Hadronic physics from the Lattice

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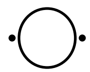
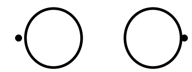
Flavour singlets - Introduction

- What role do $\bar{q}q$ quarks play in hadronic physics?
- Do $\bar{q}q$ quarks contribute to electromagentic properties of hadrons; to the mass; to the spin; ...?
- Dark matter: $\bar{s}s$ in nucleon target can be important in scattering dark matter particles in detector.

Lattice QCD is a first-principles method of attack. We also have control over whether interaction is connected or disconnected: What can we learn?

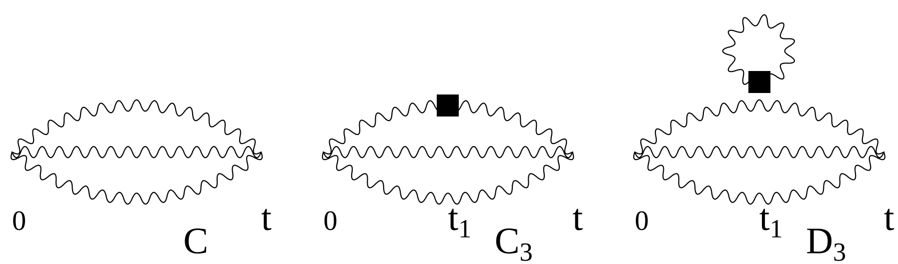
Disconnected diagrams

The correlator of a meson (created by $\bar{\psi}\Gamma\psi$) on a lattice can have two components:

connected C  disconnected D 

The disconnected contribution is only present for flavour-singlet mesons.

For baryon matrix elements (C_3), there will be a connected and disconnected contribution (D_3 for flavour-singlet currents).



Disconnected diagrams II

In lattice QCD it is straightforward to evaluate these disconnected diagrams. BUT they are very noisy. Basically because D fluctuates with a constant variance as the separation of the two disconnected components is increased.

Once the disconnected loops are evaluated, they can serve for many different physics projects:

- Flavour singlet meson spectra: η', f_0, OZI violating contributions,..
- OZI-violating meson decays
- Dependence of hadron masses on the sea-quark mass.
- Flavour singlet matrix elements of hadrons.

Mesons

Vector mesons

$\rho(775);\ \omega(782);\ \phi(1020);\ K^*(893)$

If no disconnected contribution (OZI rule) $m(\omega) = m(\rho)$

$2m(K^*) \approx m(\rho) + m(\phi)$

Pseudoscalar mesons

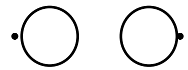
$\pi(135);\ \eta(548);\ \eta'(958);\ K(496)$

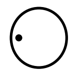
η is approximately $\eta_8 = (uu + dd - 2ss)/\sqrt{6}$

Big disconnected contribution (Topological charge density contribution?)

Lattice QCD?

Evaluation 1

Disconnected correlator 

So want to evaluate  at each point.

Stochastic method: volume source $\xi_s(x)$ with elements random in phase (length 1) for each x (space-time-colour-spin) and sample s.

$$\phi_s(x) = M^{-1}_{xy} \xi_s(y)$$

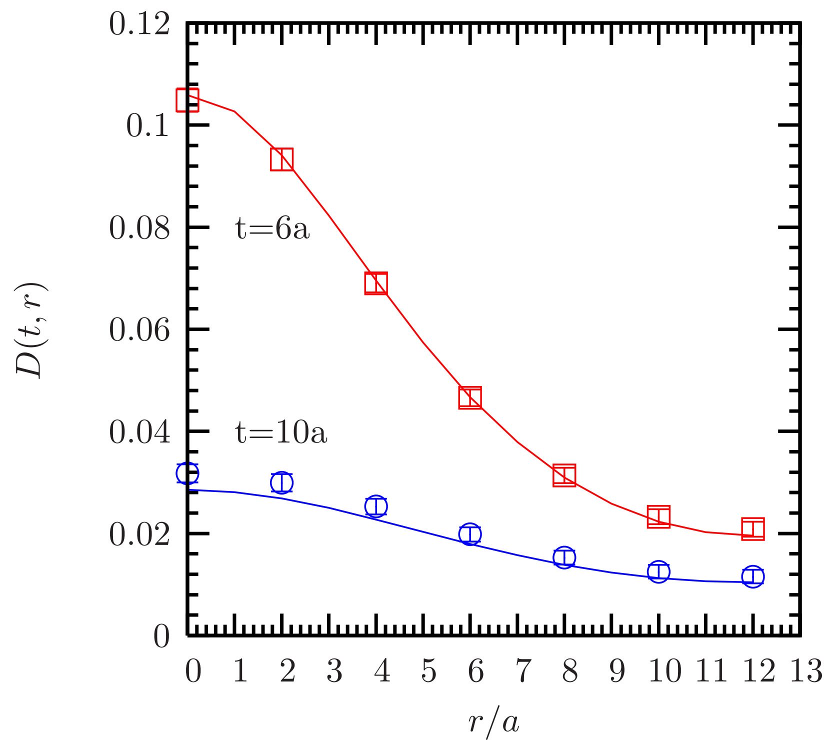
Then $\xi_s(z)^* \phi_s(x)$ is an unbiassed estimator of quark propagator M^{-1}_{xz}. As well as term required there are $N-1$ extra terms (of random sign from sum over y). One can average over stochastic samples (s) to improve signal/noise.

Variance reduction methods allow to improve this substantially either by making use of known contributions (e.g. hopping parameter methods) or by using Ward identities (e.g. TM for difference of quark masses $\pm i\mu_q$).

With enough samples s we can make the stochastic error smaller than the intrinsic variance arising from the underlying gauge fields.

Evaluation 2

But the required disconnected correlator is still very noisy.
Momentum zero correlator $D(t) = \Gamma M_{xx}^{-1} \Gamma M_{zz}^{-1}$ summed over all space at sink and source. The signal is peaked at small relative space separation, $\mathbf{r} = \mathbf{x} - \mathbf{z}$, but the noise is similar for all space separations.



The signal to noise is much better using just the point-to-point correlator.

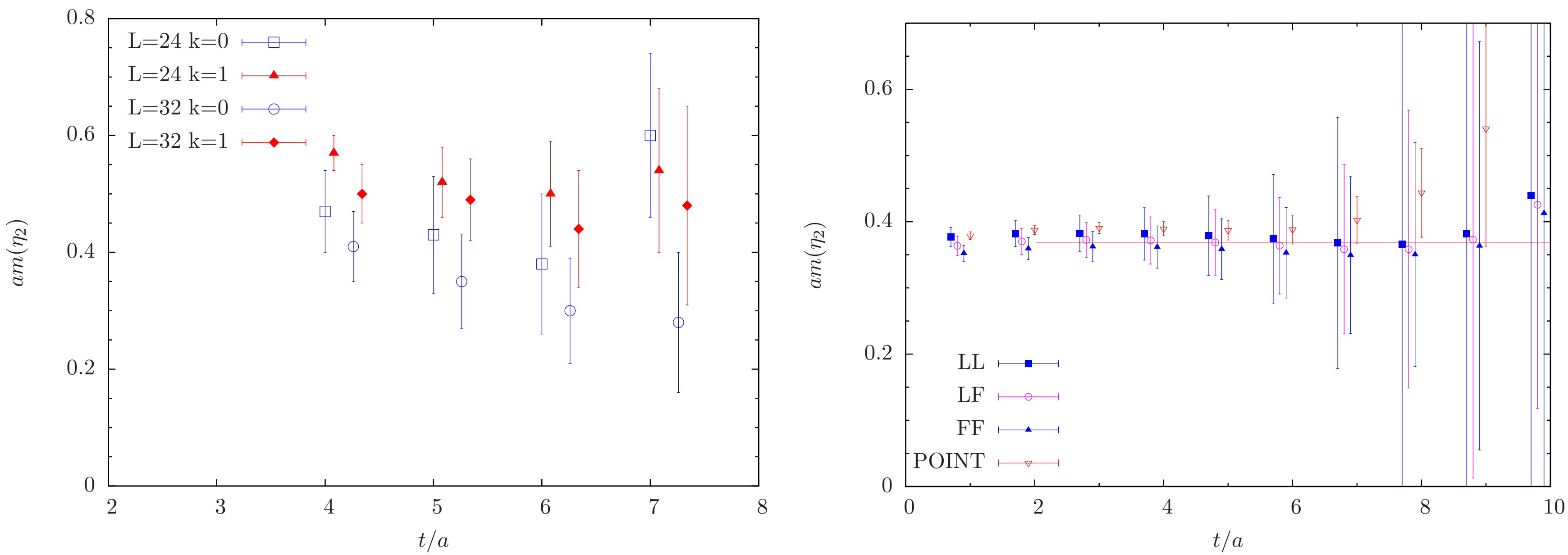
Evaluation 3

This, in principle, has contributions from all momenta: but for an $O(4)$ invariant operator at source and sink it can be parametrised in terms of a mass (m) and a coupling (c) for each relevant state.

$$C(t, \mathbf{r}) = \frac{1}{L^3 T} \sum_{p_0, \mathbf{p}} \frac{c e^{-i p_0 t - i \mathbf{p}.\mathbf{r}}}{\hat{p}^2 + m^2},$$

This allows m to be extracted using fits to the point-to-point correlators (connected + disconnected). We used $\mathbf{r}/a = 2$ along axes (6 points averaged).

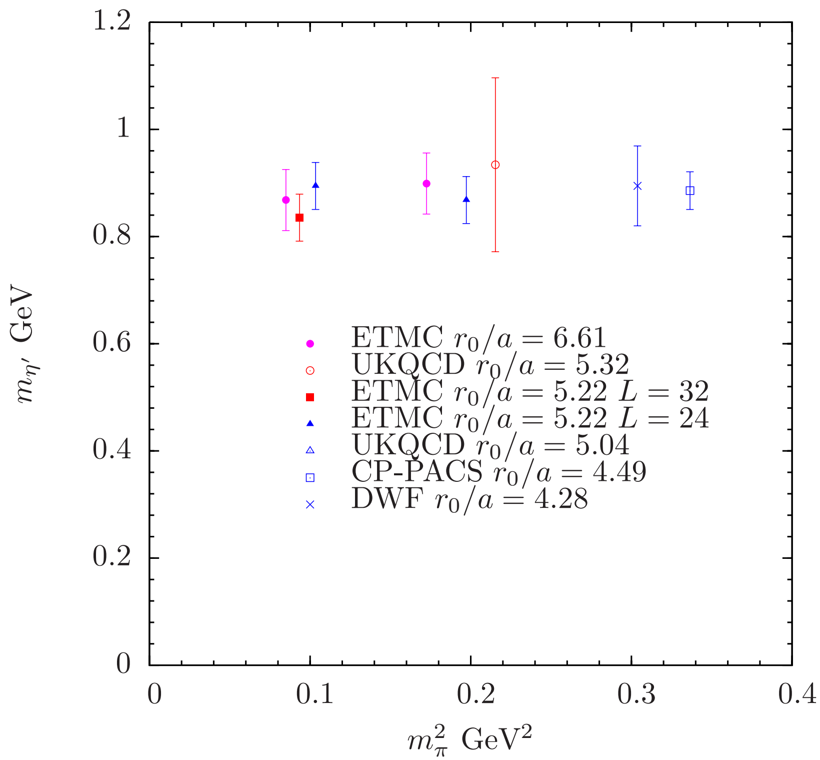
Lattice results: η'



Effect of replacing C by $C(\mathrm{groundstate})$ and of using point-to-point correlators.

Lattice results: η'

Currently lattice studies reach quark masses corresponding to a pion mass of 300 MeV. A compilation from ETMC and others ($N_f = 2$):



This shows that the flavour singlet mass goes to a non-zero constant as $m_\pi \to 0$, estimated as 0.87(7) GeV.
(expect approx RMS η and η' masses 0.78 Gev)

η_c ψ mass difference

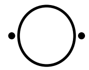
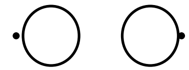
Disconnected contributions can make an impact on the η_c ψ mass difference: since they can contribute O(20) MeV to the η_c. CM+ McNeile 0402012 This is large compared to the experimental mass difference (117 MeV).

Perturbative estimates Follana et al 0610092 suggest η_c is moved down 2.5 MeV whereas lattice estimates Levkova DeTar 0910.3271 find it moved up by 3 MeV.

CAVEAT: η_c mixes with η, η' and decay products ($\eta\pi\pi$ etc.)

ω - ρ **mass difference**

Flavour singlet vector meson mass matrix:

m connected  x disconnected 

$$\begin{matrix} m_{nn} + 2x_{nn} & \sqrt{2}x_{ns} \\ \sqrt{2}x_{ns} & m_{ss} + x_{ss} \end{matrix}$$

$n = (\bar{u}u + \bar{d}d)/\sqrt{2}$

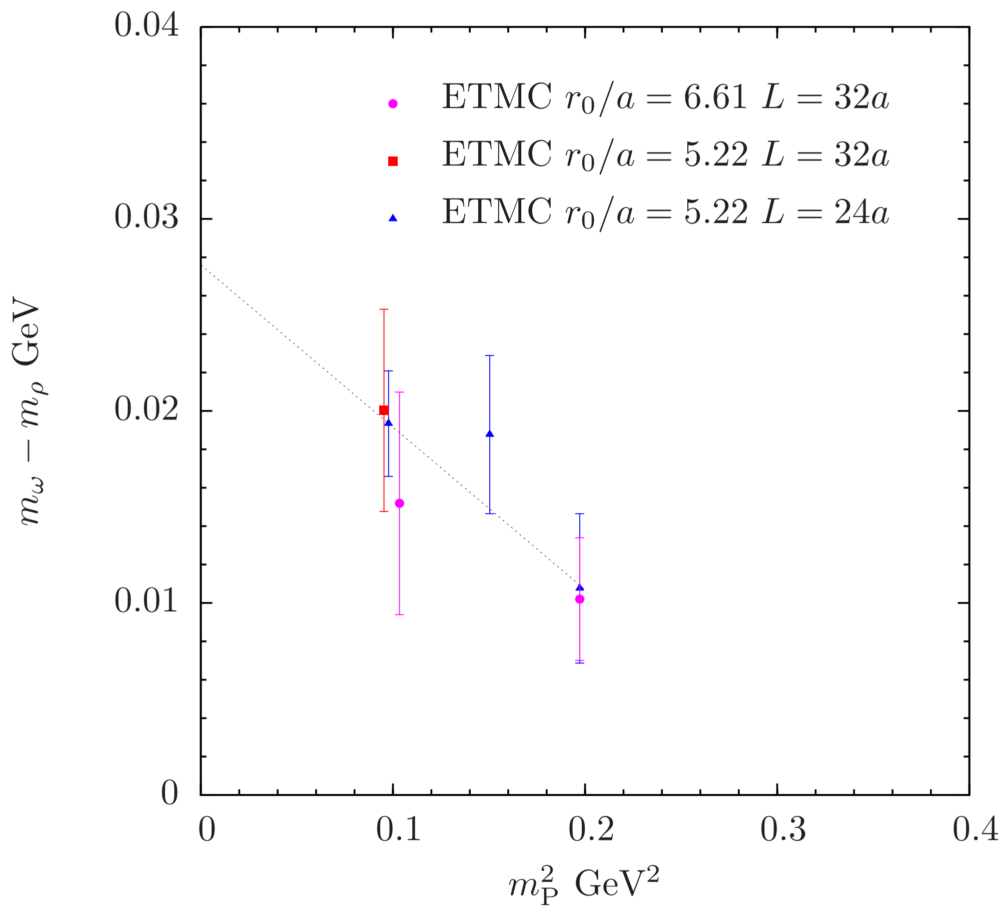
$m_{nn} = m_\rho$; $m_{ns} = m(K^*)$

$m_{ss} \approx 2m_{ns} - m_{nn} = 2m_{K^*} - m_\rho \to 1.012$ GeV.

Approximately $m(\omega) = m(\rho) + 2x_{nn}$ and $m(\phi) = m_{ss} + x_{ss}$.

PDG: $m(\omega) - m(\rho) = 7$ MeV but ρ is very wide

ω - ρ mass difference



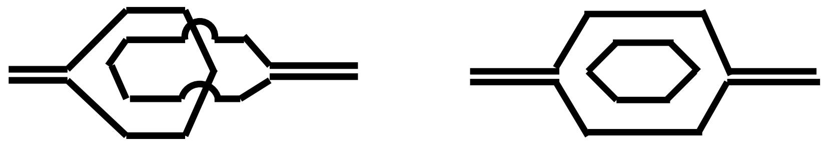
First lattice determination Craig; Chris; Carsten.

Chiral limit $m(\omega) - m(\rho) = 27(10)$ MeV; expt. 7.2 MeV with isospin violating contribution expected to be a few MeV.

Models for vector meson masses

Models for quark-mass dependence:

Disconnected diagram can come from 2-body intermediate state:



$V \to VP$ and $V \to PP$ contribute

	Vπ	$\pi\pi$
ω	3	0
ρ	1	1
m_P dep:	m_P^3	$m_P^4 \log m_P$

Gives big constraints on models of quark mass dependence of ρ.

(needed for extrapolation to physical ρ mass)

ω - ρ mixing

u-d mass difference breaks isospin invariance.

Correlator matrix ($\rho = (uu - dd)/\sqrt{2}$; $\omega = (uu + dd)/\sqrt{2}$)

$$C(\rho,\rho) = (C_{uu} + C_{dd})/2 + (D_{u|u} - D_{u|d} - D_{d|u} + D_{d|d})/2$$

$$C(\rho,\omega) = (C_{uu} - C_{dd})/2 + (D_{u|u} + D_{u|d} - D_{d|u} - D_{d|d})/2$$

$$= (C_{uu} - C_{dd})/2 + (D_{u|u} - D_{d|d})/2$$

$$C(\omega,\omega) = (C_{uu} + C_{dd})/2 + (D_{u|u} + D_{u|d} + D_{d|u} + D_{d|d})/2$$

where we have used $D_{u|d} = D_{d|u}$.

Thus the ω to ρ cross-correlator is given by a difference of correlators with u and d quarks. This is accessible from lattice studies with $m_u = m_d$.

ω - ρ mixing

The ω-ρ mass matrix mixing element is given by

$T_{\omega\rho} = (\frac{dm_\rho}{dm_q} + \frac{dm_\omega}{dm_q})\ \frac{m_u - m_d}{4}$ with $m_q = (m_u + m_d)/2$.

$\frac{dm_\rho}{dm_q} + \frac{dm_\omega}{dm_q} = 2\frac{dm_\rho}{dm_q} + \frac{d(m_\omega - m_\rho)}{dm_q}$

We measure second term and find that lattice QCD naturally produces effects of the correct size to explain the observed ω-ρ mixing (responsible for ripple in $\pi\pi$ spectrum of ρ near m_ω due to $\omega \to \pi\pi$ contribution).

$T = -3.1(3)$ MeV from experiment and including EM effects gives QCD contribution of circa -4 MeV. Lattice plus u/d quark mass ratio (ChPT) gives -3.6 MeV from first term and we find second term small (less than 7%) relative to that.

ω - ρ summary

First lattice determination of ω-ρ mass difference.

Small effect but non-zero.

(Also gives non-strange component of ϕ and strange component of ω)

Full lattice determination of QCD contribution from quark mass difference (u-d) to ω-ρ mixing.

Lattice QCD - Baryons

$C(t) = \langle \text{Vacuum}|\text{Baryon}(0)\text{Baryon}(t)|\text{Vacuum}\rangle$

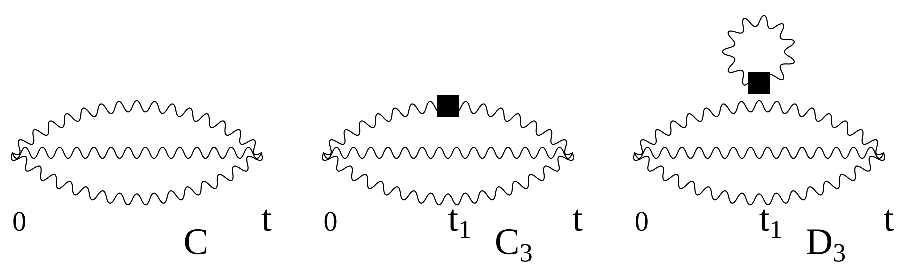
with operator "Baryon" made out of lattice quark and gluon fields creating a baryon of required quantum numbers.

$C_3(t) = \langle \text{Vacuum}|\text{Baryon}(0)J(t_1)\text{Baryon}(t)|\text{Vacuum}\rangle$

with $0 < t_1 < t$ and local current J made out of quarks and antiquarks (eg. the Vector current $\bar{\psi}\gamma_\mu\psi$).

Then from C and C_3, one can extract the required baryonic matrix element (up to Z-factor to take account of lattice regularisation scheme for current J).

For a flavour-singlet J, C_3 has disconnected diagram D_3:



$\overline{s}s$ in the Nucleon.

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$$

ChPT gives $y = 0.2(2)$, Lattice (naively): $y = D_3/(C_3 + D_3) \approx 0.6$. Problem (hep-lat/0109028) is that on a (Wilson) lattice $\bar{s}s$ mixes with I. Equivalently, fixed $\kappa_{\rm valence}$ does not correspond to fixed valence quark mass as $\kappa_{\rm sea}$ varies. Correcting this gives $y = -0.3(3)$.

TMQCD offers technical advantages: evaluate y for the pion: $D_3/C_3 = 0.0(0.05)$, so SMALL effect. here pion masses approx 300 Mev. Above pion estimate used same mass quarks in pion as in disconnected loop. Need $N_f = 2 + 1$, with strange sea, nucleon, more statistics,.... In progress.

$\bar{s}s$ in the Nucleon II.

Other recent lattice estimates:

$$f_s = \frac{m_s \langle N|\bar{s}s|N\rangle}{m_N}$$

is relevant for dark matter scattering.

0.070(22) Clover-Wilson Bali et al. 0911.2407

0.34(5) Anisotropic Wilson Babich et al. 0901.4569

0.015(28) Overlap ($N_f = 2$, $Q_{top} = 0$) JLQCD 0910.5036

0.063(11) Staggered plus Feynman-Hellman MILC 0905.2432

Conclusion

Gluonic (QCD) modifications to the quark model of hadrons:

- Glueballs (scalar mesons)
- hybrid mesons (spin-exotic $\bar{q}qG$)
- OZI rule violating contributions: disconnected quark loop contributions to hadrons:
  pseudoscalar versus vector mesons
  baryon matrix elements

Lattice QCD can extract flavour singlet quantities - explore proton spin etc,....
Watch this space..

TM Disc method

For twisted mass QCD $M_{u/d} = M \pm i\gamma_5\mu$ and consider the case $\Sigma X(1/M_u - 1/M_d)$.

(1) $$(M_d - M_u) = -2i\mu\gamma_5$$

combined with

(2) $$(1/M_d)(M_d - M_u)(1/M_u) = 1/M_u - 1/M_d$$

Hence

(3) $$1/M_u - 1/M_d = -2i\mu(1/M_d)\gamma_5(1/M_u)$$

This can serve as a method of variance reduction because the explicit factor of μ reduces the magnitude of the fluctuations. However, the most important factor is that this expression can be evaluated very effectively with no further inversions. Now $M_u^\dagger = \gamma_5 M_d \gamma_5$, so

(4) $$\sum X(1/M_u - 1/M_d) = -2i\mu \sum X\gamma_5(1/M_u)^\dagger(1/M_u)$$

Basically this identity, in the style of the GMO relation, has a RHS which has a sum of mesonic propagation from the site where X is applied to all lattice sites. This can be evaluated efficiently using the ‘one-end-trick’. Thus since $\phi = (1/M_u)\xi$ and $\phi^* = \xi^*(1/M_u)^\dagger$,

(5) $$\sum X(1/M_u - 1/M_d) = -2i\mu \sum \langle \phi^* X\gamma_5\phi\rangle_r$$

TM Disc method II

This will have an implicit sum over the sources in ϕ and ϕ^* which automatically picks out the required volume sum. This further reduces the variance because the signal (V terms) to noise (V^2 terms contributing as V on average) is of order 1 which is much improved compared to the usual case (with signal 1 versus V noise terms giving signal to noise ratio of $1/\sqrt{V}$).

For example, the special case when $X = i\gamma_5$ is

$$\mathrm{Im}\gamma_5 G = \sum i\gamma_5 (1/M_u - 1/M_d) = 2\mu \sum \langle \phi^* \phi \rangle_r \tag{6}$$

Consider the evaluation of $\mathrm{Im}\gamma_5 G$ at $\beta = 3.9$ and $\mu = 0.004$. With a conventional stochastic volume source the error from the stochastic evaluation with 24 samples is 69.5 while the above method yields 11.5. For comparison the intrinsic variation of the signal (from the underlying gauge configuration) has standard deviation of 18 in the same units. Thus the variance reduction method is very powerful and effectively reduces the stochastic error so that it is smaller than the intrinsic variation. The cost is 24 inversions per gauge configuration, a number of inversions similar to that used in obtaining the connected meson correlators.