Excited hadrons in $n_f = 2$ QCD

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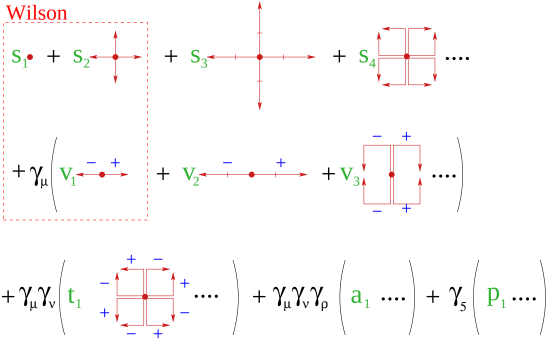
- **Action and configurations**
- Analysis
- Mass spectrum

Chirally Improved Dirac operator

General ansatz for fermion action:

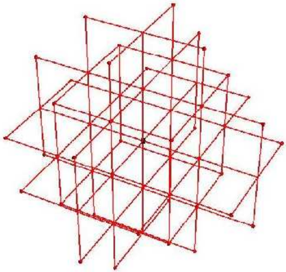
$$D_{mn} = \sum_{\alpha=1}^{16} \Gamma_\alpha \sum_{p \in \mathcal{P}^{\alpha}_{m,n}} c_p^\alpha \prod_{l \in p} U_l \, \delta_{n,m+p}$$



(Gattringer, PRD63(2001)114501)

Insert the ansatz in the Ginsparg-Wilson-equation, truncate the length of the contributions (to, e.g., 4) and compare the coefficients!

Leads to a set of (e.g. 50) algebraic equations, which can be solved (norm minimization).



***Quenched* experience:**

- D_{CI} with hypercubic smearing
- Small $\mathcal{O}(a^2)$ corrections for mass spectra
- Renormalization $Z_A/Z_V \approx 1.04$
- Eigenvalues close to circle

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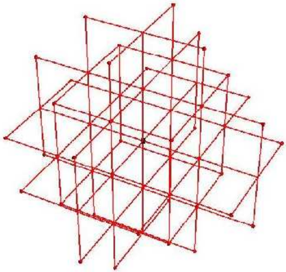

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Study with 2 dynamical CI fermions

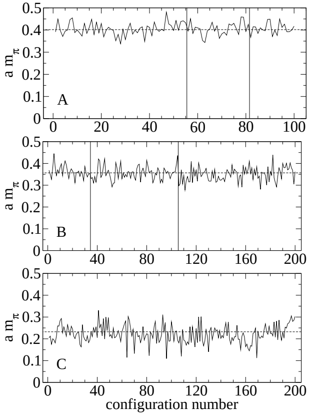
- Chirally improved fermions (D_{CI}), $n_f = 2$ light quarks
- $1\times$ Stout smearing
- Lüscher-Weisz gauge action
- Hybrid Monte Carlo simulation
  - Hasenbusch mass preconditioning, 2 pseudofermions
  - Chronological inverter (min residue extrap.)
  - Mixed precision inverter (cf. Dürr et al., PRD79, 014501)
  - Each unit MD time between 70 and 100 steps
  - Here: Three ensembles for $16^3 \times 32$:

set	β_{LW}	m_0	t_{MD}	config's	a[fm]	m_π[MeV]	m_{AWI}[MeV]
A	4.70	-0.050	600	100	0.151(2)	525(7)	42.8(4)
B	4.65	-0.060	1200	200	0.150(1)	470(4)	34.1(2)
C	4.58	-0.077	1200	200	0.144(1)	322(5)	15.3(4)

- Eigenvalues close to circle but wider distribution
- $Z_A/Z_V \approx 1.07$

Pion mass

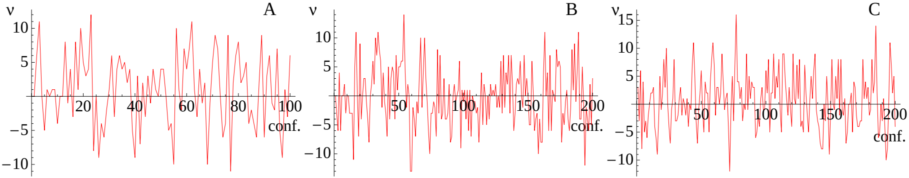
Time sequence and histogram of the configuration pion mass



Topological sectors

Frequent tunneling:

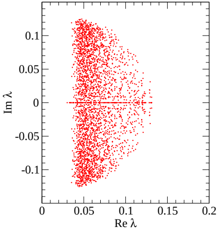
Time sequence and histogram of “topological charge” (number of small real modes counted with *sign*$\langle\gamma_5\rangle$) for the ensemble B (truncation!)



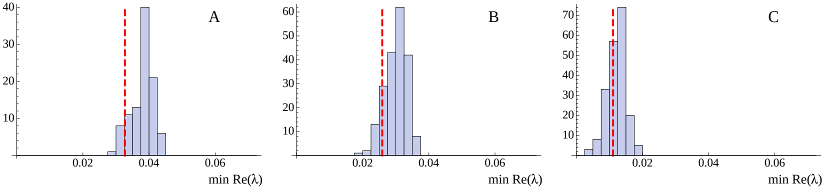
Smallest eigenvalue

Eigenvalue spread is wider than for the quenched case (diff. smearing and action parameters)$\rightarrow$

$16^3 \times 32$, ensemble A, 20 configs.:



Histogram of the smallest Re(λ):



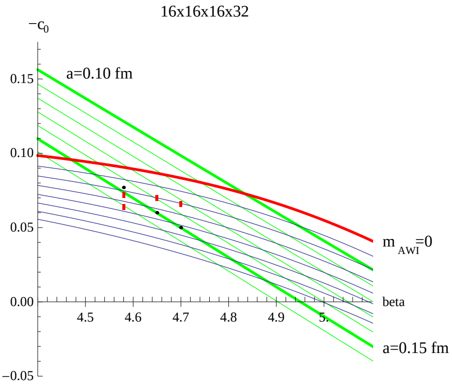
(m_{AWI}: broken red lines at 42.8 MeV, 34.1 MeV, 15.3 MeV)

Parameters: Lattice spacing and m_{AWI}

Lattice spacing determined from potential (Sommer parameter $r_0 = 0.48$ fm)

m_{AWI} defined from the axial W.I.

$$\frac{\langle \partial_t A_4(\vec{p}=\vec{0},t)\,P(0)\,\rangle}{\langle\, P(\vec{p}=\vec{0},t)\,P(0)\,\rangle} \equiv 2\,m_{AWI} = 2\,m^{(ren)}\,Z_P/Z_A$$



- Action and configurations
- **Analysis**
- Mass spectrum

Variational method

(cf., Michael, Lüscher/Wolff; recently: Blossier et al.)

- Each channel: Several interpolators O_i
- Compute all cross-correlations

$$C(t)_{ij} = \langle O_i(t)\overline{O_j}(0)\rangle = \sum_n \langle 0|O_i|n\rangle\langle n|O_j^\dagger|0\rangle e^{-t\,E_n} \ .$$

- Solve the generalized eigenvalue problem:

$$C(t)\,\vec{u}^{(n)}(t) = \lambda^{(n)}(t)\,C(t_0)\,\vec{u}^{(n)}(t)$$

Then

$$\lambda^{(n)}(t) \propto e^{-t\,E_n}\left(1+\mathcal{O}\left(e^{-t\,\Delta E_n}\right)\right) \ .$$

- The eigenvectors are “fingerprints” of the state

Hadron interpolators

Hadron operators are built on spatially 3x HYP smeared gauge configurations.

We need several interpolators!

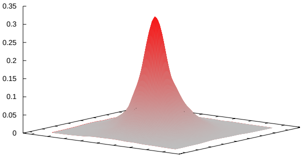
- Different Dirac structure
- Quark sources:
  - Point
  - Wall
  - **(Spatially) smeared, covariant**
  - Laplacian eigenvectors (Distillation method, Peardon et al.)
  - **Derivative sources**

Smeared quark sources

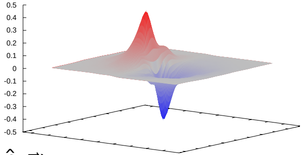
- Jacobi smeared quark sources, e.g.,
  $u_s \equiv (S\,u)_x$

$$S = M\,S_0 \quad \text{with} \quad M = \sum_{n=0}^{N} \kappa^n H^n$$

$$H(\vec{n},\vec{m}) = \sum_{j=1}^{3} \Big[U_j(\vec{n},0)\,\delta(\vec{n}+\hat{j},\vec{m}) + U_j(\vec{n}-\hat{j},0)^\dagger\,\delta(\vec{n}-\hat{j},\vec{m}) \Big] .$$



- Fewer quark propagators
- Combination allows nodes in the interpolating operators
- Derivative quark sources W_{d_i}:



$$D_i(\vec{x},\vec{y}) = U_i(\vec{x},0)\delta(\vec{x}+\hat{\imath},\vec{y}) - U_i(\vec{x}-\hat{\imath},0)^\dagger\delta(\vec{x}-\hat{\imath},\vec{y}) ,$$

$$W_{d_i} = D_i\,S_w \ .$$

Interpolating field operators

Examples for mesons:

$$\overline{u}_s \, \Gamma \, d_s$$

where u_s, d_s denote smeared and/or derivative smeared quarks

E.g. for π:

$\overline{u}_n \, \gamma_5 \, d_n$, $\overline{u}_n \, \gamma_5 \, d_w$, $\overline{u}_w \, \gamma_5 \, d_w$,
$\overline{u}_n \, \gamma_t \gamma_5 \, d_n$, $\overline{u}_n \, \gamma_t \gamma_5 \, d_w$, $\overline{u}_w \, \gamma_t \gamma_5 \, d_w$,
$\overline{u}_{\partial_i} \, \gamma_i \gamma_5 \, d_n$, $\overline{u}_{\partial_i} \, \gamma_i \gamma_5 \, d_w$,
$\overline{u}_{\partial_i} \, \gamma_i \gamma_t \gamma_5 \, d_n$, $\overline{u}_{\partial_i} \, \gamma_i \gamma_t \gamma_5 \, d_w$,
$\overline{u}_{\partial_i} \, \gamma_5 \, d_{\partial_i}$, $\overline{u}_{\partial_i} \, \gamma_t \gamma_5 \, d_{\partial_i}$

Examples for N (and Σ, Ξ):

$$N^{(i)} = \epsilon_{abc} \, \Gamma_1^{(i)} \, u_a \left(u_b^T \, \Gamma_2^{(i)} \, d_c - d_b^T \, \Gamma_2^{(i)} \, u_c \right)$$

with the choices

	$\Gamma_1^{(i)}$	$\Gamma_2^{(i)}$
$i = 1$	1	$C\gamma_5$
$i = 2$	γ_5	C
$i = 3$	i	$C\gamma_4\gamma_5$

projected to parity (and spin)

Example: Angular momentum content of ρ

Type:	*Interpolator:*	*Chiral classification:*
Vector	$O_\rho^V = \overline{u}(x)\gamma^i d(x)$	$(0,1) \oplus (1,0)$
Pseudotensor	$O_\rho^T = \overline{u}(x)\gamma_4\gamma^i d(x)$	$(\frac{1}{2},\frac{1}{2})_b$

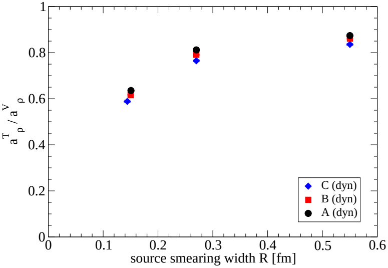
- If both operators couple to ρ: Chiral symmetry is broken (for $a \to 0$ only V couples: pert. th. RG)
- Unitarily equivalent to angular momentum $\{^{2S+1}L_J\}$ basis in CMS: $|^3S_1\rangle$, $|^3D_1\rangle$

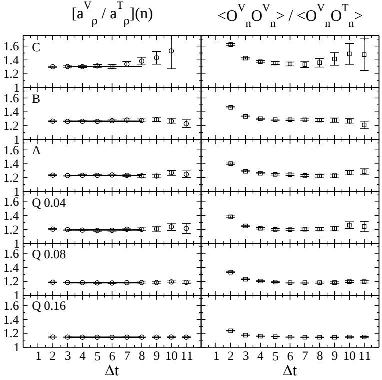
Smearing size defines infrared scale: Allows study of IR scale dependence of Vector vs. Tensor contribution and thus S-wave and D-wave admixture.

Contribution of lattice operator i to eigenstate (n)
$a_i^{(n)} = \langle 0|O_i|n\rangle$, eigenvectors $\vec{u}^{(n)}$.
Then

$$\frac{C(t)_{ij} u_j^{(n)}}{C(t)_{kj} u_j^{(n)}} \sim \frac{a_i^{(n)}}{a_k^{(n)}}$$

gives the ratio of coupling of different lattice operators to the eigenstate.





Glozman et al., PRL103(09)121601; ArXiv:0909.2939

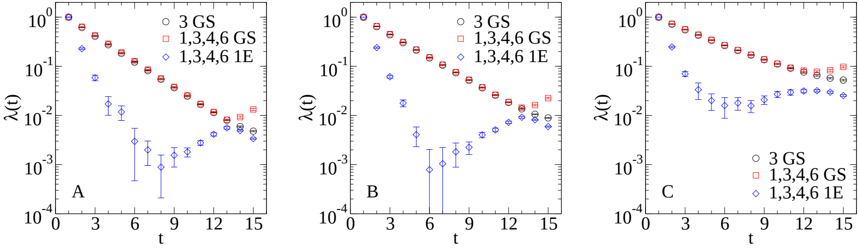
$R \rightarrow 0$: only $(0,1) \otimes (1,0)$, $S/D = \sqrt{2}$

large R: mainly S-wave

- Action and configurations
- Analysis
- **Mass spectrum**

0^{-+} : $\pi^{\pm}(140)$, $\pi^{\pm}(1300)$

Multi-operator (variational) analysis at *small* pion masses: the back-running pion limits the observation range for the excited state!

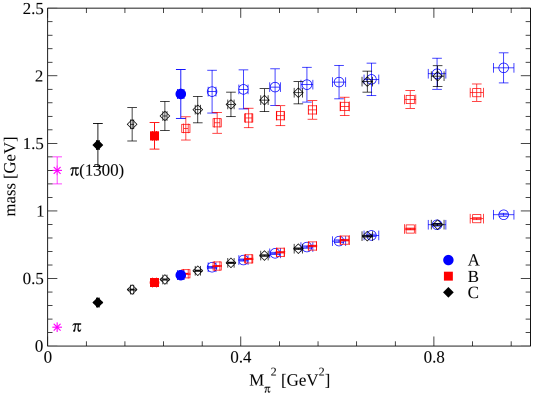


(Fig. from PRD79, 054501 (2009))

This can be cured with larger time-size: for physical pion masses one needs at least $N_t = 64$ for $a = 0.15$ fm, $N_t = 128$ for $a = 0.075$ fm

0^{-+} : $\pi^{\pm}(140)$, $\pi^{\pm}(1300)$

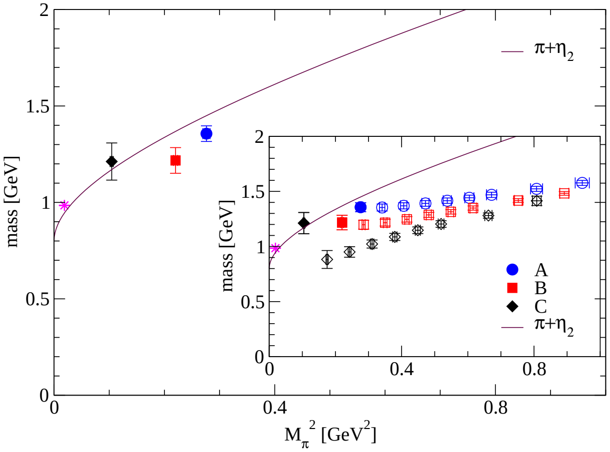
Choosing the cosh-fit interval on basis of eigenvalues: Excited pion state, including partially quenched data (open symbols)



0^{++} : $a_0(980)$, $a_0(1450)$

...compared to $\pi\eta_2$ channel (mass of η_2 estimated)... compatible with Jansen et al., arXiv:0906.4720

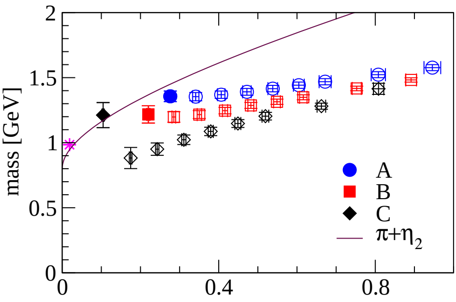
...tetraquark state?



0^{++} : $a_0(980)$, $a_0(1450)$

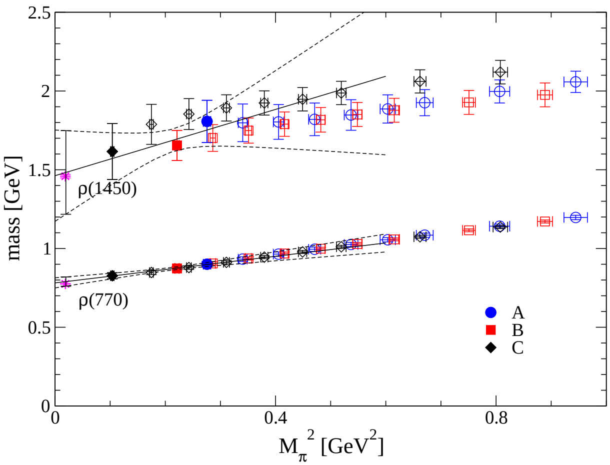
Dynamical points (full symbols) and partially quenched points (open symbols): the partially quenched states may couple to pairs of pseudoscalars (VS)(VS), cf. Prelovsek et al. PRD70, 094503 (2004) (cancels for fully dynamical case)?

But: Broad range of values for different sets of interpolators!

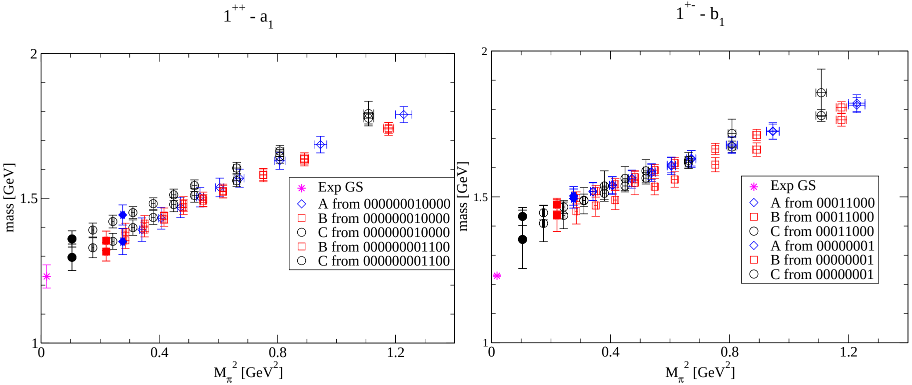


1^{--} : $\rho^{\pm}(770)$, $\rho^{\pm}(1450)$

- No decay yet (p-wave)
- More contamination with higher excitations, thus $t_0 = 2$ is preferable. Optimal combination chosen for each data set.
- 2nd excitation $\rho(1720)$ signal is seen for some combinations of interpolators
- Exotic (1^{-+}) channel is too noisy.

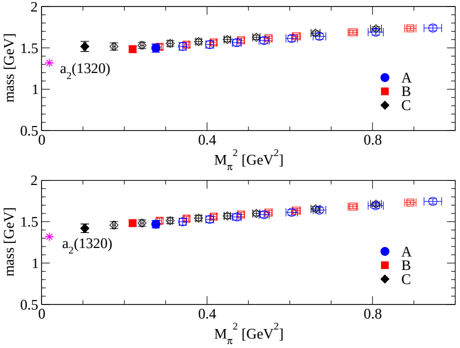


1^{++} : $a_1(1260)$ and 1^{+-} : $b_1(1235)$



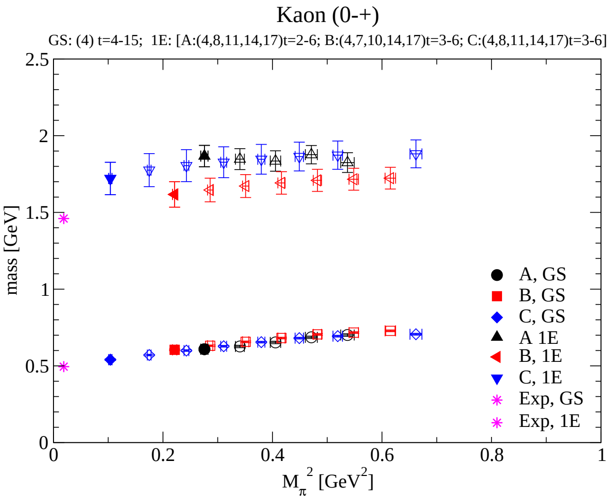
Good signal for ground state *needs* interpolators with derivative sources; note: $m_{latt,b_1} \approx m_{latt,a_0} + 200$ MeV

2^{++} : $a_2(1320)$



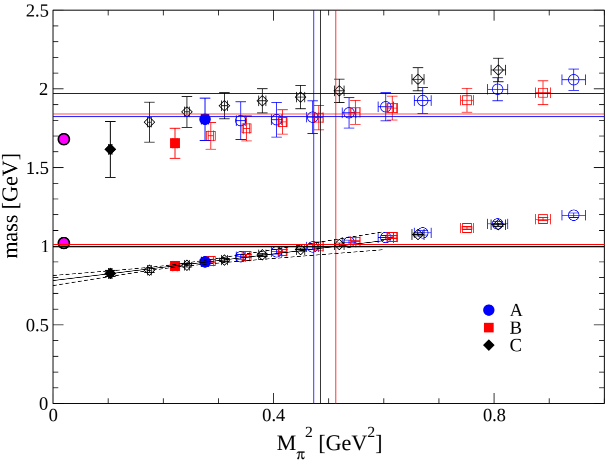
Both representations agree; variational method necessary - one needs more than one interpolator; derivative sources (e.g. $\epsilon_{ijk}\, \overline{u}_{\partial_i}\, \gamma_j\, d_n$) are necessary.

0^- : $K(490)$ and $K(1460)$

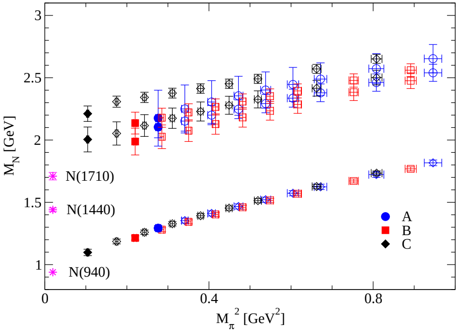


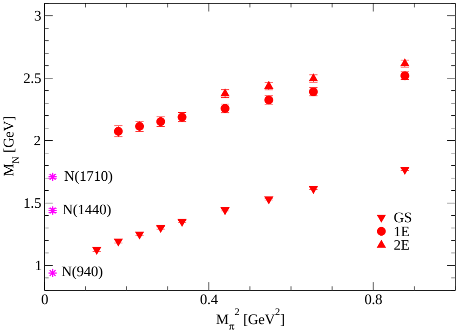
1^{--} : $\phi^{\pm}(1020)$, $\phi^{\pm}(1680)$

- $I = 0$, disconnected contributions omitted
- 1st excitation $\phi(1680)$ signal too high (cf. ρ)



$\frac{1}{2}^+$: $N(940)$, $N(1440)\,P_{11}$, $N(1710)\,P_{11}$



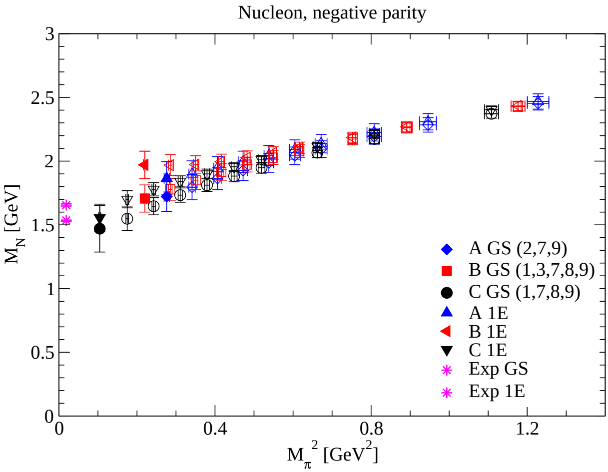


Two excitations (higher one vague), too high up! Roper?

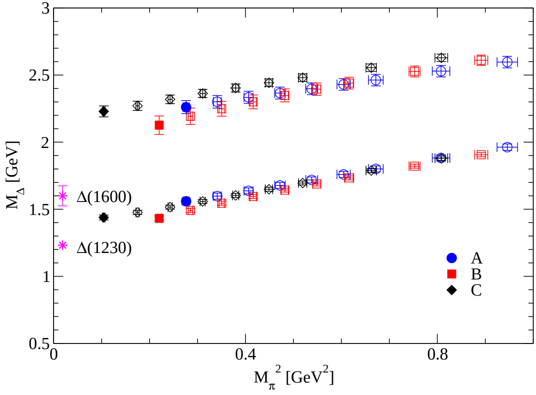
Quenched results (Burch et al., PRD 74 (2006) 014504);
cf., Mahbub arXiv:0905.3616

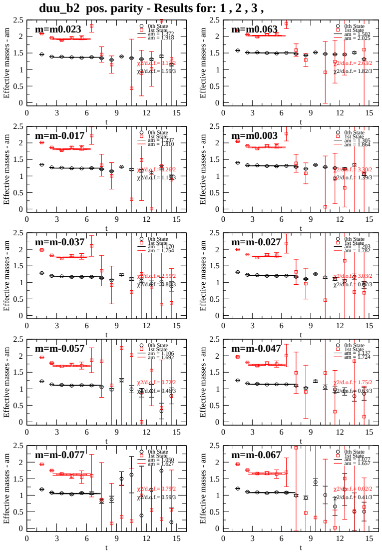
$\frac{1}{2}^-$: $N(1535)\,S_{11}$, $N(1650)\,N_{11}$

Two states seen, mass splitting seen, but not clearly resolvable; dynamical points for lowest mass (set C) have short fit range



$\frac{3}{2}^+$: $\Delta(1232)\, P_{33}$, $\Delta(1600)\, P_{33}$

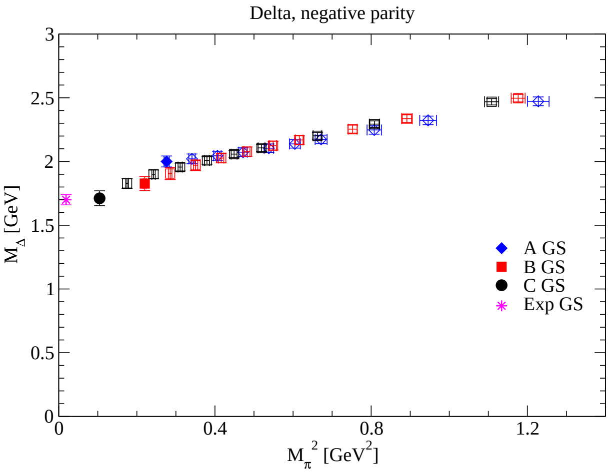




Excited Δ clearly seen, but too high (squeezed?)

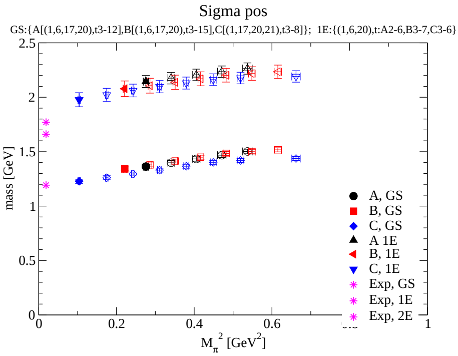
M_Ω is used to fix strange (only valence) quark mass.

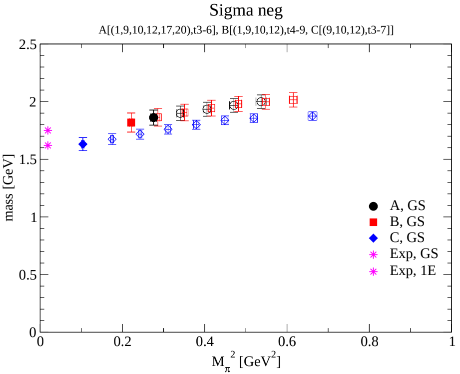
$\frac{3}{2}^{-}$: $\Delta(1700)\,P_{33}$



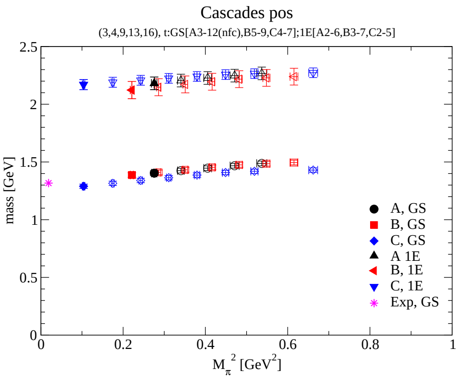
Axial charge for N, Σ and Ξ see ArXiv 0910.4190.

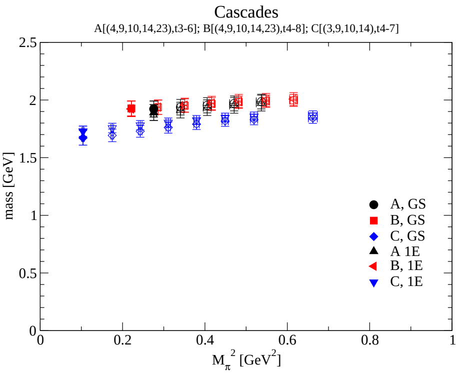
$\frac{1}{2}^+$: Σ(1190), Σ(1660) $\frac{1}{2}^-$: Σ(1750)



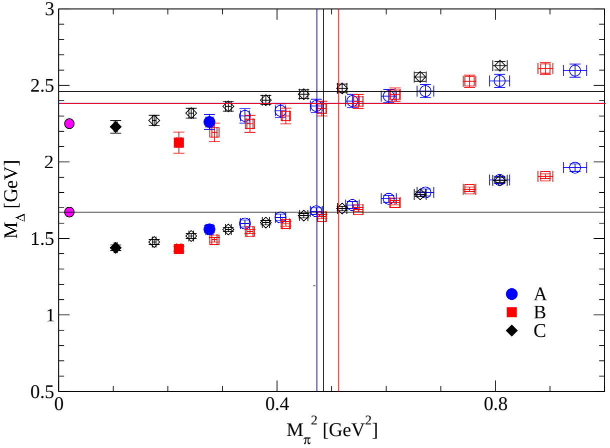


$\frac{1}{2}^+$: Ξ(1320), Ξ(?) $\frac{1}{2}^-$: Ξ(?)





$\frac{3}{2}^+$: Ω(1672), Ω(2250)?



Conclusion

- Excellent ground state masses for (in lattice units) small lattices and all mesons: 0^{-+}, 1^{--}, 1^{+-}, 1^{++}, 2^{--}, 2^{-+}, 2^{++}
- a_0 behaves unlike other mesons
- Weak signals for excited states towards smaller pion masses
- Pion excitations need large time size
- Ground state baryons fine, excited state baryons too high: volume squeezing (Lm_π= 6.36, 5.63, 3.66)?
- Other approaches with more interpolators (cf. distillation method)?
- Results for Baryon axial charge (Mohler at LAT09)
- Ongoing: enhancing statistics and parameter sets