

All-to-all Propagators in Lattice Hadron Spectrum Calculations

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Germany

OUTLINE

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Hadron
Spectrum
Calculations

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Background

Distillation -
An Exact
All-to-all
Method

Variance-
Reduced
Stochastic
LapH (VRSL)

1 BACKGROUND

2 DISTILLATION - AN EXACT ALL-TO-ALL METHOD

3 VARIANCE-REDUCED STOCHASTIC LAPH (VRSL)

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THE HADRON SPECTRUM COLLABORATION

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- Dedicated to mapping out the low-lying excited hadron spectra.
- Members:

Jefferson Lab: H. W. Lin, S. D. Cohen, J. Dudek, R. G. Edwards, B. Joo, D. G. Richards

Carnegie Mellon: J. Foley, C. Morningstar, D. Lenkner, C. H. Wong

DESY, Zeuthen: JB

U. of Maryland: E. Engelson, S. Wallace

U. of the Pacific: K. J. Juge

Tata Inst., Mumbai: N. Mathur

Trinity College, Dublin: M. J. Peardon, S. M. Ryan

LATTICE QCD BACKGROUND

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- Correlation functions between **hadron operators** are an ensemble average over gauge configurations.

$$\frac{\langle 0 | \mathcal{O}_i[\psi, \bar{\psi}, U](t) \overline{\mathcal{O}}_j[\psi, \bar{\psi}, U](t_0) | 0 \rangle}{\langle 0 | 0 \rangle} = \langle F_{ij}[M^{-1}(U), U] \rangle_U$$

- $M^{-1}(U)$, is a $(V \times L_t \times N_{spin} \times N_{color})$ -dimensional matrix. Can only solve equations like

$$M(x, y)\phi(y) = \eta(x) \rightarrow \phi(x) = M^{-1}(x, y)\eta(y)$$

- 'Point-to-all' $\Rightarrow \eta(x) \propto \delta(x, x_0)$
'All-to-all' \Rightarrow Use of $M^{-1}(x, y) \forall x, y$

HADRON SPECTRA FROM LATTICE QCD

- Spectral decomposition of two-point correlation functions:

$$\langle 0 | \mathcal{O}(t + t_0) \bar{\mathcal{O}}(t_0) | 0 \rangle = \sum_n | \langle 0 | \mathcal{O}(t_0) | n \rangle |^2 \exp[-E_n t]$$

- Difficult (or impossible) to fit sub-leading exponentials, instead form a **matrix of two-point correlators**:

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t + t_0) \bar{\mathcal{O}}_j(t_0) | 0 \rangle$$

- Define **new operators** $\Omega_n(t)$ such that:

$$\langle 0 | \Omega_a(t + t_0) \bar{\Omega}_b(t_0) | 0 \rangle = \delta_{ab} \lambda_a(t)$$

- Fit $\lambda_n(t)$ with a single exponential to obtain E_n

THE NEED FOR ALL-TO-ALL PROPAGATORS

- Finite-volume stationary states are comprised of resonance states as well as scattering states.
- Some resonance states may have multi-particle content.

$$\langle 0|B(\mathbf{p} = 0, t)\bar{B}(\mathbf{p} = 0, t_0)|0\rangle = \quad (1)$$
$$\frac{1}{V^2} \sum_{\mathbf{x}, \mathbf{y}} \langle 0|\varphi_B(\mathbf{x}, t)\bar{\varphi}_B(\mathbf{y}, t_0)|0\rangle$$

$$B(\mathbf{p}, t)M(-\mathbf{p}, t) = \frac{1}{V^2} \sum_{\mathbf{x}, \mathbf{y}} \varphi_B(\mathbf{x}, t)\varphi_M(\mathbf{y}, t)e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \quad (2)$$

- Sum over \mathbf{y} can be eliminated in Eq. 1, but not in correlation functions containing the operator from Eq. 2. Solving $M\phi = \eta$ for all \mathbf{y} is not feasible.

SIMULATION DETAILS

- **Anisotropic Wilson Gauge** action with spatial rectangle. (C. Morningstar, M. Peardon '99)
- **Anisotropic Clover-Wilson** quark action, with tadpole improvement
- Spatial links are stout smeared (Morningstar, Peardon '04), parameters are tuned non-perturbatively (Edwards, Lin, Joo '08).
- $N_f = 2 + 1$, $a_s = 0.12\text{fm}$, $a_t = a_s/3.5$. Range of **pion masses**: $\approx 700\text{MeV} - 230\text{MeV}$, and **lattice sizes** $12^3 \times 96$ to $32^2 \times 256$.

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LAPH (LAPLACIAN HEAVISIDE) SMEARING

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- Quark smearing damps out unwanted excited states by applying $S = \exp[\frac{\sigma^2}{4} \Delta]$.

$$S_{ab}[U; t](\mathbf{x}|\mathbf{y}) = \sum_{i=1}^N e^{\frac{\sigma^2}{4} \lambda_i[U; t]} v_a^{(i)}[U; t](\mathbf{x}) v_b^{*(i)}[U; t](\mathbf{y})$$

- Truncate** at a finite number ($n_{max} \ll N$) of eigenmodes (M. Peardon, JB, et al. (2009))

$$\tilde{M}_{(a\alpha|b\beta)}^{-1}(\mathbf{x}, t|\mathbf{x}_0, t_0) = \sum_{i=1}^{n_{max}} \sum_{\bar{i}=1}^{n_{max}} v_a^{(i)}[t](\mathbf{x}) K_{\alpha\beta}^{(i)(\bar{i})}(t|t_0) \times v_b^{*(\bar{i})}[t_0](\mathbf{x}_0)$$

with

$$K_{\alpha\beta}^{(i)(\bar{i})}(t|t_0) = v_c^{*(i)}[t](\mathbf{y}) M_{(c\alpha|d\beta)}^{-1}(\mathbf{y}, t|\mathbf{z}, t_0) v_d^{(\bar{i})}[t_0](\mathbf{z})$$

VOLUME DEPENDENCE

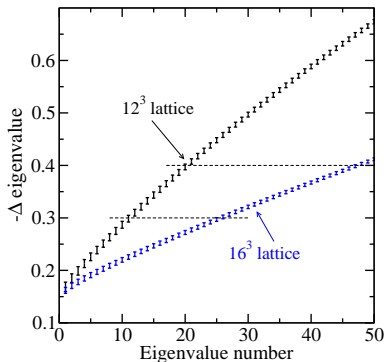
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- $N_f = 2 + 1$ ensembles:
 $12^3 \times 96$ and $16^3 \times 128$
with $a_s = 0.12\text{fm}$,
 $a_t = a_s/3.5$,
 $m_\pi \approx 700\text{MeV}$
- Examine the number of eigenvalues in $[0.3, 0.4]$:
 $n_{12}/n_{16} = (12/16)^3$
- For fixed λ_{max} , $n_{max} \propto V$

LAPH CONCLUSIONS

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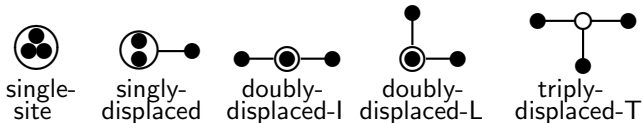
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- *Exact* (smeared) all-to-all propagator for a finite number of inversions!
- n_{max} (and thus required number of inversions) grows linearly with the volume.
- Cannot completely separate initial and final degrees of freedom.
- Still useful for smaller volumes ($L_s < 2 - 2.5\text{fm}$). Results!

IT'S ALL ABOUT THE OPERATORS

- Spatially extended operators have overlap with orbital and radial excitations.
- Operators must transform irreducibly under lattice symmetries.
- Generate a large set of operators from simple elemental building blocks C. Morningstar, et al. (2005)



- Select $\mathcal{O}(10)$ with good low-lying state overlap which form a well-conditioned correlator matrix.

RESULTS: BARYON AND MESON SPECTRA

- 100 configurations from $N_f = 2 + 1$, $16^3 \times 128$ ensemble, with $L_s \approx 2\text{fm}$, $a_s \approx 0.12\text{fm}$, $a_t = a_s/3.5$, and $m_\pi \approx 380\text{MeV}$. Also, $n_{max} = 32$.
- Operators transform under lattice irreps: 'g' \rightarrow +ve parity, 'u' \rightarrow -ve parity. $J = 1/2 \rightarrow G_1$, $J = 3/2 \rightarrow H$, $J = 5/2 \rightarrow G_2$ and H .
- About 12 (single-hadron) operators chosen in each channel.

NUCLEON (JB) AND DELTA (E. ENGELSON) SPECTRA

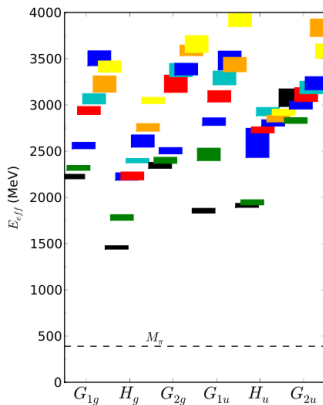
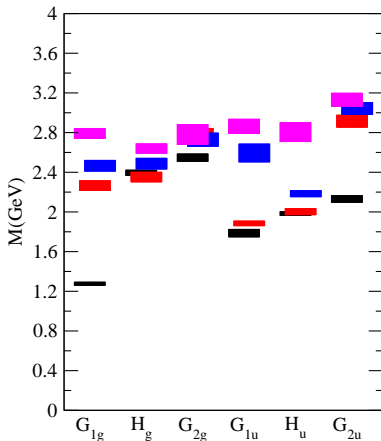
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Σ (D. LENKNER) AND Ξ (C. H. WONG) BARYON SPECTRA

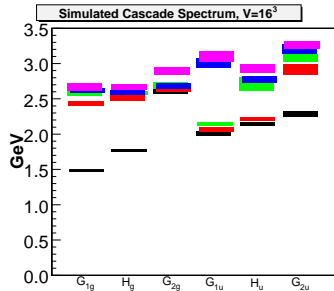
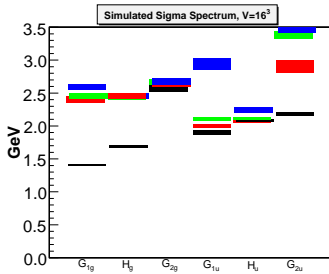
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$N_f = 2 + 1$ RESULTS: $I^G = 1^-$ AND $I^G = 1^+$ MESON SPECTRUM (C. H. WONG)

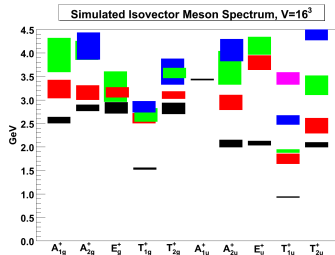
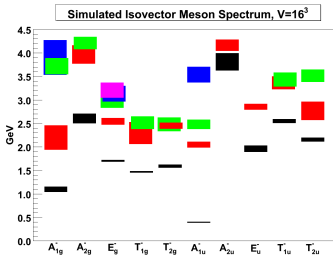
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STOCHASTIC ALL-TO-ALL

- Generate N_r independent stochastic sources

$$\eta_{c\alpha}^{(r)}(\mathbf{x}, t) \in Z_4$$

and solve $M\phi^{(r)} = \eta^{(r)}$ for $\phi^{(r)}$.

- Can estimate the quark propagator:

$$M^{-1} \approx \frac{1}{N_r} \sum_{r=1}^{N_r} \phi^{(r)} \eta^{(r)*}$$

- Variance Reduction: **Dilute** each noise source
 $\eta^{(r)}[d] = P[d]\eta^{(r)}$, with $\sum_{d=1} P[d] = 1$ and
 $P[d]P[d'] = \delta_{dd'}P[d]$.

NOISE IN THE SUBSPACE ONLY

- A [new way](#) to introduce noise:

$$\eta_{c\alpha}^{(r)}(\mathbf{x}, t) = \sum_{n=1}^{N_{ev}} a_{\alpha n}^{(r)}[t] v_c^{(n)}[U; t](\mathbf{x})$$

- Dilute in the subspace only

$$\eta_{c\alpha}^{(r)[d]}(\mathbf{x}, t) = \sum_{n=1}^{N_{ev}} a_{\alpha n}^{(r)[d]}[t] v_c^{(n)}[U; t](\mathbf{x})$$

$$a_{\alpha n}^{(r)[d]}[t] = P_{(\alpha n|\alpha' n')}^{[d]}(t|t') a_{\alpha' n'}^{(r)}[t']$$

- Is VRSL more efficient than conventional dilution? What about the volume dependence?

VRSL vs. CONVENTIONAL DILUTION

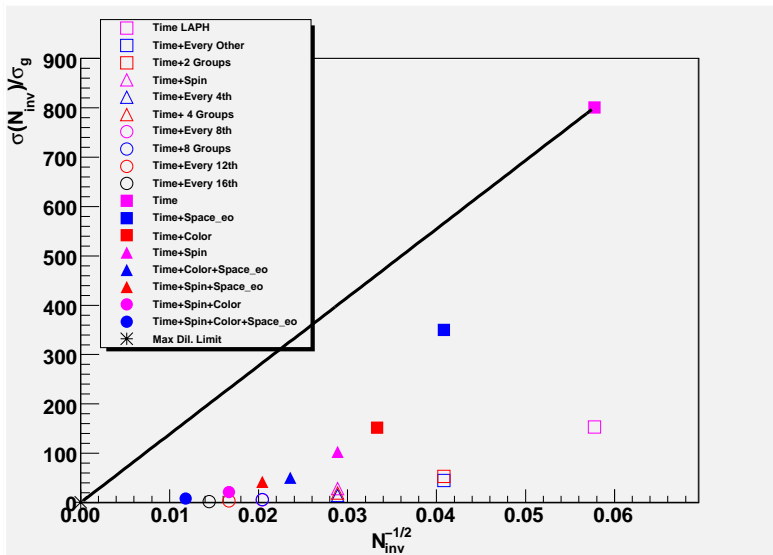
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VRSL VOLUME DEPENDENCE

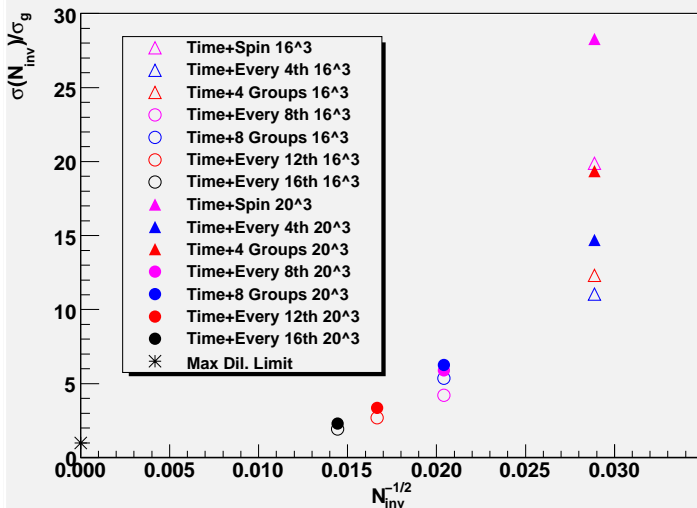
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SAME-TIME ('DISCONNECTED') DIAGRAMS

- Required for **flavor-singlet** quantities and **single hadron/multi-hadron** correlators.
- For 'Connected' correlators, only need $\eta(\mathbf{x}_0, t_0)$ for a single t_0 but here we need many t_0 's.
- Exact distillation result on all t_0 's for a small lattice: 100 cfgs., $N_f = 2 + 1$, $12^3 \times 96$, $m_\pi \approx 700\text{MeV}$, $n_{max} = 12$, $N_{inv} = 4608$.
- Try different **time**, **spin**, and **eigenvector** dilution schemes. (C. H. Wong, M. Peardon)

DISC. TERM FROM η' MESON CORRELATOR - 192 INVERSIONS

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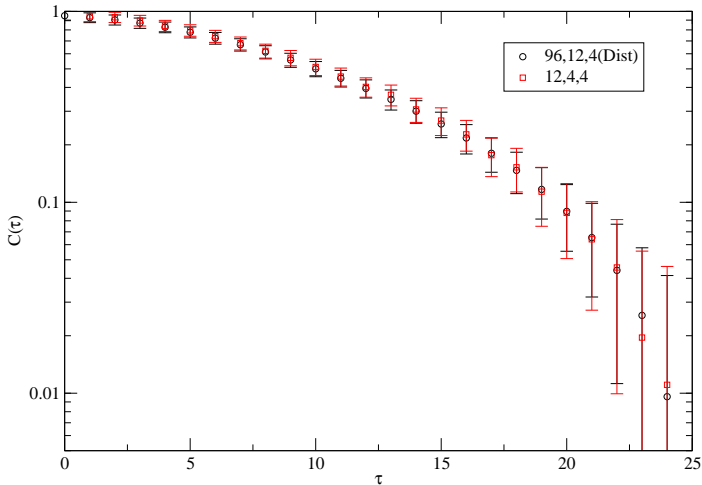
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Eta' , $V=12^3 \times 96$, in the Chosen Dilution Scheme

No. of Interlace Projectors in (time, vector, spin) in the labels



DISC. TERM FROM σ MESON CORRELATOR - 192 INVERSIONS

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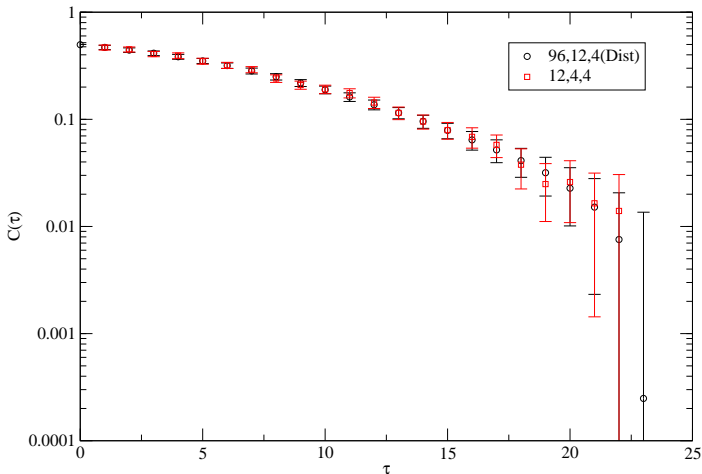
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Sigma, $V=12^3 \times 96$, for the Chosen Dilution Scheme

No. of interlace projectors in (time, vector, spin) in the labels



SINGLE TERM FROM SCALAR MESON DECAY - 192 INVERSIONS

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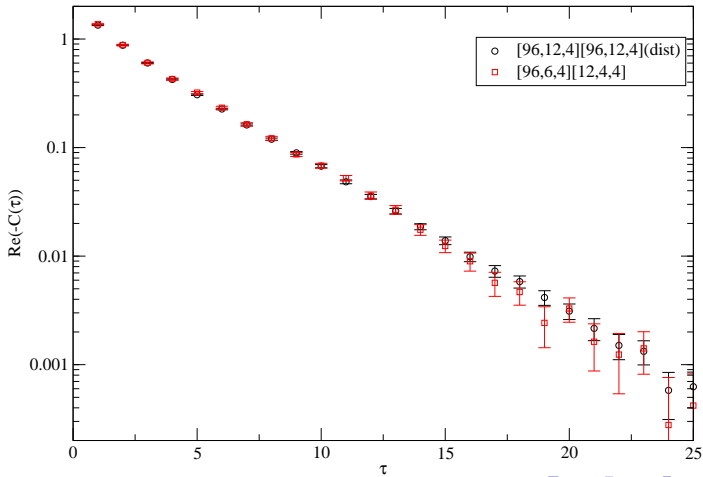
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Scalar Decay, $V=12^3 \times 96$, for the Chosen Dilution Scheme

No. of Interlace Projectors in $[t,v,s][t,v,s]$ for the labels (1st \square is t_0-t_1 , 2nd \square is t_0-t_0)



VRSL CONCLUSIONS

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- VRSL is **more efficient than conventional dilution**. For a fixed number of inversions, VRSL gives a considerably lower error.
- The smeared subspace is low-dimensional, so the gauge-noise (σ_g) limit can be obtained with a MUCH smaller number of projectors.
- Unlike exact method, **volume dependence seems mild**.
- A moderate level of dilution projectors works for **disconnected terms**.

FUTURE PLANS

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- Employ the VRSL method to study multi-hadron (specifically nucleon-pion) operators.
J. Foley: Group theory for moving hadrons
J. Juge, C. H. Wong, J. Bulava: First Multi-pion results
- Repeat the spectrum calculations for a variety of larger volumes (up to $32^3 \times 256$, $L_s \approx 3.8\text{fm}$) with lighter quarks (pion mass down to $\approx 230\text{MeV}$) including multi-hadron operators.
- Differentiate resonances from multi-hadron states and identify resonance quantum numbers.