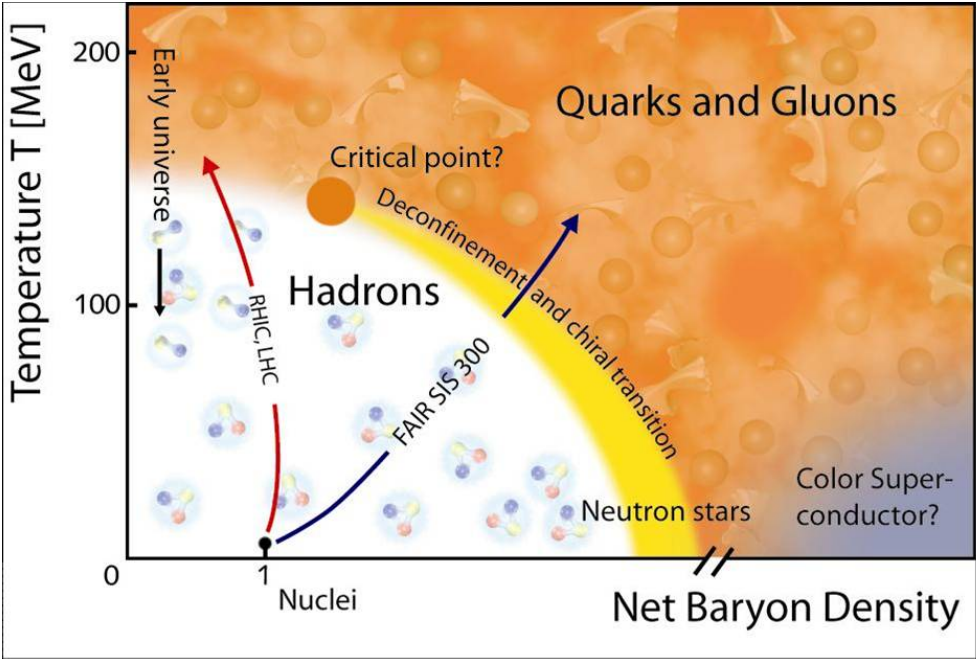
Coherent center domains in QCD and the deconfinement transition

christof.gattringer@uni-graz.at

Julia Danzer, Christof Gattringer, Alexander Schmidt

Motivation and basic facts

Something we would like to understand in detail ...



Center symmetry of gluodynamics

- For a fixed time t_0 the temporal gauge variables are multiplied with a center element $z \in \{1, e^{\pm i2\pi/3}\}$:

$$U_4(\mathbf{x}, t_0) \quad \longrightarrow \quad z\, U_4(\mathbf{x}, t_0)$$

- The path integration measure and the gauge action are invariant under the center transformation.

- The Polyakov loop ...

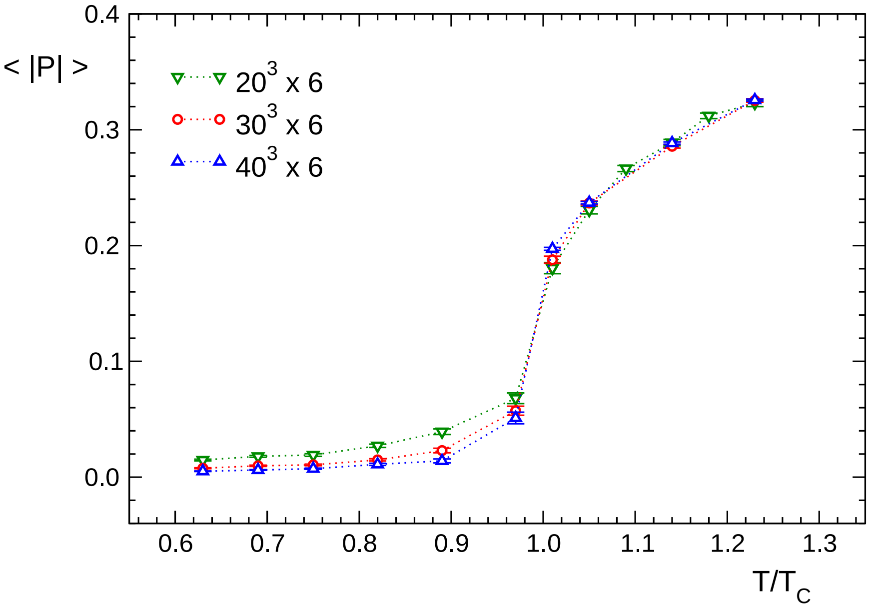
$$L(\mathbf{x}) \;=\; \text{Tr} \prod_t U_4(\mathbf{x}, t) \quad , \quad P \;=\; \frac{1}{V} \sum_{\mathbf{x}} L(\mathbf{x})$$

... is an observable which transforms non-trivially

$$L(\mathbf{x}) \quad \longrightarrow \quad z\, L(\mathbf{x}) \quad , \quad P \quad \longrightarrow \quad z\, P\ .$$

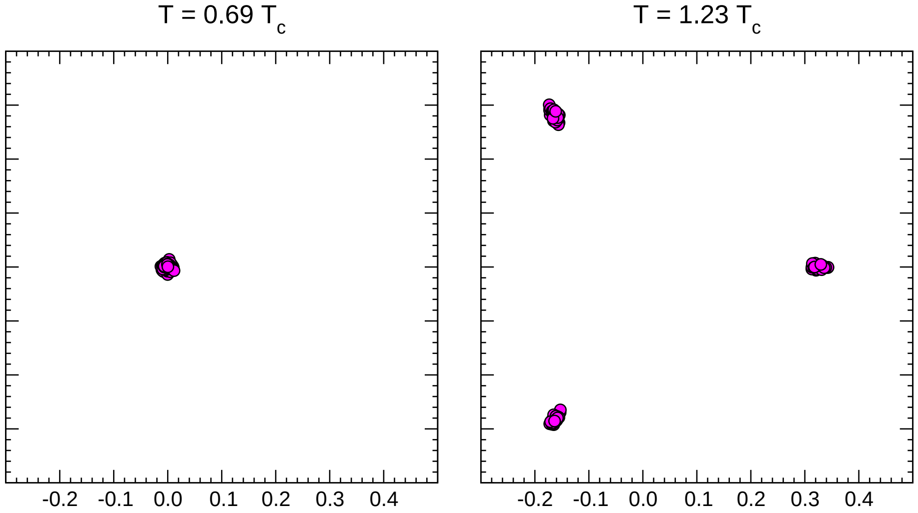
- A non-vanishing expectation value $\langle P \rangle \;=\; \langle L(\mathbf{x}) \rangle \neq 0$ signals that the center symmetry is broken spontaneously.

Polyakov loop as function of temperature



Spontaneous breaking of center symmetry

- The spontaneous breaking of center symmetry may lead to any of the three center sectors, as seen in scatter plots of P:



The relation to confinement

- After suitable renormalization $\langle P \rangle = \langle L(\mathbf{x}) \rangle$ may be related to the free energy F_q of a single quark:

$$\langle P \rangle \; = \; \langle L(\mathbf{x}) \rangle \; \propto \; e^{-\beta F_q} \; = \; \begin{cases} = 0 & \text{for} \quad T < T_c \\ \neq 0 & \text{for} \quad T > T_c \end{cases}$$

- Thus the Polyakov loop is also an order parameter for confinement.

- In pure gauge theory the deconfinement transition is linked to the spontaneous breaking of the center symmetry.

Svetitsky-Yaffe conjecture, spin systems, cluster percolation

Svetitsky-Yaffe conjecture

An influential idea for understanding the phase transition of pure gluodynamics is the Svetistky-Yaffe conjecture (1981):

- At T_c the critical behavior of SU(N) gauge theory in $d+1$ dimensions can be decribed by a d - dimensional spin system with a $\mathbb{Z}_N$ - invariant effective action. The spins are related to the local loops $L(\mathbf{x})$.

- Leading term of the effective action:

$$S[L] \;=\; -\,\tau \sum_{\langle \mathbf{x},\mathbf{y} \rangle} \Big[\, L(\mathbf{x})L(\mathbf{y})^{\star} \,+\, L(\mathbf{y})L(\mathbf{x})^{\star} \,\Big]$$

Properties of spin systems in QCD?

- The critical behavior of spins systems is well understood.

- At T_c the Svetitsky-Yaffe conjecture links them to the critical behavior of QCD.

- Can we identify characteristic properties of spin systems directly in QCD?

- Are these properties important only at T_c, or is there a range of temperatures where they play a role?

- Here we focus on clusters with coherent spin values and their percolation properties near T_c.

Percolation in spin systems

- For many spin systems clusters of coherent spins can be defined which percolate at the temperature of the magnetic transition.

- These clusters do not simply bind parallel spins, but have a more complicated structure (Fortuin-Kasteleyn clusters).

- For the example of the Ising model two parallel spins are connected only with probability

$$p_{FK} = 1 - e^{-2\beta}$$

- As an alternative strategy the binding probability p has been considered as a free parameter and for a critical value of p (or even in an interval of p - values) the percolation and magnetic transitions coincide.

Previous work in SU(2)

- Previously the cluster/percolation properties have been studied only for SU(2) gauge theory (Fortunato, Satz, ...).
- The analysis was done indirectly through the identification of the corresponding Ising-type of spin model.
- In this approach the key problem is the matching of the spin system coupling β (which is needed for $p_{FK} = 1 - e^{-2\beta}$) to the SU(2) couplings.
- This matching was attempted with strong coupling expansion for small N_t and via a Schwinger-Dyson approach.
- No analysis of discretization/scaling effects.
- No studies for SU(3).

Local properties of the Polyakov loop

Setting and goal of our analysis

- We study clusters and critical percolation directly in SU(3) lattice gauge theory without intermediate spin system step.

- For that purpose we analyze properties of the local loops $L(\mathbf{x})$.

- Technicalities:
  - Lüscher-Weisz gauge action.
  - Lattice sizes: $20^3 \times 6$ ... $40^3 \times 10$.
  - Temperatures: $T \in [\,0.63\,T_c\,,\,1.32\,T_c\,]$.

C. Gattringer, PRL in print, arXiv:1004.2200

The local Polyakov loop $L(\mathbf{x})$

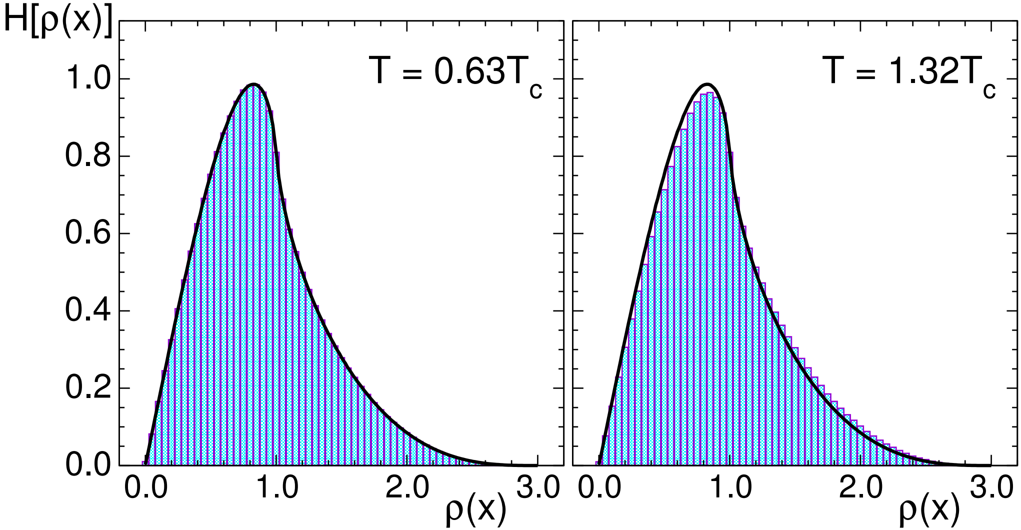
- For the analysis of local properties of $L(\mathbf{x})$ we define

$$L(\mathbf{x}) \;=\; \rho(\mathbf{x})\,\mathbf{e}^{\,\mathbf{i}\,\varphi(\mathbf{x})}$$

- We study histograms of $\rho(\mathbf{x})$ and $\varphi(\mathbf{x})$ to see how the local loops $L(\mathbf{x})$ give rise to the behavior of the averaged loop

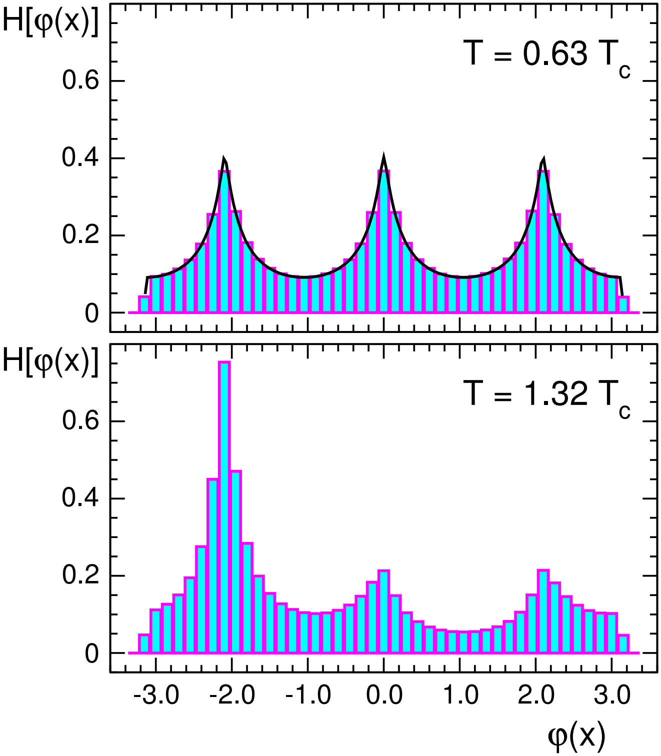
$$P \;=\; \frac{1}{V}\sum_{\mathbf{x}} L(\mathbf{x})$$

Histograms for the modulus of the local loops



The values $\mid L(\mathbf{x}) \mid = \ \rho(\mathbf{x})$ are distributed with Haar measure. The distribution is insensitive to temperature and thus cannot give rise to the first order jump of $\langle P \rangle \neq 0$ at T_c.

Histograms for the phase of the local loops



Center sectors of the local Polyakov loop

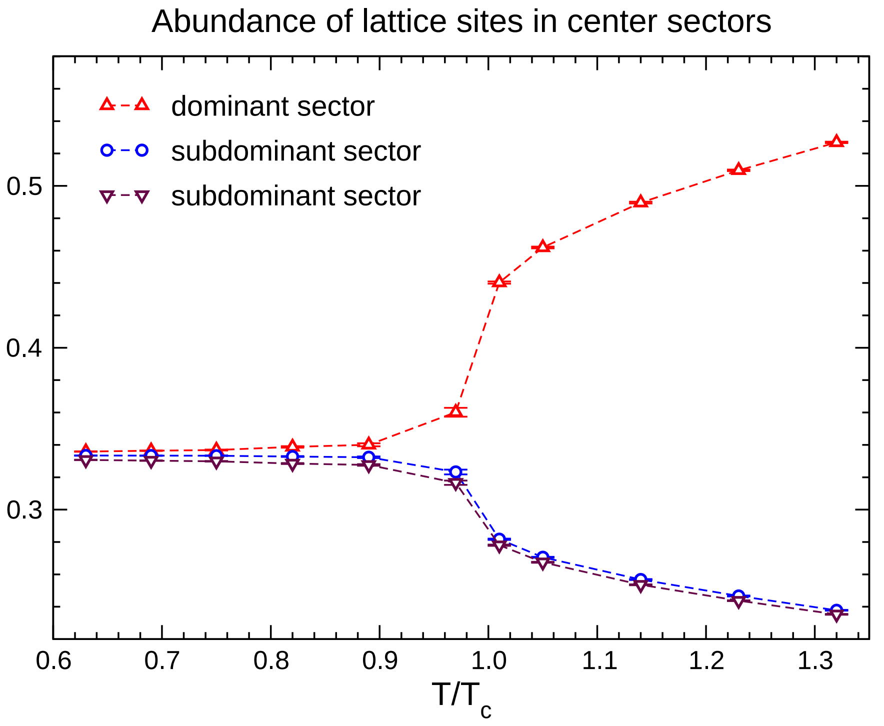
- The phases of the local Polyakov loop $L(\mathbf{x})$ have preferred values in the vicinity of all center elements.

- These center sectors are clearly seen also below T_c where the distribution is found to be the Haar measure distribution.

- Below T_c all three sectors are equally populated.

- The emergence of a non-vanishing $\langle P \rangle$ is driven by an increasing population of one of the sectors.

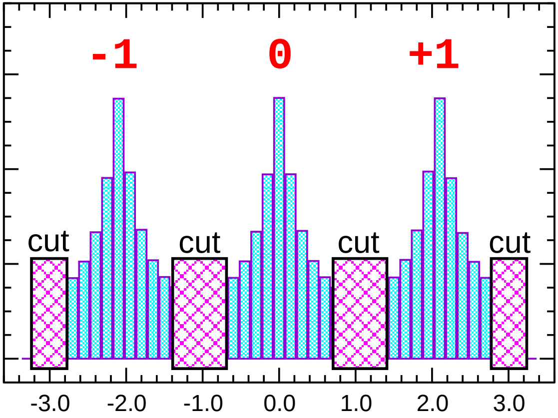
Center sectors across the phase transition



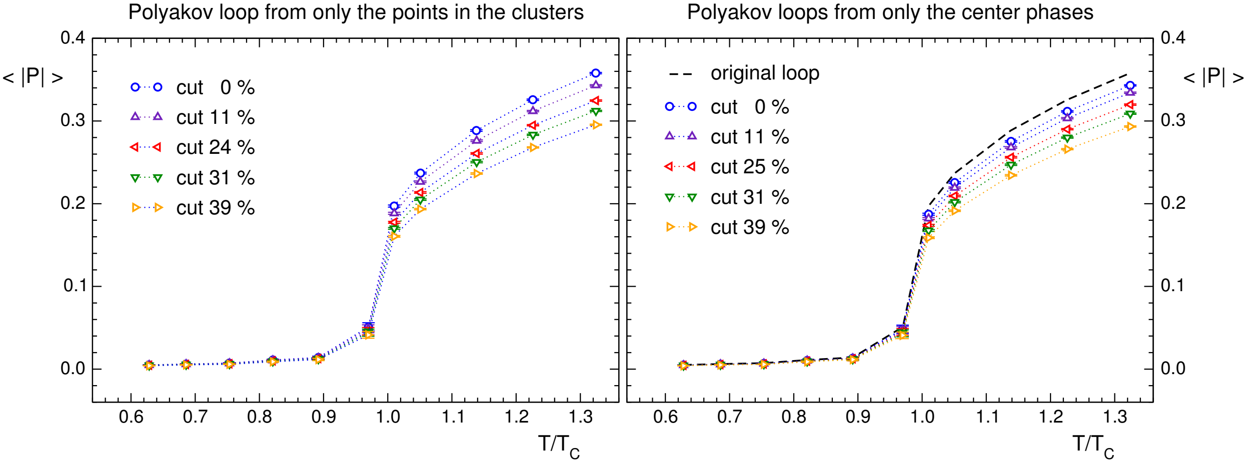
Cluster- and percolation properties of center domains

Center domains of the local Polyakov loop

- The behavior of the Polyakov loop P across the phase transition is driven by the dynamics of the phases of the local loops $L(\mathbf{x})$.
- We assign sector numbers `-1, 0, +1` to the three sectors and study properties of the corresponding clusters.
- We study the effect of a cut on the fluctuations.

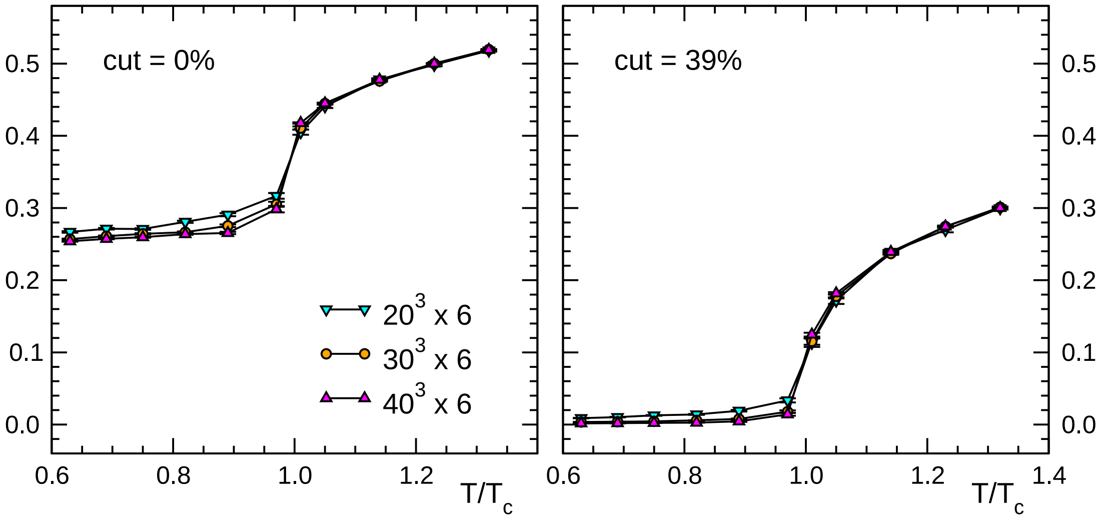


Cut leaves physics essentially unchanged



Behavior of the largest cluster

Size of largest cluster normalized with the volume



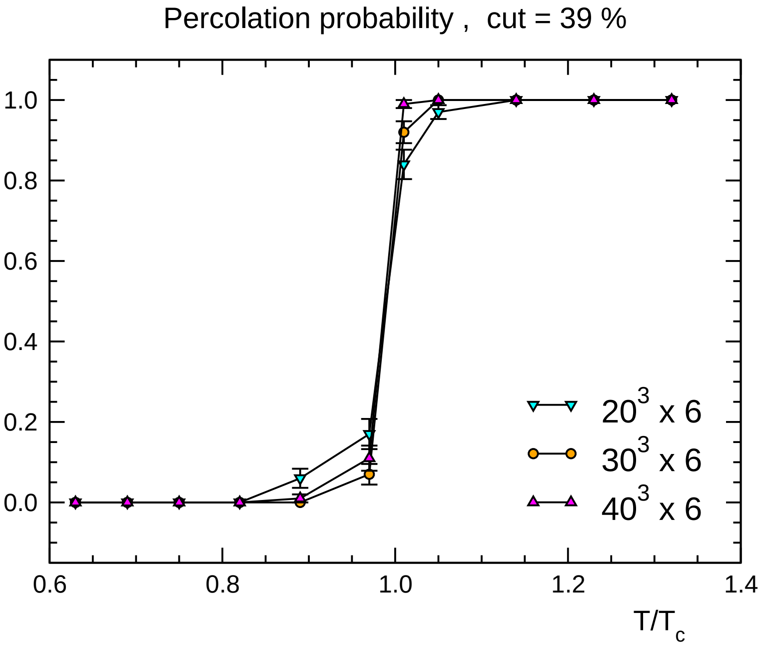
Aspects of the percolation phenomenon

- In 3 dimensions the critical occupation probability for site percolation is $p_c = 0.3116$.

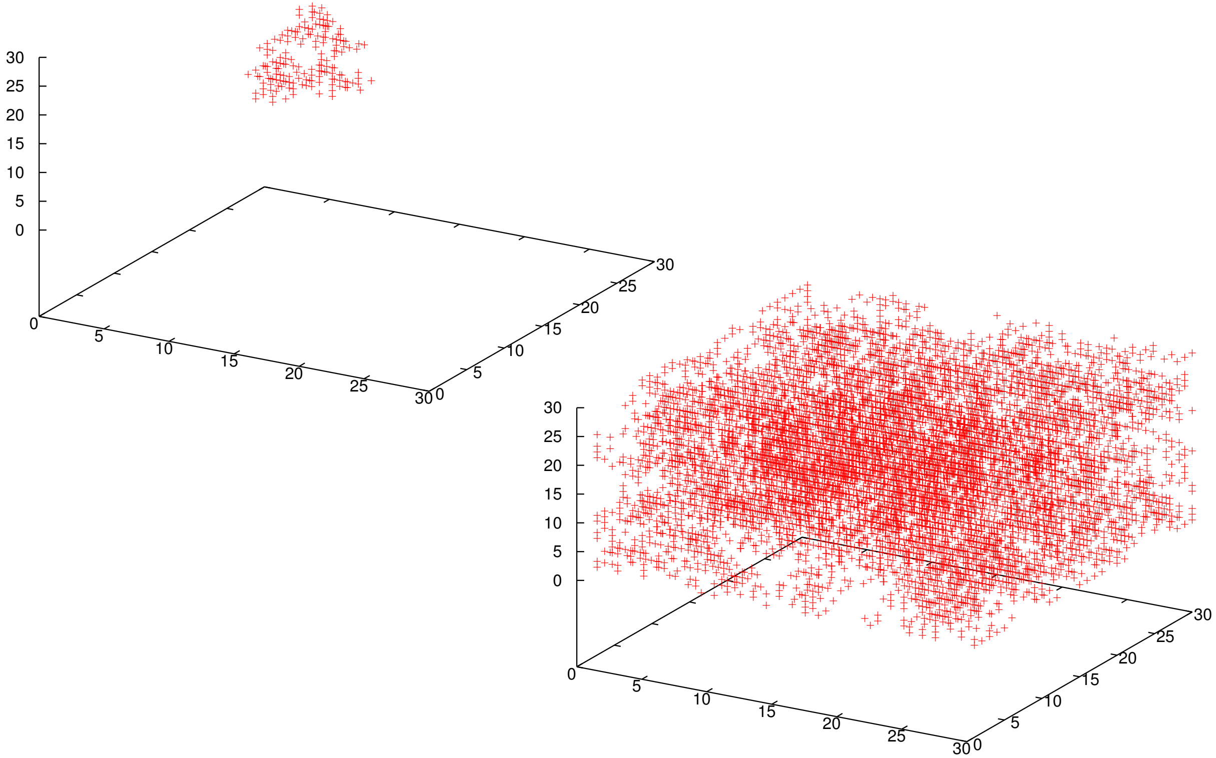
- Thus without a cut on the incoherent fluctuations we always encounter a percolating cluster.

- When we cut off the incoherent fluctuations, we find that the phase transition is signalled by the onset of percolation.

Percolation at T_c

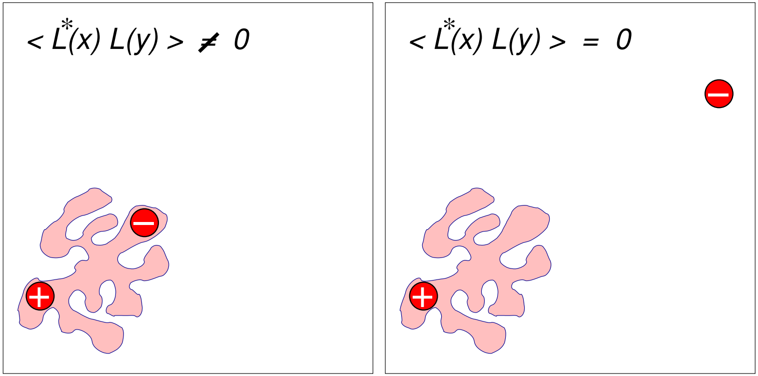


Clusters below and above T_c



A geometrical picture for confinement and the deconfinement transition

$T < T_c$

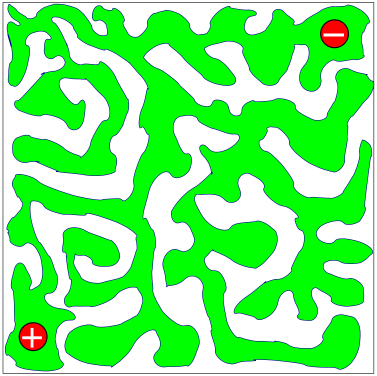


$< L^*(x)\, L(y) > \; \neq \; 0$

$< L^*(x)\, L(y) > \; = \; 0$

Below T_c two static sources (= local loops) have a non–vanishing expectation value only if they fit into the same cluster, such that the phases cancel.

$T > T_c$



When the clusters percolate the sources can be put at arbitrary distances.

Some aspects left out here

- Scaling properties of the clusters.
- Physical size of clusters.
- Results for SU(2).
- Relation to topological excitations.

Very preliminary results for dynamical ensembles

In collaboration with S. Borsanyi, Z. Fodor, S. Katz, K. Szabo.

Canonical determinants

- When quarks are included the fermion determinant appears as an additional weight factor.

- Fugacity expansion of the fermion determinant ...

$$\det[D(\mu)] \;=\; \sum_q e^{\,\beta\mu q}\,\det[D]^{(q)}$$

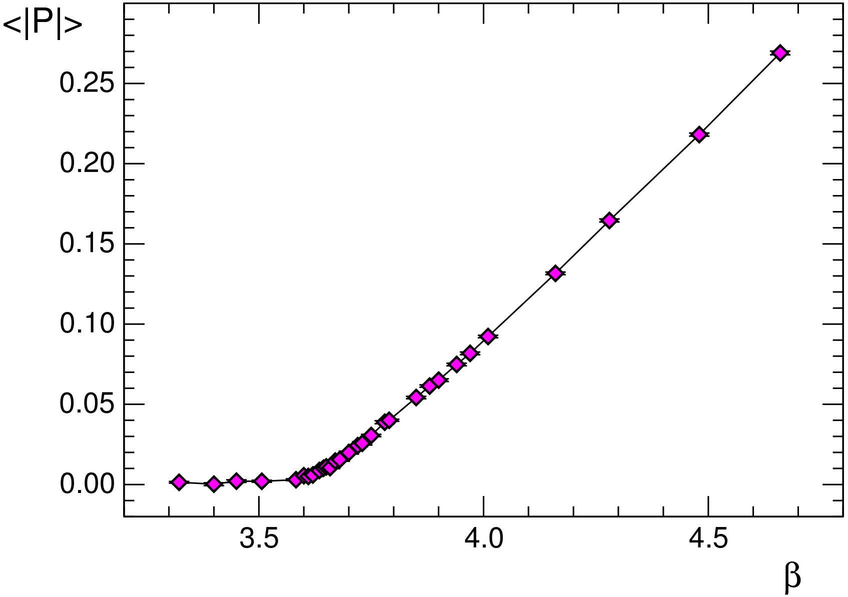
... decomposes it into canonical determinants $\det[D]^{(q)}$ for q quarks.

- The canonical determinants have simple center transformation properties:

$$\det[D]^{(q)} \;\longrightarrow\; z^{\,q\,mod\,3}\,\det[D]^{(q)}$$

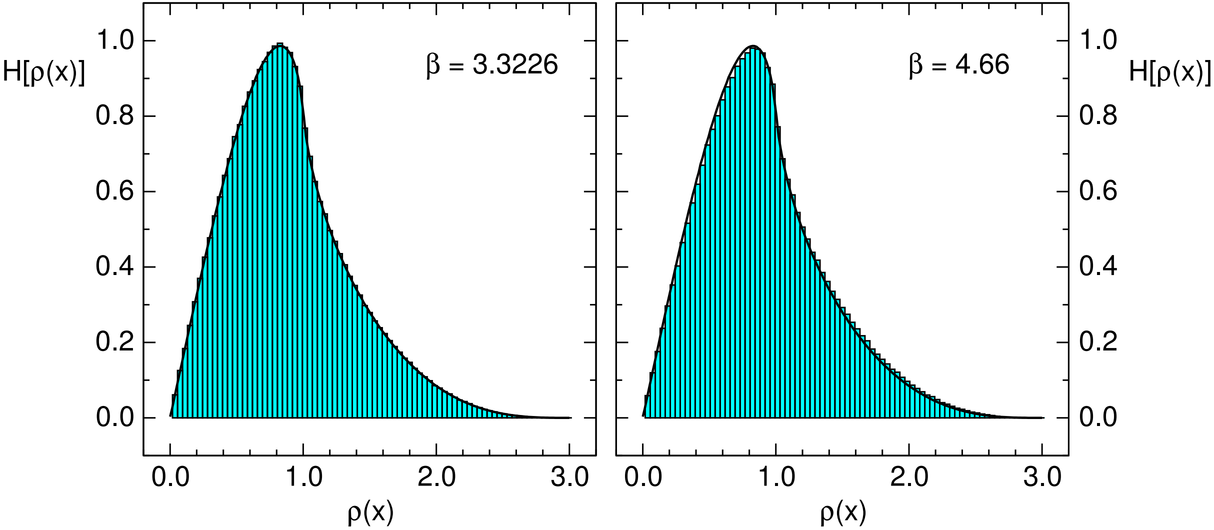
- Canonical determinants with non-zero triality break the center symmetry.

Expectation value of the Polyakov loop



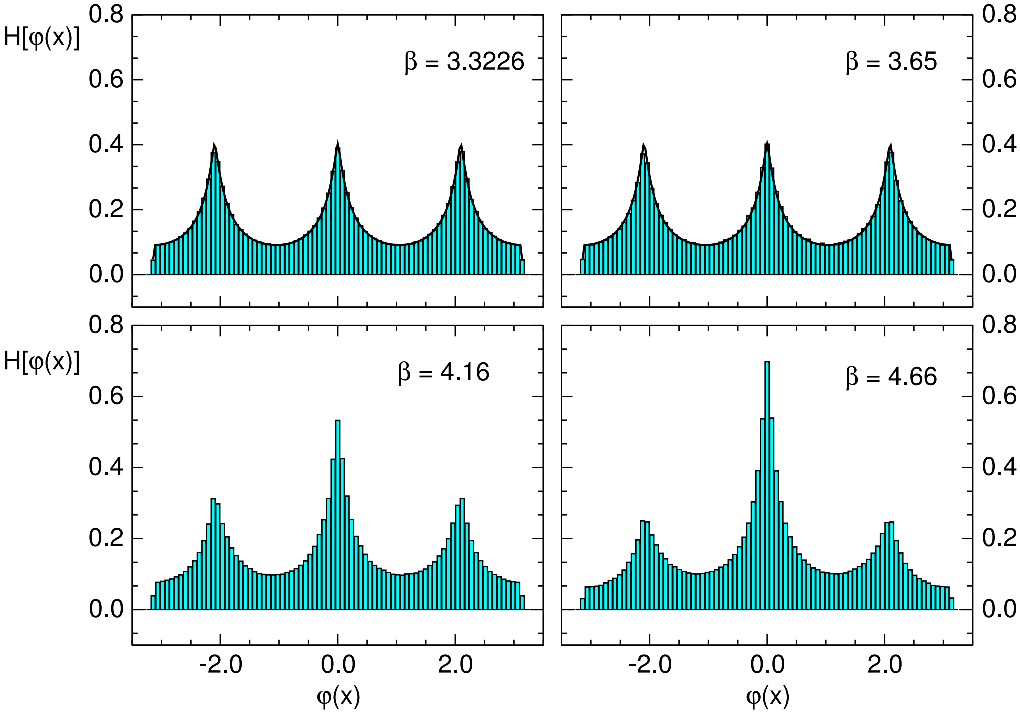
Two flavors of staggered fermions, size $24^3 \times 8$, critical coupling $\beta_c \sim 3.65$.

Distribution of the modulus of the local loop



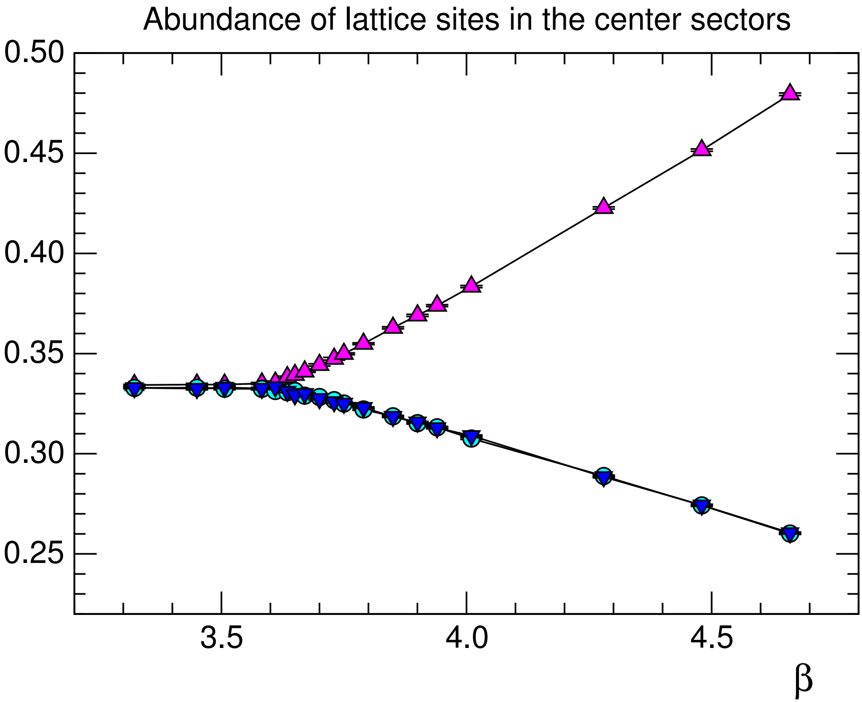
Again we find temperature independence of the distribution.

Distribution of the phase of the local loop

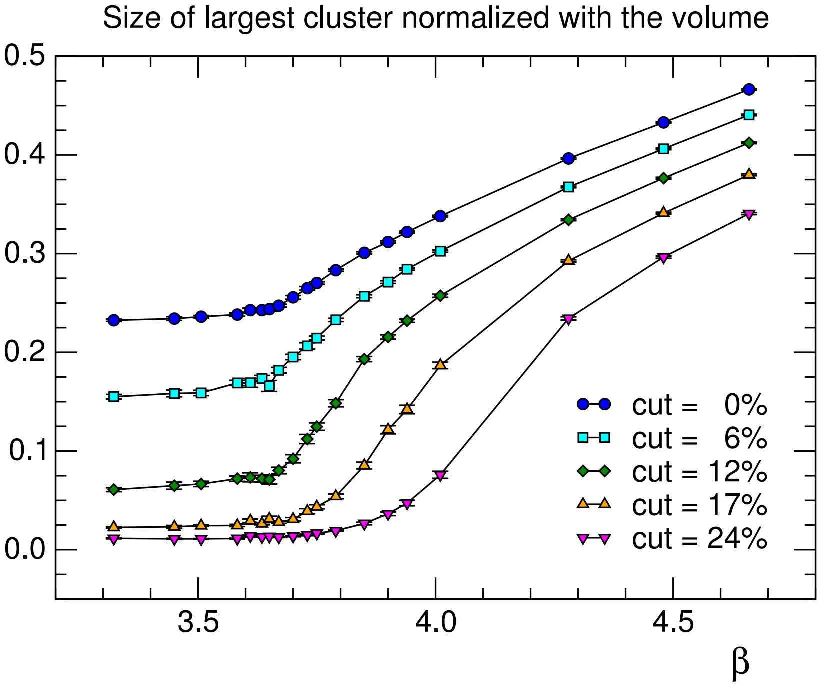


Below T_c center breaking is very small, above T_c the real sector is enhanced.

Abundance of the center sectors

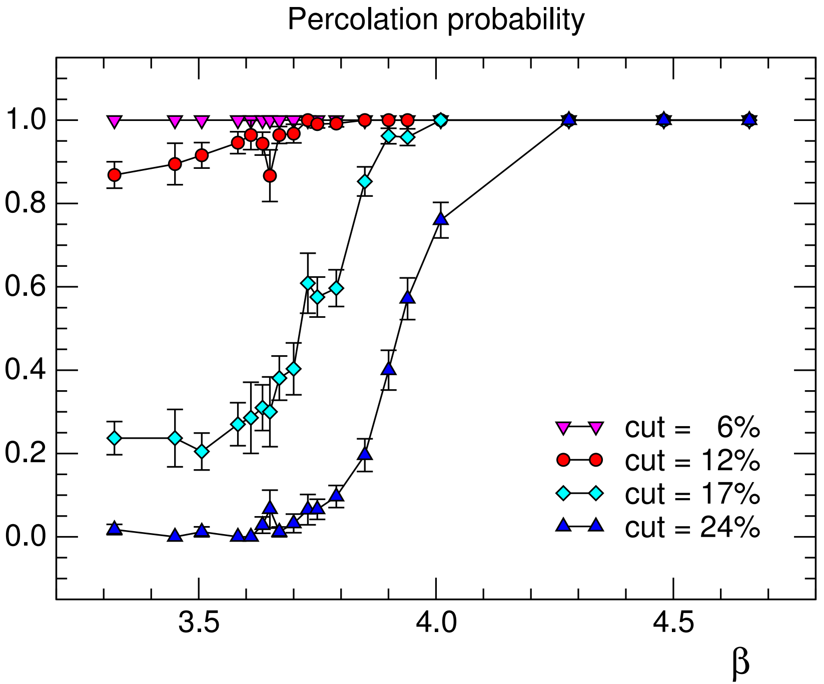


Size of largest cluster



Percolation properties unclear – a second volume is needed.

Percolation probability



Percolation properties unclear – a second volume is needed.

Summary

- We study the role of center symmetry in the QCD finite temperature transition by analyzing local Polyakov loops.

- For all temperatures the phases of the local loops have preferred values near all center values $1, \pm i2\pi/3$.

- The phases form spatially localized clusters.

- For pure gauge theory one of the sectors starts to dominate at the phase transition and the corresponding clusters show percolation.

- For the dynamical case it is currently unclear whether there is a percolation phenomenon for suitably defined clusters.