Finite Size Scaling Beyond the Standard Model

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Outline

- Introduction – "standard continuum stories" for BSM physics
- Our model, and its status
- Letting the simulation volume do the work for us
- Consequences

Much work done in collaboration with B. Svetitsky, Y. Shamir (Tel Aviv), A. Hasenfratz (Boulder)

Possibilities for Strongly Coupled Physics Beyond the Standard Model

Many people desire to replace fundamental Higgs field by something more "natural"

- New strongly interacting sector
- Higgs v replaced by $\langle \bar{Q}Q \rangle$
- $W - \pi - W$ coupling $\rightarrow M_W^2 = (\frac{g_2}{2})^2 f_\pi^2$

Many variations on this theme.

Typically encounter QCD-like dynamics, but continuum calculations are all semi-analytic and uncontrolled.

- Gap equations, ladders . . .
- AdS-CFT-brane inspired

Lattice people solved QCD, what about here?

Actually, it's a different situation

- QCD had context for analyzing results
  - Confinement
  - Chiral symmetry breaking
- The goal of lattice QCD work was (mostly) numbers
- Here, we are first trying to create context, then compute numbers

Multiple possibilities, inspired by the perturbative $\beta-$ function

$$\beta(g^2) = \frac{dg^2}{d\log(\mu^2)} = \frac{b_1}{16\pi^2}g^4 + \frac{b_2}{(16\pi^2)^2}g^6 + \ldots \quad (1)$$

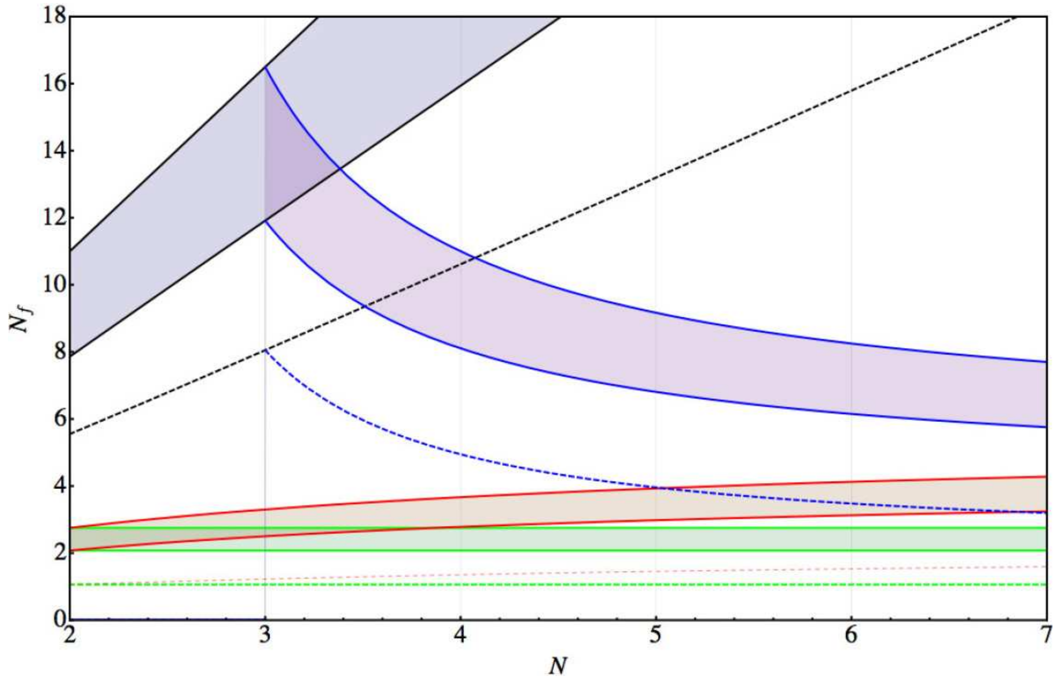
$$b_1 = -\frac{11}{3}N_c + \frac{4}{3}N_f T(R)$$

$$b_2 = -\frac{34}{3}N_c^2 + N_f T(R)(\frac{20}{3}N_c + 4C_2(R))$$

- For large enough $N_f T(R)$, $b_1 > 0$: trivial theory
- For small enough $N_f T(R)$, b_1 and $b_2 < 0$: QCD-like theory?
  - This is "technicolor:" techni-pions eaten by W's, techni-particles as new physics
  - Coupling runs slowly, or "walks" (while PT makes sense)

Inside the conformal window

- In between, $b_1 < 0$, $b_2 > 0$: possibility of an IR attractive fixed point (IRFP) – $\beta(g^{*2}) = 0$
  - No confinement
  - No chiral symmetry breaking
  - No particles
  - If $g \longrightarrow 0$ at cutoff, Λ parameter governs short distances – but not long distances
- This sounds like a disaster for phenomenology but people actually build models based on this phase
  - "conformal technicolor"
  - "hidden valley" scenarios
  - "unparticle" scenarios
- Usually, BSM literature goes off IRFP by introducing nonzero m_q. Physics still "at IRFP"
- Solvable model here: large N_c, fixed N_f/N_c can put $g^{*2} \sim O(\epsilon)$ – FP is perturbative



The usual figure in the continuum literature (Dietrich & Sannino, PRD 2007)

- bands show model predictions for conformal window vs (N_f, N_c, and fermion rep)
- Colors for different fermion representations

IS THIS PICTURE TRUE?

How does physics (masses, couplings,anomalous dimensions) vary across the parameter space?

Our model and its status

- We're simulating $SU(3)$ gauge fields, $N_f = 2$ flavors of sextet fermions
  - Thought it would "walk" and it's simple
- Long story about technical details, which I'll spare you (really boring, probably important)
- Input two bare parameters: $\beta = 6/g^2$ and variable related to quark mass m_q
  - insert another long story here...
- Do simulations at many values of the bare parameters in many simulation volumes
- Measure all kinds of things (potential, spectroscopy, running coupling, . . .)
- Look for curious features in the data

Berlin – Lattice Capital of the World

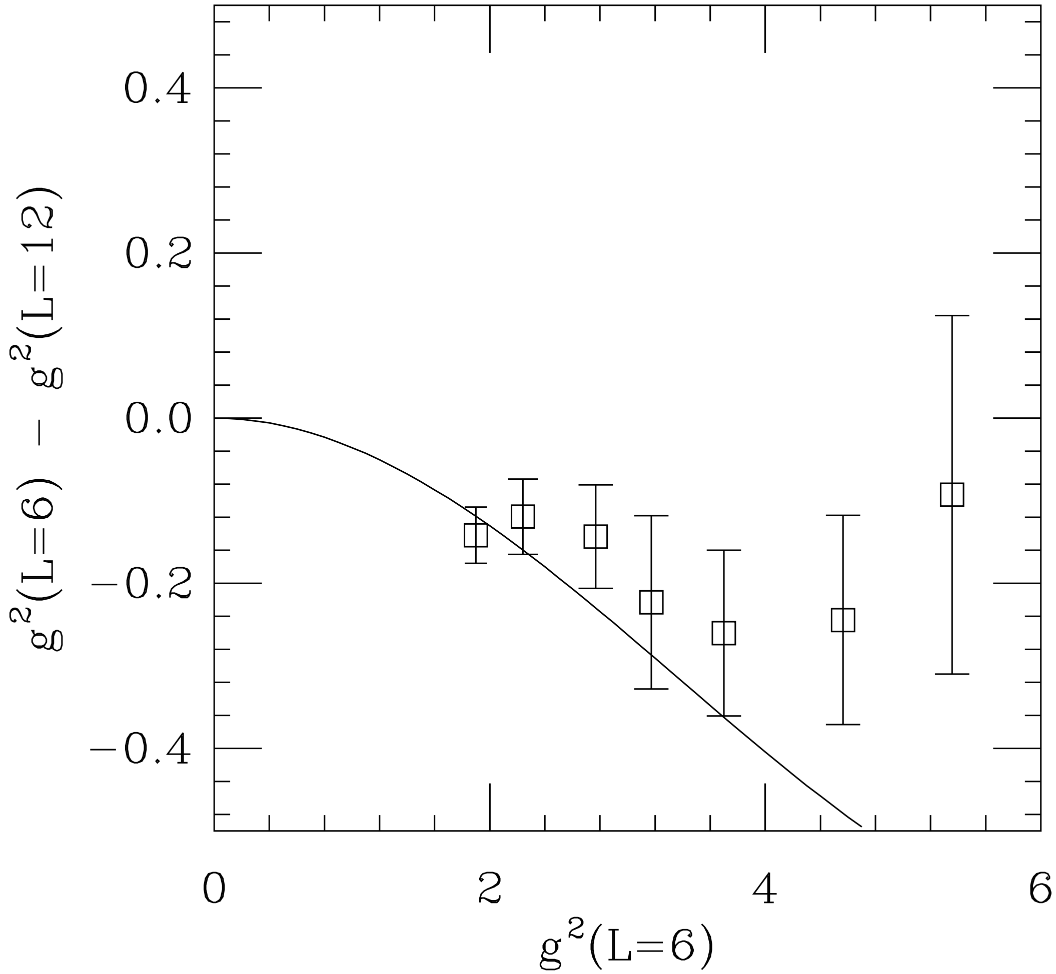
- We use clover fermions
  - Flavor counting important
  - Overlap too costly
  - But chiral symmetry is also important, clover fermions have problems
- Gauge connections use differentible hypercubic link (SS++)
- $c_{SW} = 1$
  - PT plus some QCD studies: Z-factors = 1 + tiny
  - Worry – stories about actions implicitly involve asymptotic freedom
- Another long story
  - β, κ go into computer
  - Express results using AWI quark mass (because $m_q = 0$ is "special")

$$\partial_t \sum_x \langle A_0(x,t) X(0) \rangle = 2m_q \sum_x \langle P(x,t) X(0) \rangle \tag{2}$$

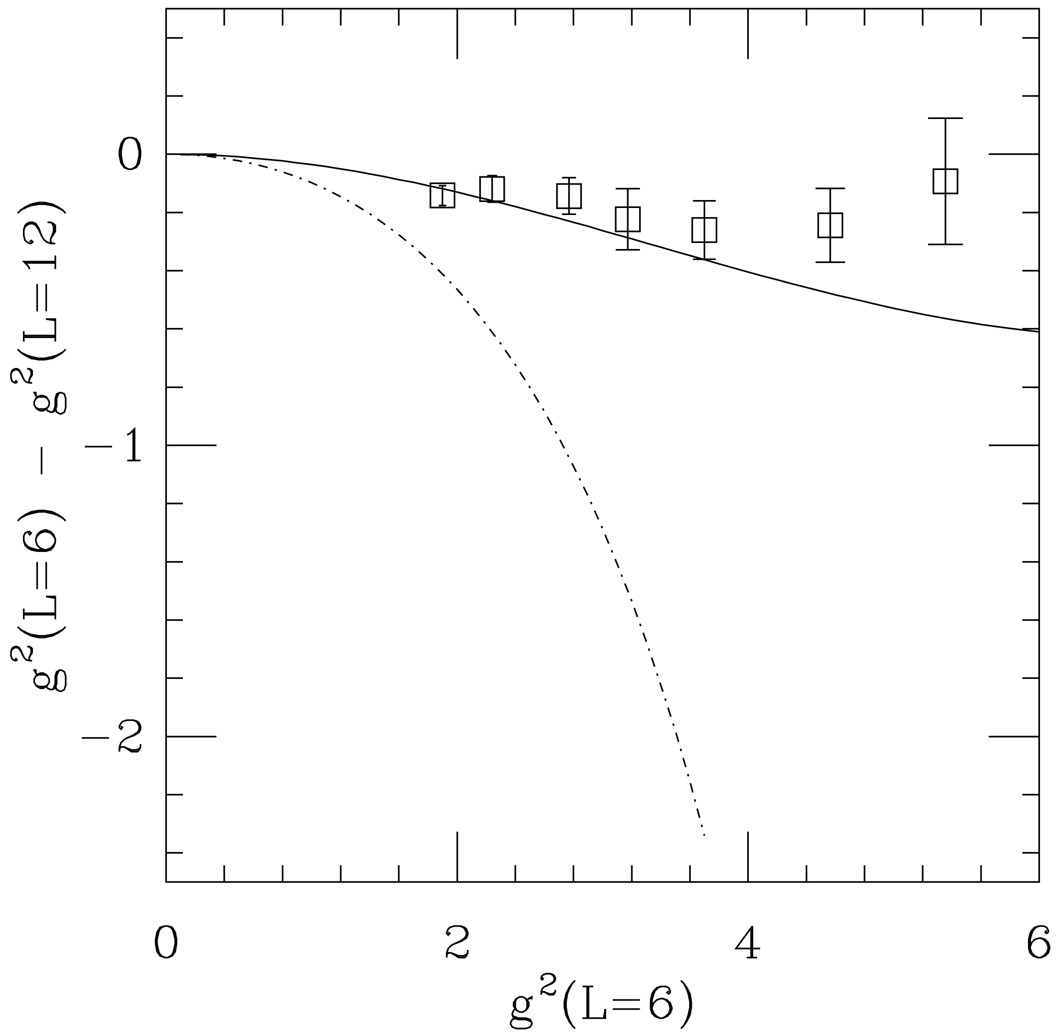
  - in WC phase, this is straightforward.

What I think is going on

- Lattice theory ($N_f = 2$, $N_c = 3$, sextet quarks) has two phases
  - A strong coupling (SC) phase which is confining and chirally broken
  - A weak coupling (WC) phase which is deconfined and chirally restored
  - A first order transition separates them (I think) meaning no continuum limit on SC side
  - Warning: this is all controversial! ("lattice artifacts...")
- (Schrodinger functional) coupling runs very slowly in WC phase
  - Last year, with different lattice action, saw zero on discrete beta fn
- In WC phase, correlation length ξ depends weakly on g^2, strongly on m_q
- In WC phase, ξ is strongly affected by size of box L when $\xi \sim L$

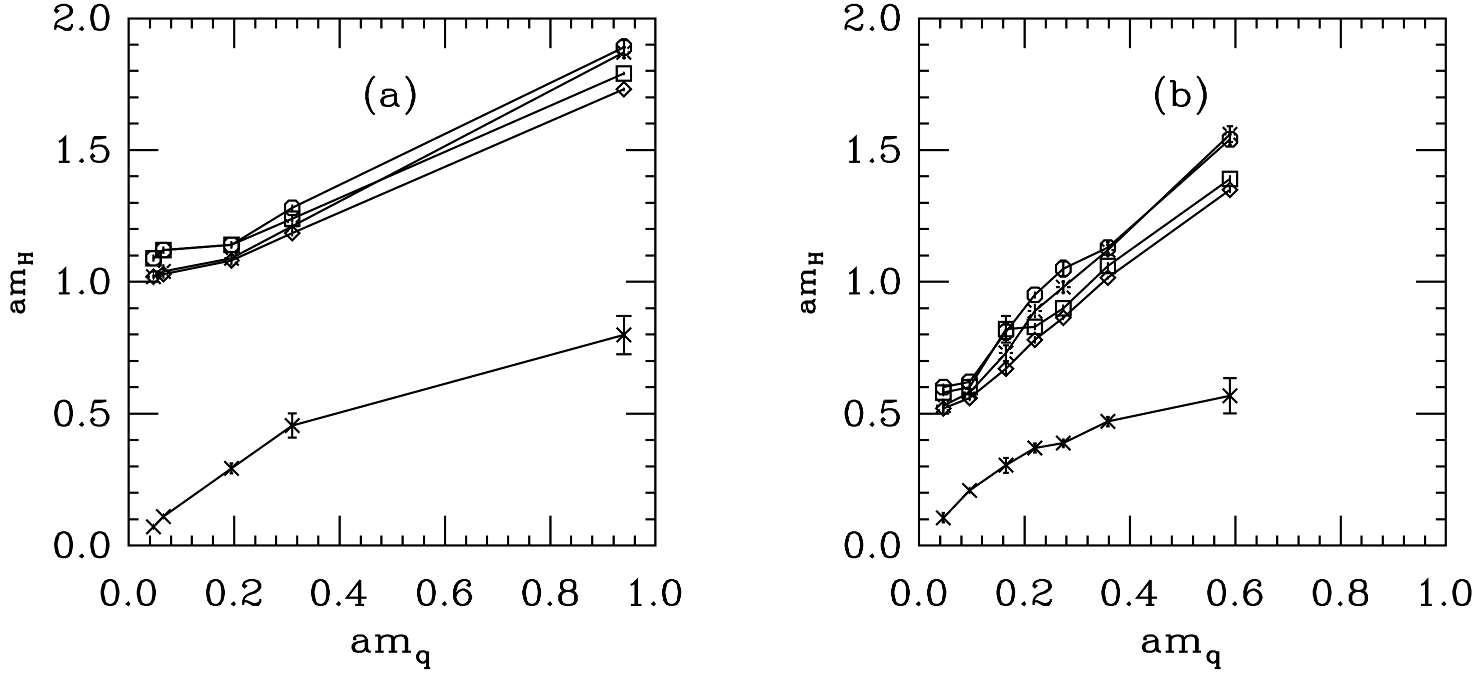


This year's "discrete beta function" or "step scaling function" (scale factor 2, lattice data vs 2-loop expectation)



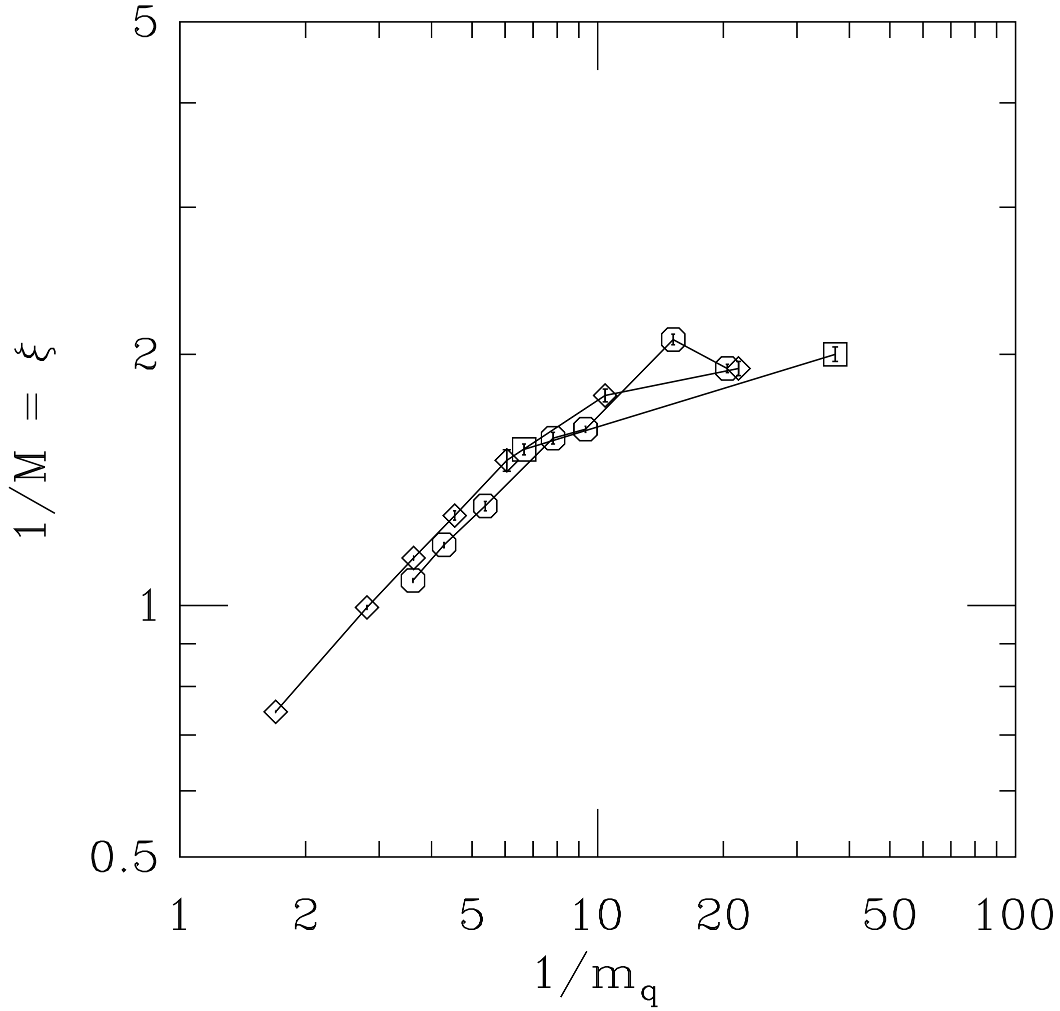
Walking vs running – dashed line is (integrated) 2-loop beta function for $N_c = 3$, $N_f = 2$ fundamentals

Illustrating chiral restoration – parity doubling



Left: $12^3 \times 6$, right, 12^4: mass vs AWI quark mass: diamonds–PS; squares–V; octagons–PV; burst–S; crosses–f_{PS}

Dependence on bare parameters



12^4 volume, several $\beta = 6/g^2$ bare couplng values

Last items suggest –

- Weak coupling phase has one relevant coupling, m_q
- g^2 is irrelevant

This implies correlation length diverges as

$$\xi \sim m_q^{-1/ym} \tag{3}$$

or

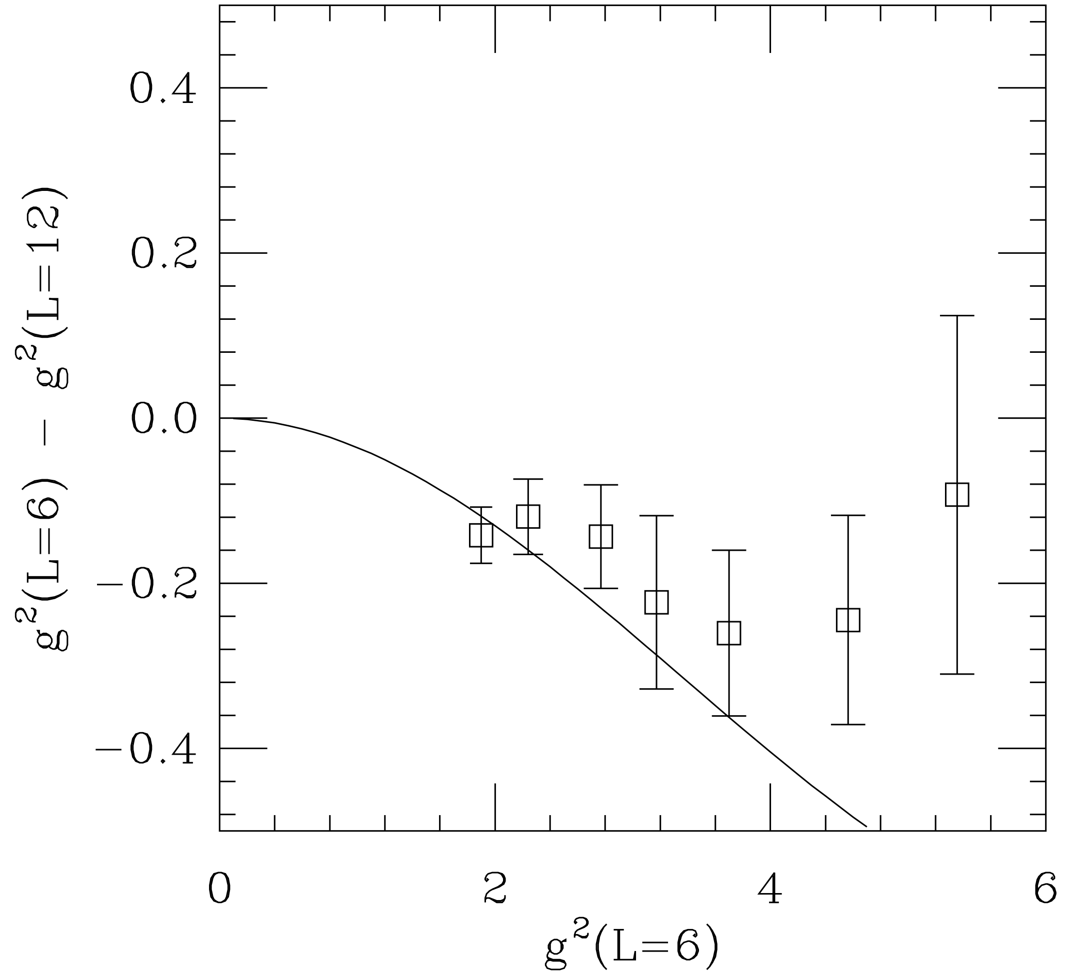
$$M^{ym} \propto m_q \tag{4}$$

- This could be absolutely true (in a real IRFP theory, $g \to g^*$ so it's irrelevant)
- This could only be approximately true – we already know, g runs slowly

If you only look over scales where g doesn't change much, you get power laws

$$\begin{aligned} \Gamma(sp) &= s^{dn}\Gamma(p) \exp \int_1^s \frac{dt}{t}\gamma(g(t)) \\ &\simeq s^{dn}\Gamma(p) s^{\gamma(g(s))} \end{aligned} \tag{5}$$

This is power law, $\Gamma(k) \sim k^{dn+\gamma}$



"Discrete beta function" to remind (under scale factor 2, coupling changes by less than ten per cent)

A reasonable approach:

- Analyze "as if it were critical"
- Look for consistency
- Repeat for several values of g – map out $y_m(g)$

Aside: a generic CFT has many exponents, I can only see the biggest one

Aside: y_m is the ingredient phenomenologists want (related to scaling dimension of $\bar{\psi}\psi$)

$$\langle\bar{\psi}\psi\rangle_{TC} = \langle\bar{\psi}\psi\rangle_{ETC} \exp \int_{TC}^{ETC} \frac{d\mu}{\mu}\gamma(\mu) \tag{6}$$

with $y_m = 1 - \gamma = 4 - d$, $d =$ scaling dimension of condensate

Theorems say $3 > d > 1$ or $1 < y_m < 3$

- $y_m = 1$ is free field
- y_m is expected to grow near the bottom of the conformal window, perhaps big y_m marks its end

Letting the box do the work – Finite size scaling

A nice trick from the stat mech literature

- (Assume linear zeroes in beta functions)
- Near a critical point, ξ diverges, $\xi \sim t^{-\nu}$
- Use ξ to set the length scale – trade ξ for relevant coupling
- This implies scaling relations between critical exponents
- A finite box size L prevents divergence of ξ
- Diverging quantities like susceptibilities now have finite peaks, peak location shifts, etc

$$f_s(t, L) = t^{\nu D} f_s(t_0, u_{i0} t^{|y_i|/y_t}, \frac{1}{L t^\nu}) \sim L^{-D} \psi(t L^{1/\nu}) \tag{7}$$

- Size of peak, motion of peak vs L gives critical exponents

$$\chi(t, \frac{1}{L}) = t^{-\gamma} \phi(\frac{1}{L t^\nu}) = L^{\gamma/\nu} \psi(t L^{1/\nu}) \tag{8}$$

- This also works for the correlation length, itself
- Aside – next leading exponent, y_g from gauge coupling, probably near zero

Finite size scaling for the correlation length

Recall $\xi \sim m_q^{-1/ym}$ only if $L >> \xi$

In a box of size L, correlation length ξ_L sees L

FSS says that if only ξ and L are large, then

$$\xi_L = LF(\xi/L) \tag{9}$$

Nicer version –

$$\xi_L = Lf(L^{ym} m_q) \tag{10}$$

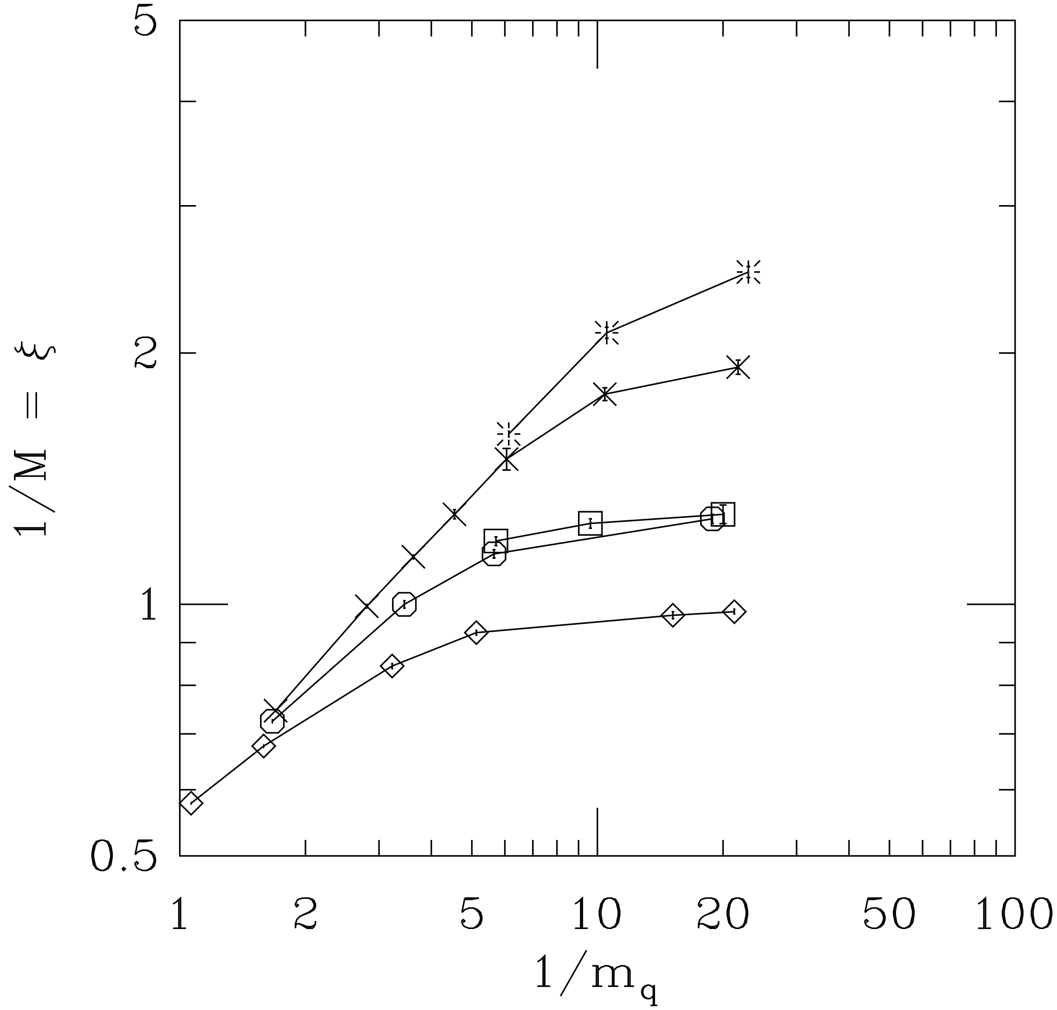
To implememt, do simulations on many volumes, same bare gauge coupling, vary m_q

Measure ξ_L always with the same boundary conditions

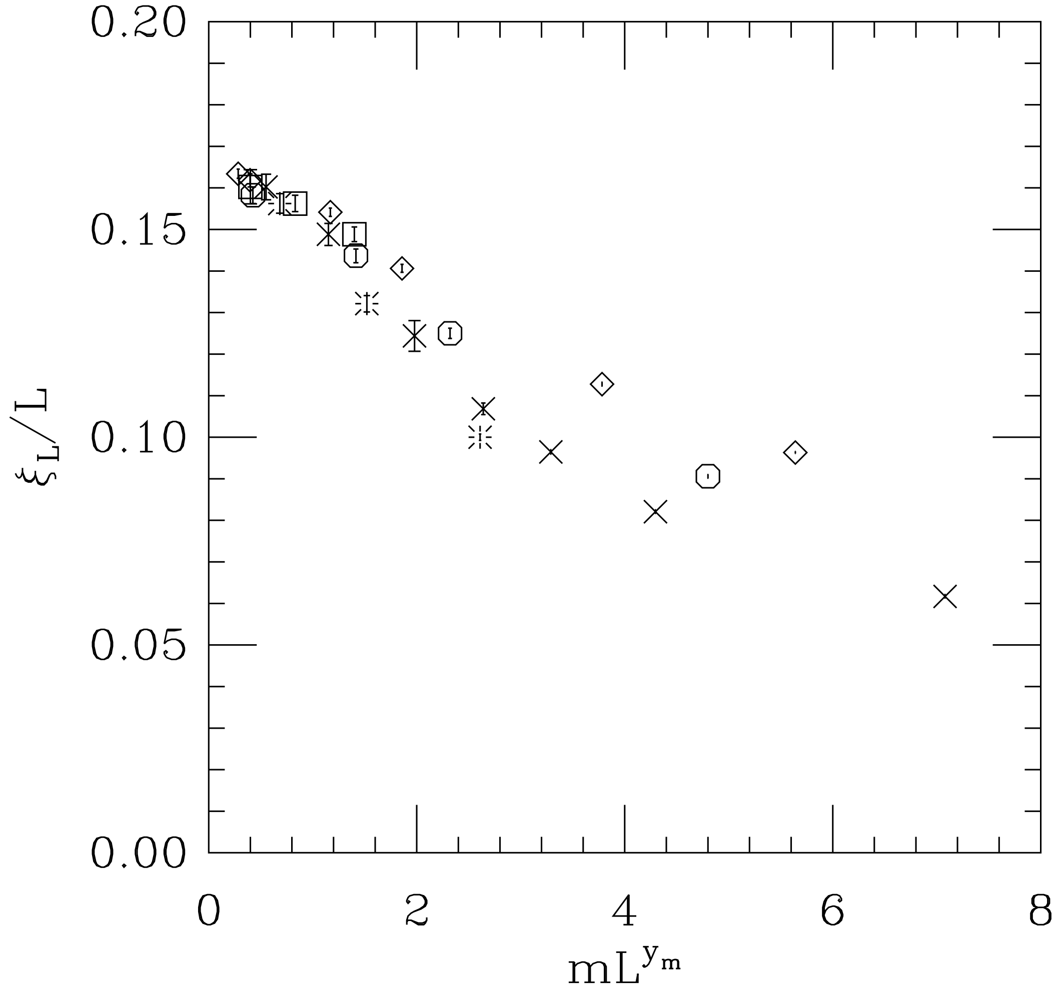
Plot, vary y_m – x-axis moves

$\rightarrow y_m \sim 1.5$, meaning $d \sim 2.5$

Dependence on lattice size, same bare parameters

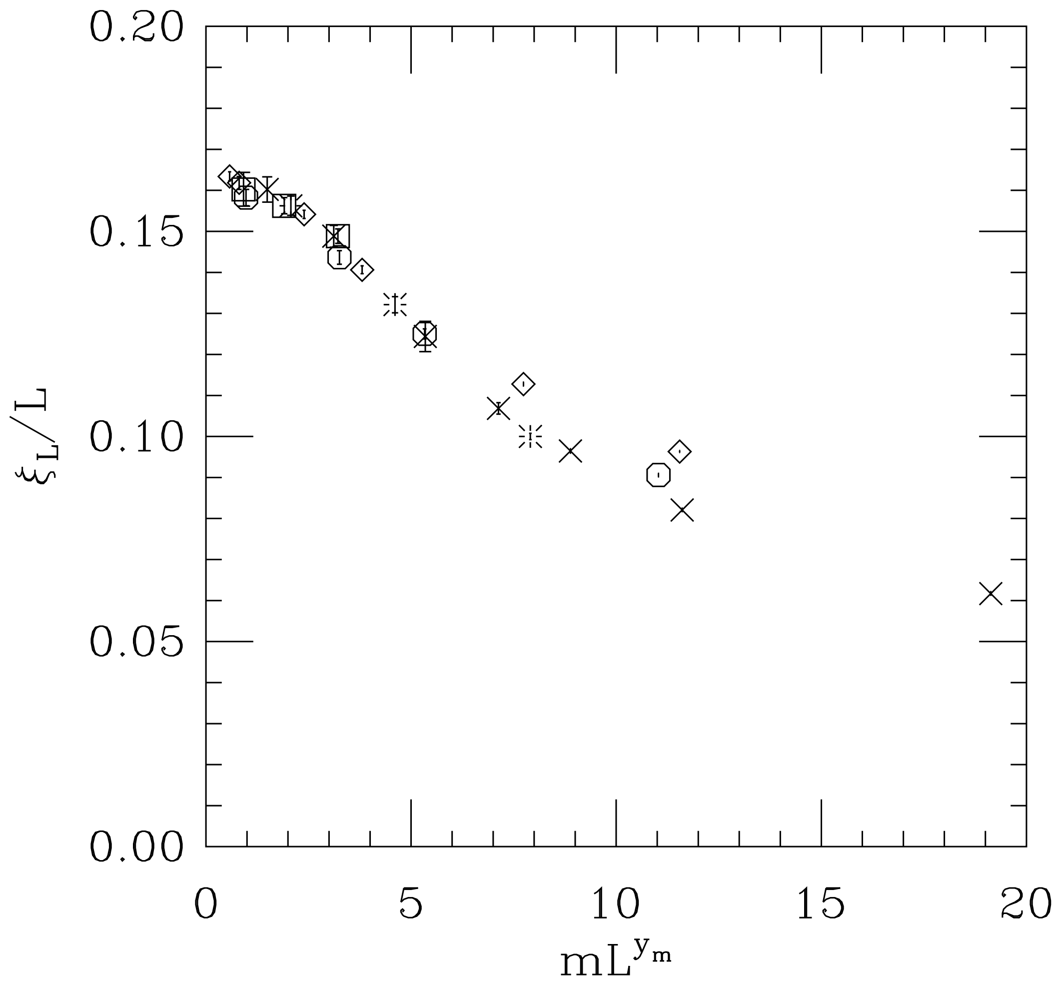


FSS rescaling



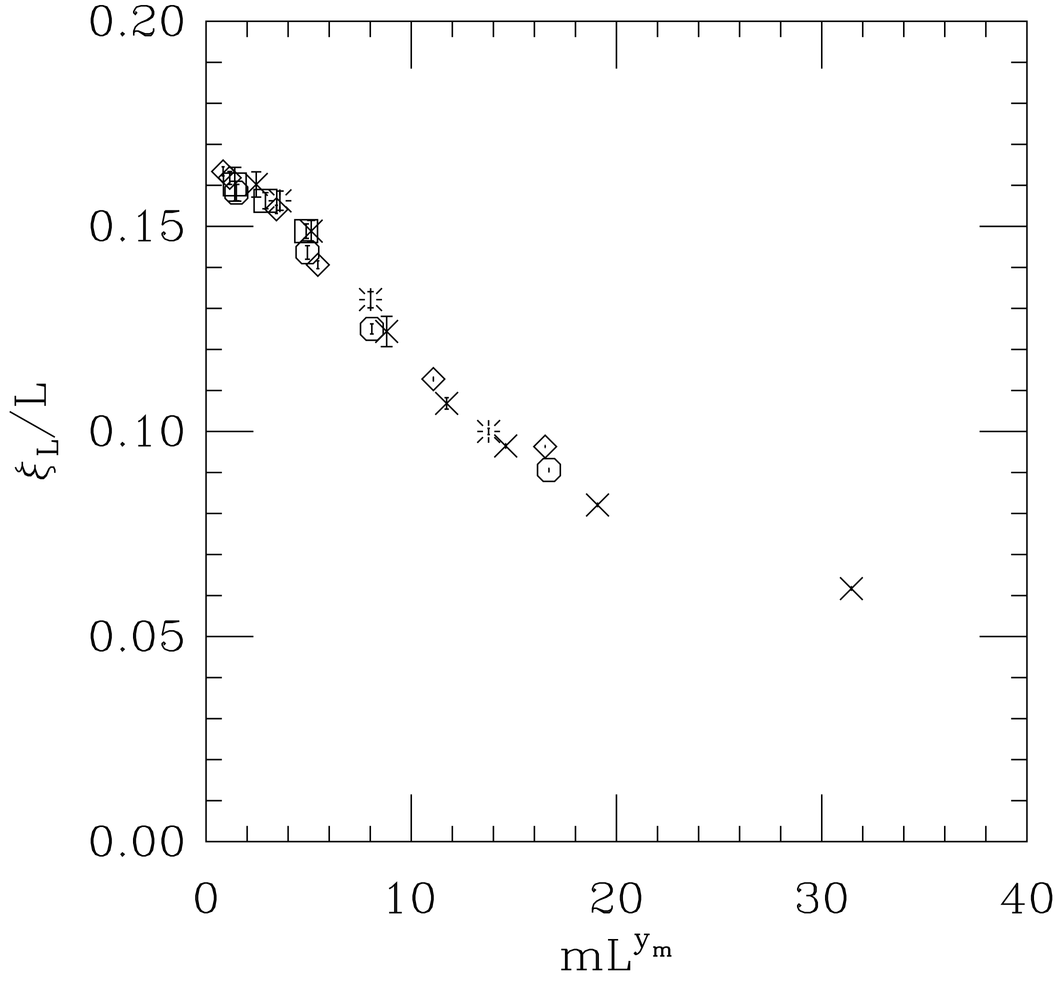
$y_m = 1.0$, L=6, 8, 12, 16

FSS rescaling



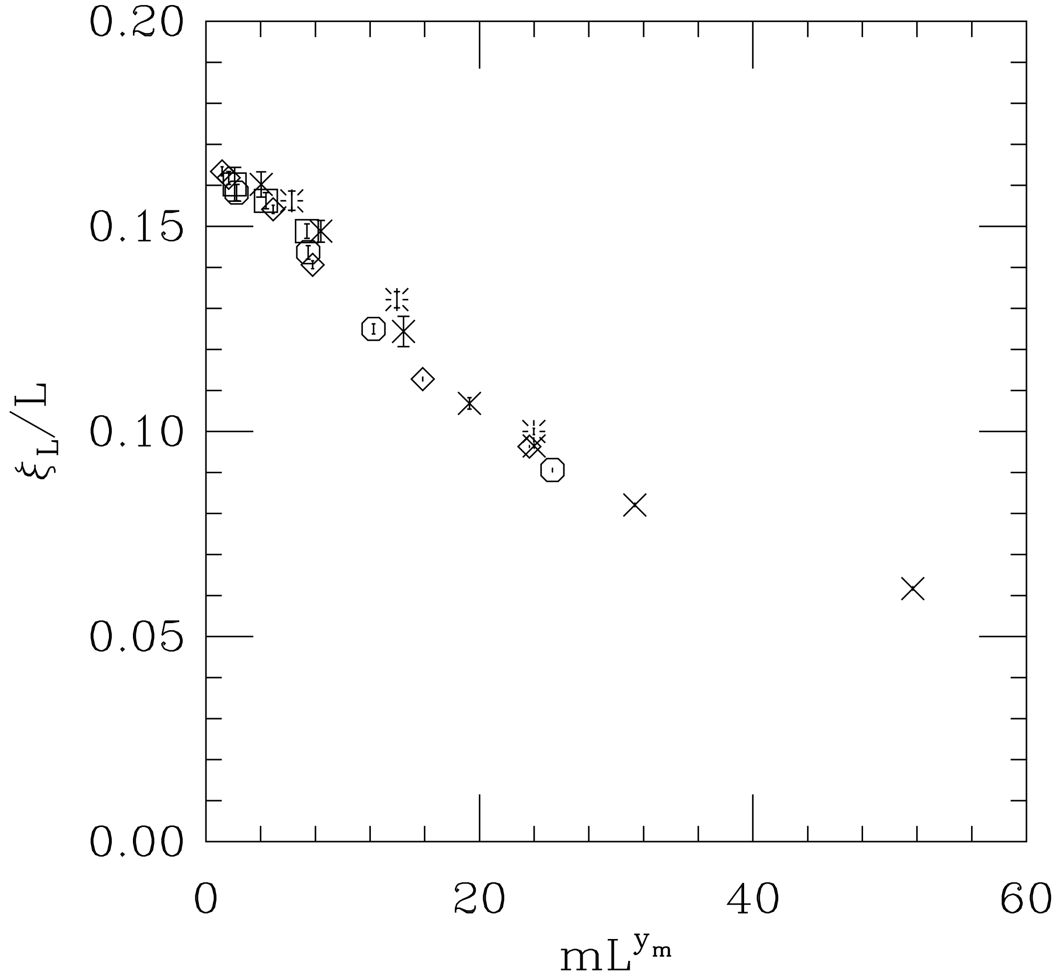
$y_m = 1.4$, L=6, 8, 12, 16

FSS rescaling



$y_m = 1.6$, L=6, 8, 12, 16

FSS rescaling



$y_m = 1.8$, L=6, 8, 12, 16

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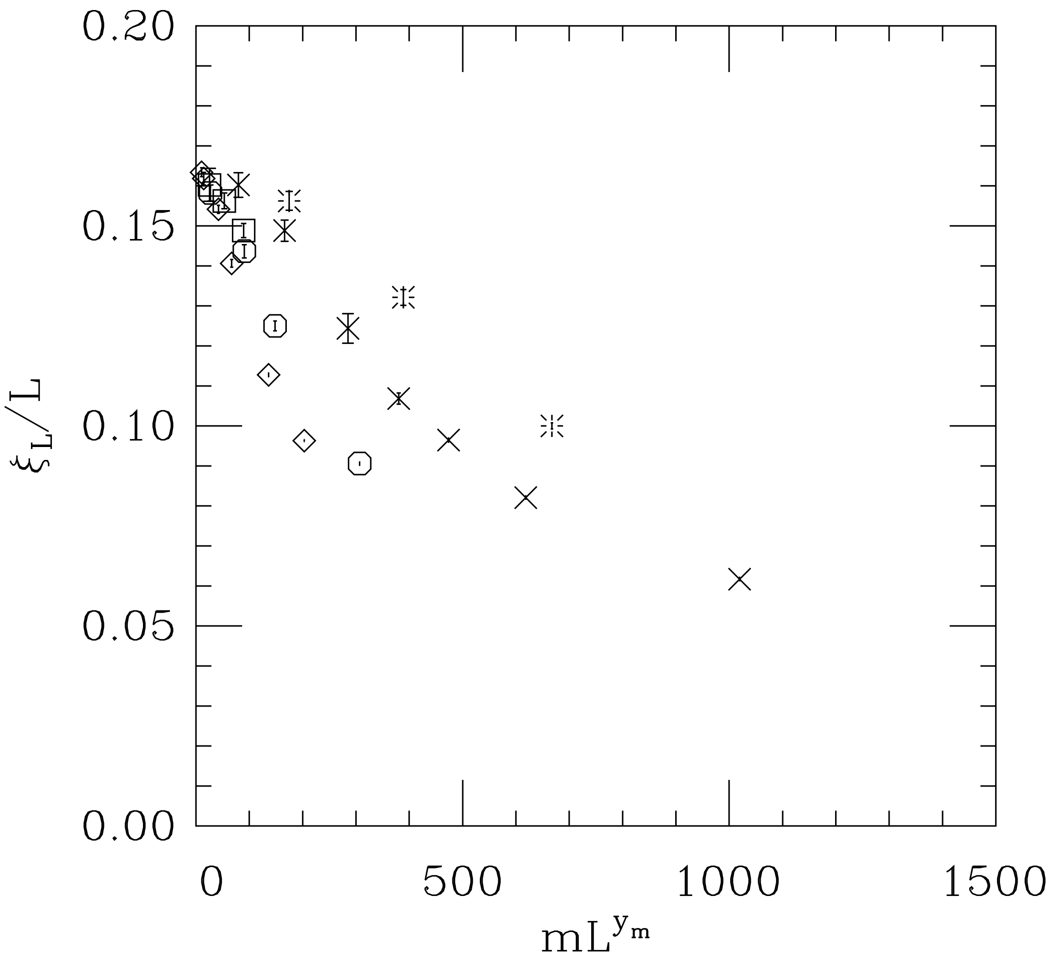
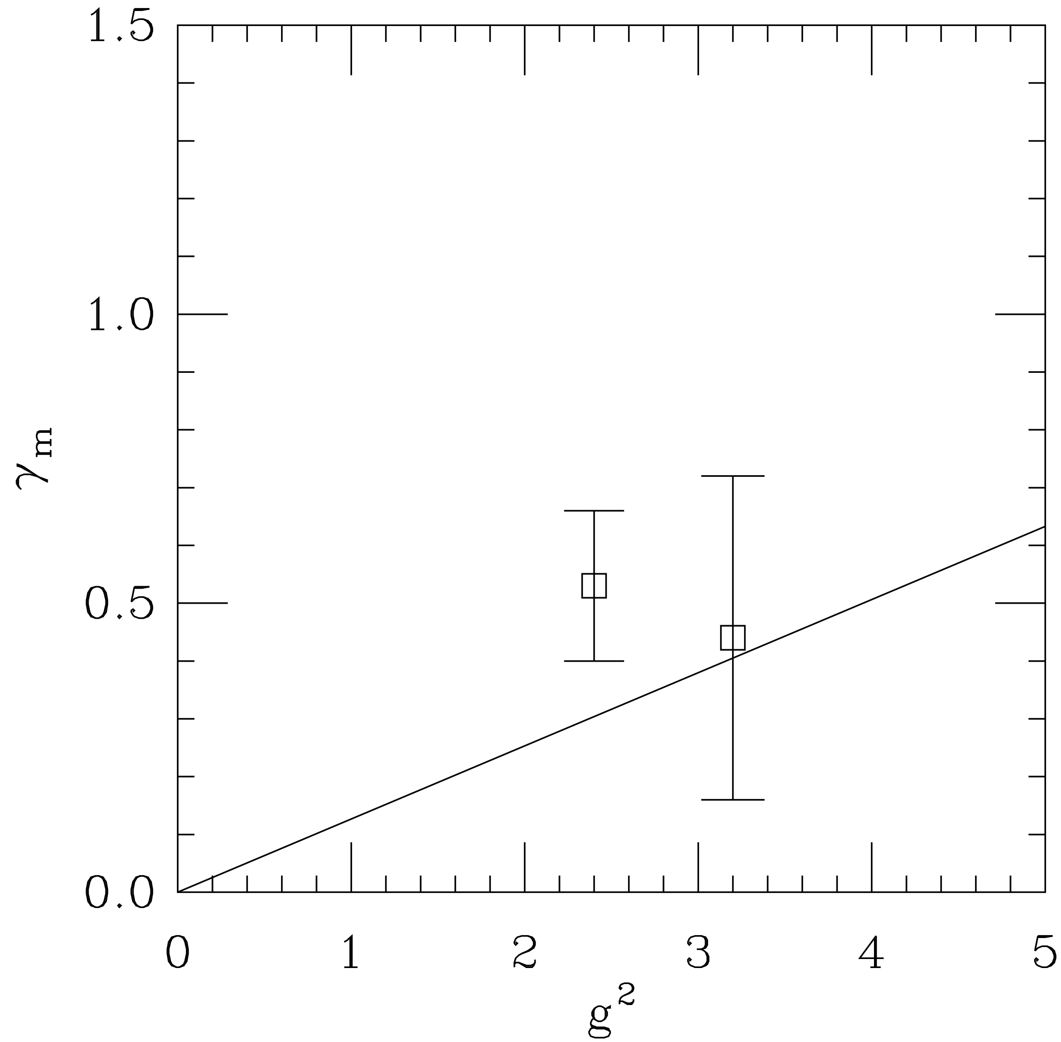
FSS rescaling



$y_m = 3.0$, L=6, 8, 12, 16

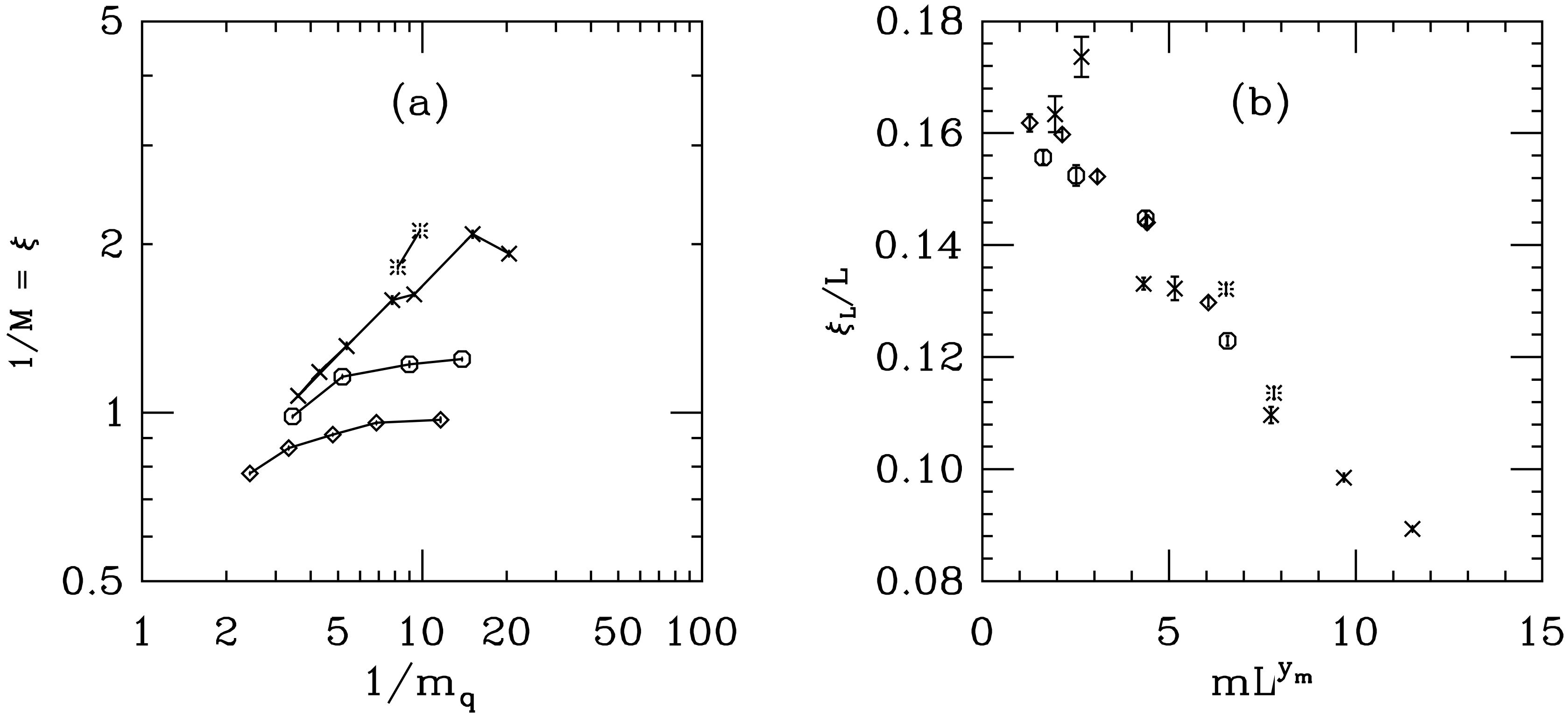
FSS status

- Have pretty complete data at one bare parameter set ($g^2 = 2.4$) – $y_m = 1.53(13)$
- Collecting data at a second bare parameter set ($g^2 = 3.2$) – $y_m = 1.44(28)$
  - Seems to have about the same y_m
- Means $y_m(g)$ depends weakly on g – FP or just slow running??
- Professional FSS analyses involve susceptibilities – quite difficult for 4-dim lattice theories due to UV fluctuations



Status last Tuesday...$y_m = 1 + \gamma_m$, $\gamma_m =$ anomalous mass dimension

FSS rescaling



Stronger gauge coupling, $y_m = 1.5$ FSS curve

The condensate, directly

Free energy per site (in dimension D) scales as (1/length)^D

Use ξ for length

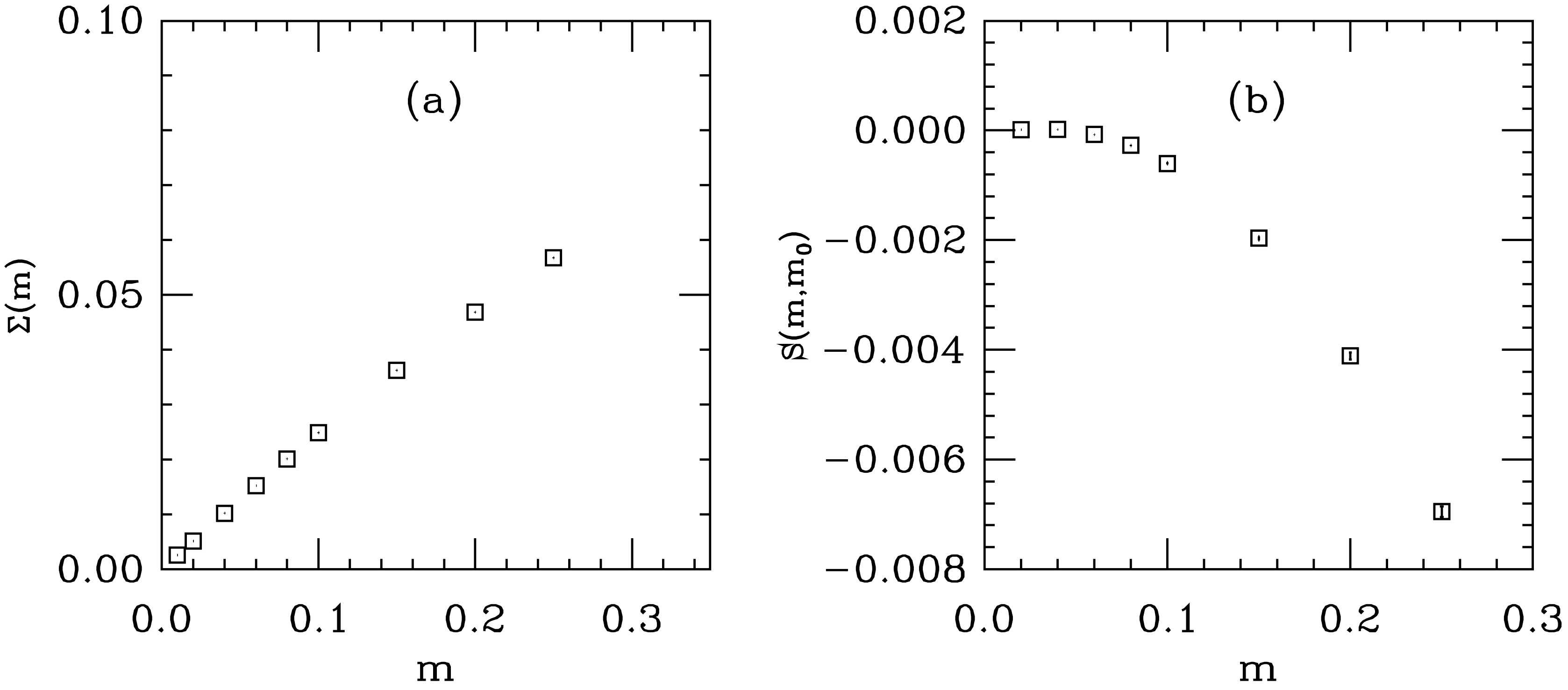
$$f_s(m_q) = m_q^{D/ym}(A_1 + A_2 m_q^{|yg|/ym} + \ldots) \quad (11)$$

$$\Sigma \equiv \langle \bar{\psi}\psi \rangle = \frac{\partial f_s}{\partial m_q} \sim m_q^{4/ym-1} \quad (12)$$

- Like a specific heat exponent
- Hard: Analytic UV terms mask nonanalytic part, $\Sigma \sim m\Lambda^2 + m^3 \log \Lambda$

Tried the direct approach – failed – have to be smarter!

Subtracted condensate



Letting the box do the work – eigenvalues of the Dirac operator

Mass dependence of condensate is related to spectral density of Dirac operator (Banks-Casher relation)

$$\Sigma(m_q) = -\int \rho(\lambda) d\lambda \frac{2m_q}{\lambda^2 + m_q^2}. \quad (13)$$

If $\Sigma \sim m_q^\alpha$, then $\rho(\lambda) \sim \lambda^\alpha$

Then finite-size scaling tells us that in a volume $V = L^D$

$$\rho(\lambda) \sim \lambda^\alpha \rightarrow \langle \lambda_i \rangle \sim (\frac{1}{L})^p \quad (14)$$

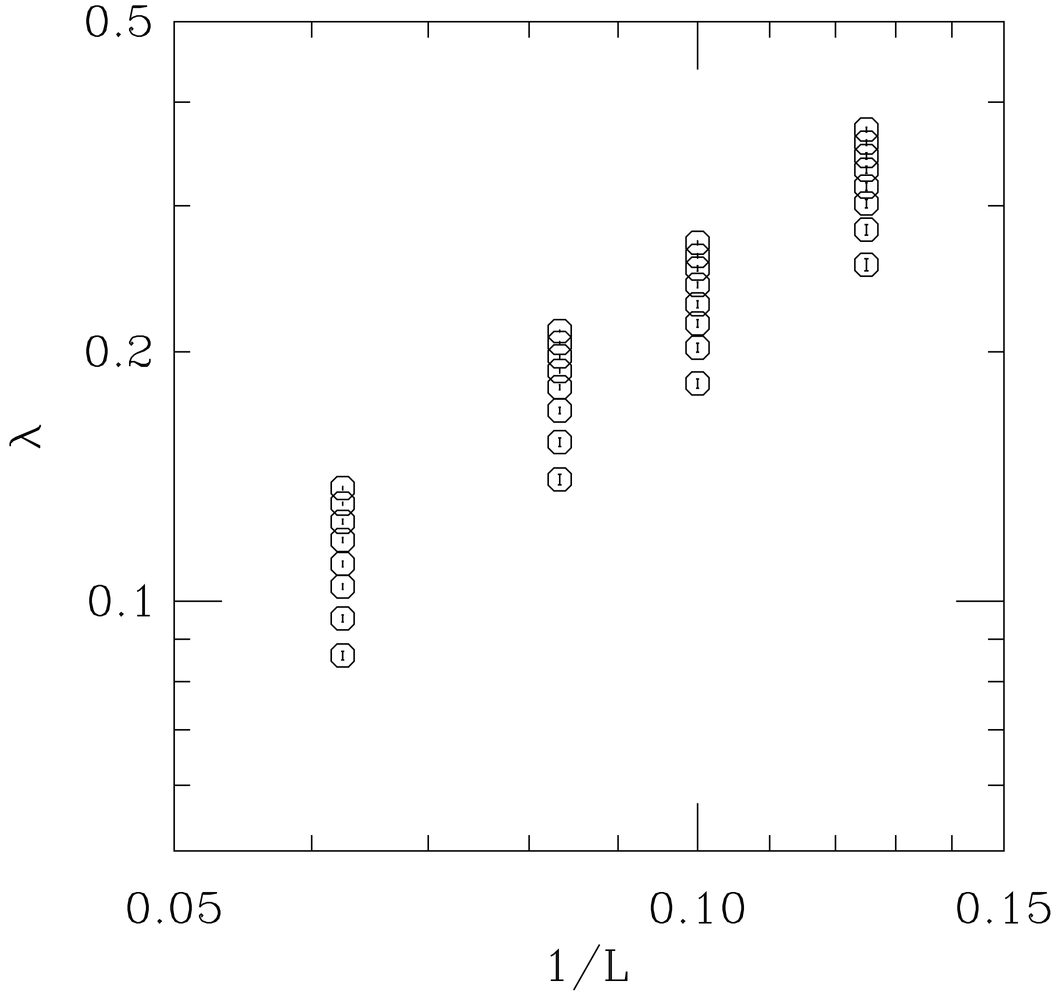
$$p = \frac{D}{1+\alpha} = y_m \quad (15)$$

so measure the eigenvalue spectrum in many volumes at one bare coupling set

- Aside: use chiral valence quarks
- Aside: bare parameters for tiny quark mass for sea quarks

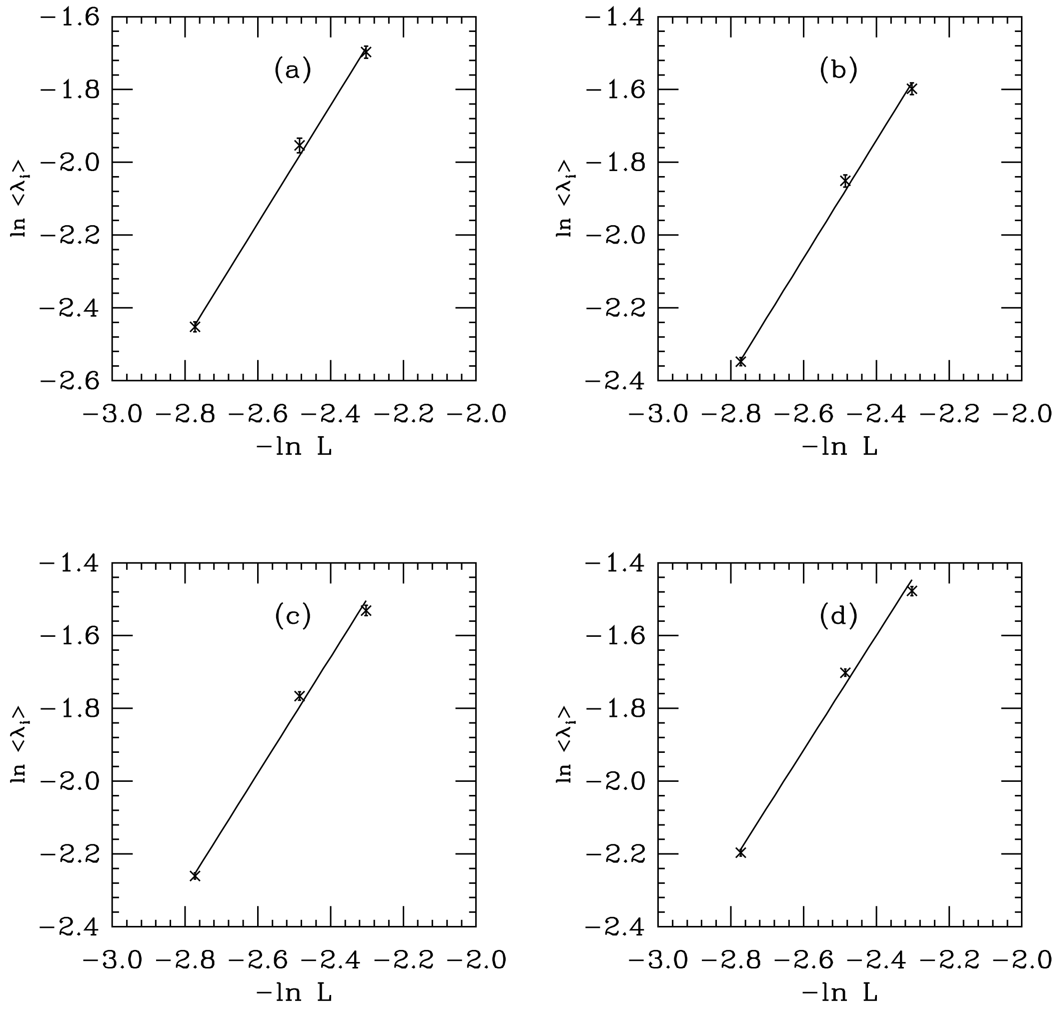
Data shows power law with $p \sim 1.6(1)$ at same coupling where FSS had $y_m \sim 1.5$!

Eigenvalues

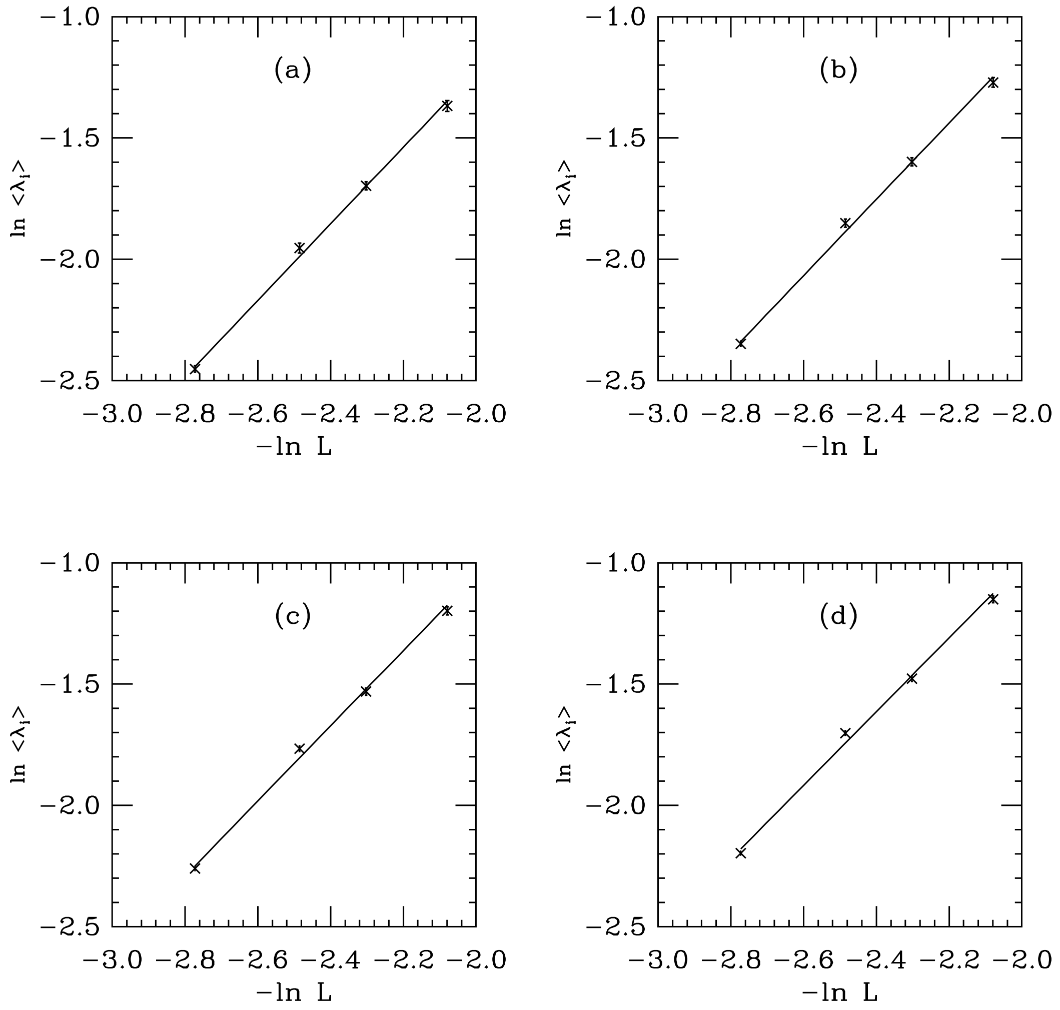


Volumes: 8^4, 10^4, 12^4, 16^4

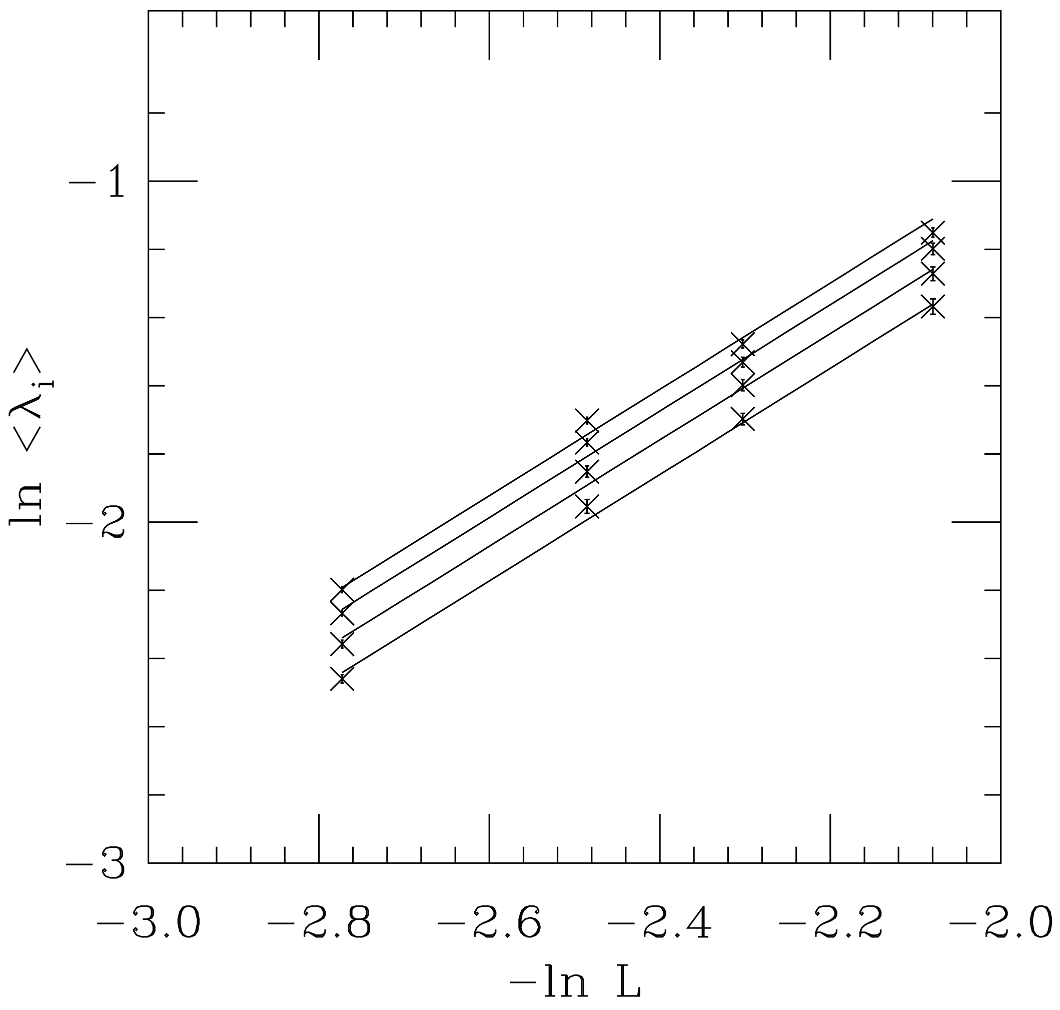
Fit several volumes



Fit several volumes



Fit several volumes



So what? possible consequences

- Really, there's no conclusion
  - I don't know if there is an IRFP or not, so I don't know if my exponents are real or effective
- If there's a FP, y_m is the interesting number – the relevant exponent
  - $y_m \sim 1.5 \rightarrow d \sim 2.5 \rightarrow \gamma_m \sim 0.5$
  - PT predicts $\gamma_m = 6C_2(R)g^2/(16\pi^2) \sim 0.3 - 0.4$ with SF g^2
- But it might not be interesting enough!
  - Remember, conformal BSM theories aren't used "as is," introduce a mass and run
  - People want d to be as small as possible, near $d = 1$

$$\mathcal{L} = \frac{\Lambda^{d_{NP}+d_{SM}}}{M^{4+d_{NP}+d_{SM}}} \mathcal{O}_{SM}\mathcal{O}_{NP} \tag{16}$$

  - "Bottom of conformal window" predicted to be $d = 2$ or $d = 1$; 2.5 is above either choice
- If we can't see the FP, then we might have a walking theory
  - $y_m(g) =$ "walking exponent" (integrate from small g^2 at ETC scale to g^2 at TC scale)
  - Walking scenario could just be entangled with lattice dirt at strong coupling
- Either way, we're still looking for a general picture
  - How do classes of models behave as N_c, N_f, representation is varied
  - How do exponents vary across the conformal window

A last thought

"The purpose of computing is insight, not numbers."–W. Hamming