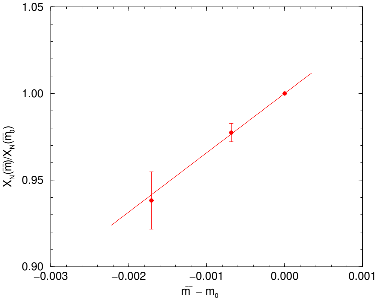
$$\frac{1}{r} = \frac{m_s^R}{m_l^R} = \frac{-2\delta m_l + (1+\alpha_Z)\overline{m}}{\delta m_l + (1+\alpha_Z)\overline{m}}$$

The only assumption is that the 'fan' plot splittings remain linear in δm_l down to the physical point.



Equation II

$$\sigma_s^{(H)} = \frac{3}{1+2r} \overline{m} M_1 - \frac{1}{r}\sigma_l^{(H)}$$



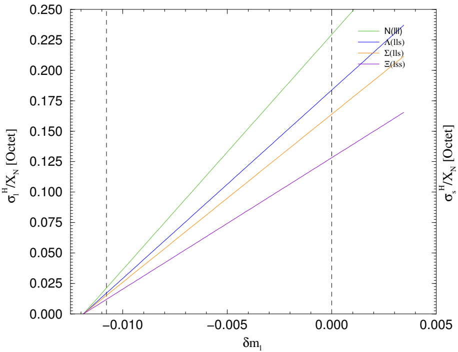
Consistent estimates:

$$\frac{1}{r^*} = \begin{cases} \frac{-2\delta m_l^* + (1+\alpha_Z)\overline{m}}{\delta m_l^* + (1+\alpha_Z)\overline{m}} & \sim 27 \\ \frac{m_s^R}{m_l^R} = \left.\frac{2M_K^2 - M_\pi^2}{M_\pi^2}\right|^* & \sim 25 \end{cases} \qquad \left.\frac{c_N \delta m_l}{X_N}\right|^* = \begin{cases} \sim 0.15 & \text{[fit]} \\ \left.\frac{M_\Xi - M_\Lambda}{X_N}\right|^* \sim 0.17 \end{cases}$$

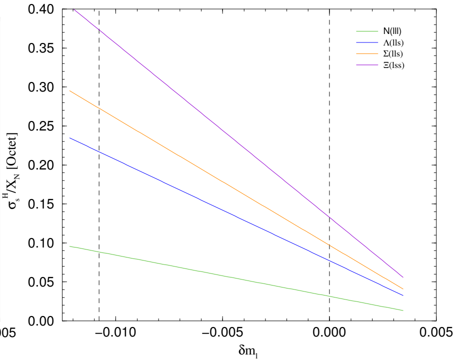
so (eg) for the nucleon

$$\sigma_l^{(N)*} \approx 26\,\mathrm{MeV} + \frac{\sigma_s^{(N)*}}{12.5}$$

- numerically (reasonably) stable result
- $\sigma_l^{(N)*} > 26\,\mathrm{MeV}$



$\sigma_l^{(H)}$, $H = N, \Lambda, \Sigma, \Xi$



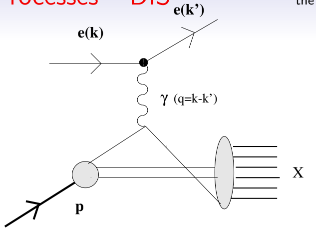
$\sigma_s^{(H)}$, $H = N, \Lambda, \Sigma, \Xi$

$\sigma_l^{(N)} \sim 30\,\text{MeV}$; $\sigma_s^{(N)} \sim 80\,\text{MeV}$

MT_c project: hep-lat/9311012, hep-lat/9311020, $\sigma_{\pi N} \sim 140$ MeV

High Energies / Hard Processes – DIS

the parton model and structure functions F



$ep \longrightarrow eX$ $\qquad Q^2 \equiv -q^2 \to \infty,\ \nu = p \cdot q \to \infty \sim en.\ transfer,\ x = -q^2/2\nu = fixed$

- α_s small, so 'quasifree' quarks (and gluons) – partons
  $F \equiv F(x)$: Bjorken scaling
  x is fraction of nucleon momentum carried by parton $\qquad p_{part} = x p_N$
- with QCD gives ($\ln Q^2$) scaling violations
- Quantitatively: [OPE] for both unpolarised/polarised processes

$$\int_0^1 dx x^{n-2} F_2^{p-n}(x, Q^2) = E_{NS}^{\overline{MS}}(\alpha_s) \langle x^{n-1} \rangle^{\overline{MS};(u-d)}(Q) \qquad n \text{ even}$$

where NS, non-singlet: $p - n$ or $u - d$

$$\langle N | \mathcal{O}_q^{\gamma;\{\mu_1 \cdots \mu_n\}} - \mathrm{tr} | N \rangle = 2 \langle x^{n-1} \rangle^{(q)} \left[p^{\mu_1} \cdots p^{\mu_n} - \mathrm{tr} \right]$$

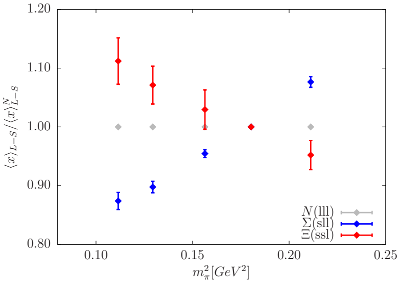
$$\mathcal{O}_q^{\gamma;\mu_1 \cdots \mu_n} = \bar{q} \gamma^{\mu_1} i \overleftrightarrow{\mathcal{D}}^{\mu_2} \cdots i \overleftrightarrow{\mathcal{D}}^{\mu_n} q$$

Hyperon quark momentum fractions

Consider

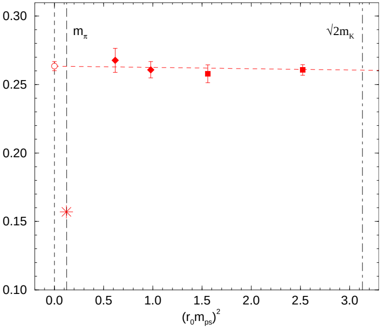
$B \sim qqq' \leftrightarrow \mathcal{O} \sim \bar{q}q - \bar{q}'q'$

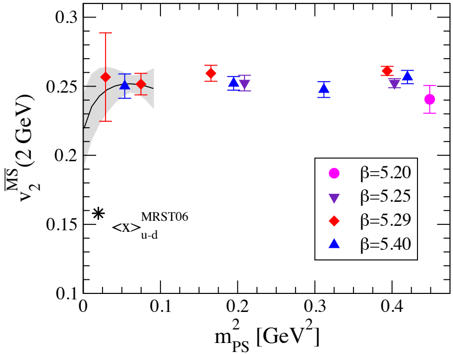
$$\frac{\langle x \rangle_{\Sigma}^{(u-s)}}{\langle x \rangle_{N}^{(u-d)}} \qquad \frac{\langle x \rangle_{\Xi}^{(s-u)}}{\langle x \rangle_{N}^{(u-d)}}$$



- Qualitative: larger momentum fraction carried by heavier quark
- presently only 24^3x48 results

But main problem: Value of lowest moment of structure function F_2^{NS}

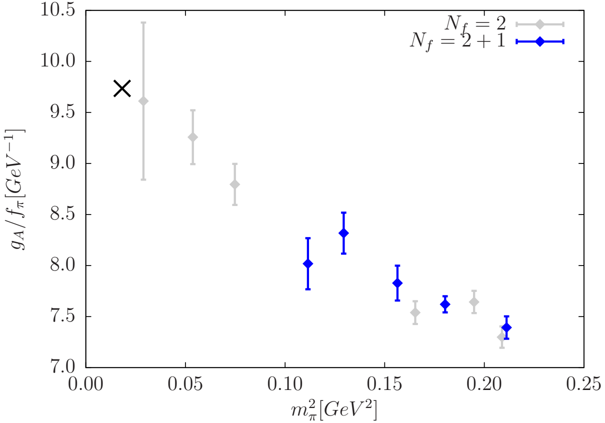




- Compare $n_f = 0$ (left) and $n_f = 2$ (right) results (!)
- Discrepancy to phenomenological value

Nucleon axial charge

⟶ hyperon axial charges



- presently only 24^3x48 results

In addition

- theoretical:
  - renormalisation
  - matrix element (and improvement coefficients) quark mass expansions (from $SU(3)$ flavour symmetry expansion)
  - ...
- phenomenological:
  - running on a $48^3 \times 96$ lattice: $m_\pi^2/X_\pi^2 \sim 0.17$, $m_\pi \sim 190\,\text{MeV}$
  - $1 + 1 + 1$ masses
  - semileptonic hyperon and kaon decays (V_{us})
  - hyperon distribution amplitudes
  - general 3-pt functions, including
    - hyperon form factors
    - spin structure of the Λ
    - ...

Conclusions

- Programme:
  Tune strange and light quark masses to their physical values simultaneously by keeping

$$\overline{m}^R = \frac{1}{3}\left(2m_l^R + m_s^R\right) = \text{const.}$$

- $m_\pi \searrow$; $m_K \nearrow$
- Can have situation with a heavy l-quark and a light s-quark
- Singlet quantities remain constant
  [helps to determine consistent scale]
- Meson octet and Baryon octet and decuplet mass spectrum
  - highly constrained fits – alternative to χPT
  - fan plots
  - little curvature seen
- *pq* results will help to give a handle on constraining $SU(3)$-flavour expansion coefficients, in particular quadratic coefficients